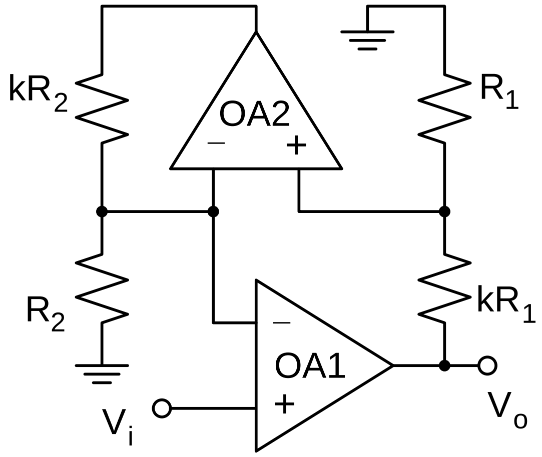
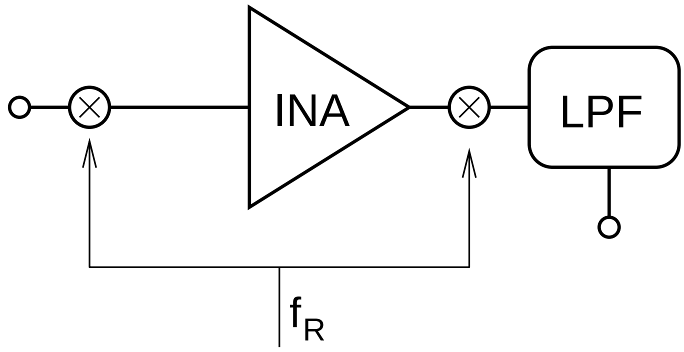
Electronics – September 11, 2015 Prof. A. Spinelli





Problem 1

The scheme in the left figure shows an amplifier with minimum phase shift. OAs have $A_0 = 10^5$ and $GBWP = 1$ MHz; component values are $R_1 = R_2 = 10$ kΩ, $k = 9$.

1. Compute the closed-loop gain of the stage (ideal case).
2. Compute the loop gain for OA1 when OA2 is ideal. What changes if we consider the finite gain of OA2?
3. Compute the total output rms noise when the input noise sources of the OAs are $\sqrt{S_V} = 10$ nV/$\sqrt{\text{Hz}}$ and $\sqrt{S_I} = 5$ pA/$\sqrt{\text{Hz}}$ (neglect resistor noise).
4. OA1 has an input resistance of 100 kΩ: compute the input resistance of the stage considering OA2 as ideal.

Problem 2

A very-low frequency signal of amplitude $A \approx 10$ µV is modulated at frequency f_R before being preamplified and sent to a lock-in amplifier (see figure on the right). The INA has an input voltage noise $S_V = S_{WN} + K/f$ with $K = 10^{-10}$ V^2 and noise corner frequency of 500 Hz, an input offset voltage $V_{OS} = 1$ mV and a bandwidth of 5 kHz.

1. Find a set of LIA parameters that provides $S/N = 1$.
2. The LIA reference input is $\cos(\omega_R t + \phi))$, where ϕ is a low-frequency term. Compute the mean and rms output values when ϕ is uniformly distributed in an interval $\pm\phi_M$.
3. The power line generates strong interferences at all harmonics of 50 Hz, superimposed onto the modulated signal. Is the solution computed in #2.1 still viable? If not, find a new set of LIA parameters that provides an attenuation of a factor of 1000 (at least) of the harmonics.
4. A square wave is used for demodulation, without a selective filter. With reference to the previous solution, discuss the choice of f_R.

Question

Describe the differences between analog and digital LIAs.

For a correct evaluation, you are asked to write your answers in a readable way; thank you
Do a good job!

Results will be posted by September 16th **Mark registration: Friday, September 18th**

Solution

Problem 1

1.1

Considering that the input terminals of the OAs are kept at the same voltage, we immediately get:

$$\frac{V_o}{V_i} = \frac{R_1 + kR_1}{R_1} = k + 1.$$

1.2

If OA2 were ideal, the transfer between its non-inverting and inverting input would be 1 and the resulting loop gain would be:

$$G_{loop} = -A_1(s)\frac{1}{k+1}.$$

If we now label $G(s)$ the transfer function between the non-inverting and the inverting input of OA2, the actual loop gain becomes

$$G_{loop} = -A_1(s)\frac{1}{k+1}G(s).$$

Now, looking at the AO2 stage, we see a standard non-inverting amplifier with the only difference that the output is taken at the voltage divider rather than at the OA output (Fig. 1, left). We can then apply the feedback theory to this stage, obtaining:

$$G(s) = \frac{G_{open}}{1 - G_{loop}} = \frac{\frac{A_2(s)}{k+1}}{1 + \frac{A_2(s)}{k+1}} \approx \frac{1}{1 + \frac{k+1}{A_0}s\tau}.$$

The resulting loop gain is shown in Fig. 1 (right), where it can be seen that OA2 introduces a second pole falling exactly at 100 kHz, the zero-dB crossing frequency of G_{loop} (i.e., giving a phase margin of 45°).

1.3

Elementary calculations lead to the output PSD

$$S_{Vo} = (S_{V_1} + S_{V_2})(k+1)^2 + S_{I_2}^{+}(kR_1)^2 = 2 \times 10^{-14} + 2 \times 10^{-13} = 2.2 \times 10^{-13}\ \mathrm{V}^2/\mathrm{Hz}.$$

The zero-dB frequency of the loop gain, previously computed, is 100 kHz, leading to:

$$\overline{V_o^2} = 2.2 \times 10^{-13}\frac{\pi}{2}10^5 \approx (0.19\ \mathrm{mV})^2.$$

1.4

In the ideal case we have $Z_i = \infty$, i.e. we need to multiply the open-loop value, Z_{open} by the factor $1 - G_{loop}$. Now, if OA2 is ideal we have simply

$$Z_{open} = R_i,$$

because OA2 keeps its inverting node at ground potential when the loop is open and $V^+(OA2) = 0$. From the expression of the loop gain already computed, we obtain:

$$Z_i = Z_{open}(1 - G_{loop}) = R_i\left(1 + \frac{A_1(s)}{k+1}\right),$$

which is equal to 1 GΩ at low frequencies.

As a reference, we now address the (non-straightforward) case of finite gain of OA2. Referring again to the block shown in Fig. 1 (left), it is easy to see that its output impedance is given by:

$$Z_{out} = \frac{R_2 \parallel kR_2}{1 + \frac{A_2}{1+k}},$$

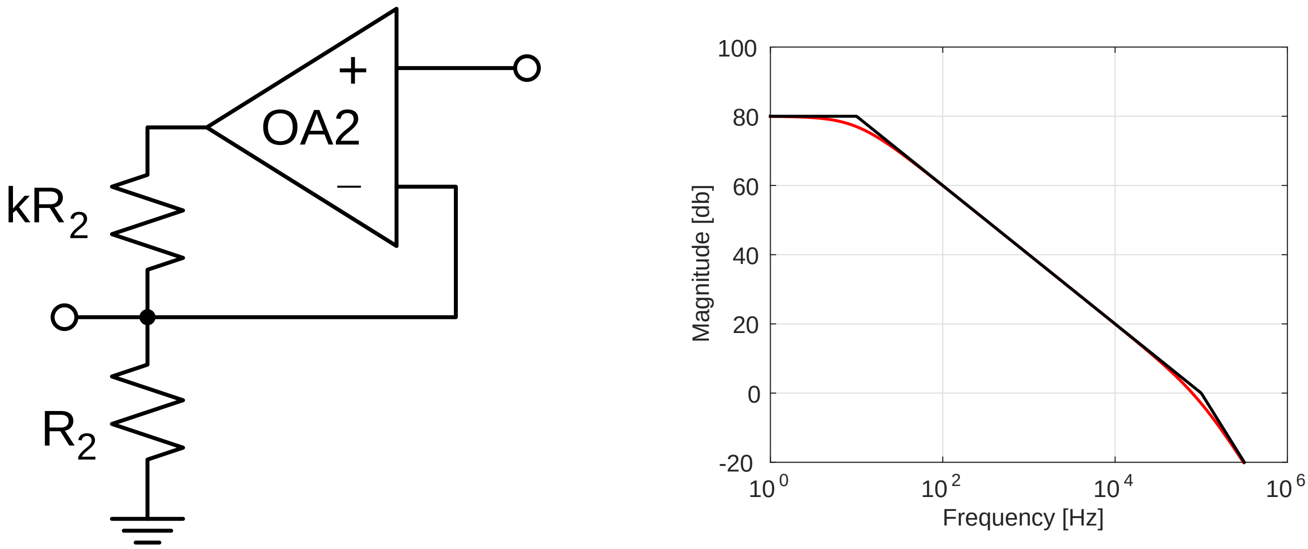


Figure 1: Left: Scheme of the OA2 block used in calculations of G_{loop} (and later Z_{in}); Right: Bode magnitude diagram of the loop gain when OA2 is taken into account.

so that the correct expression for Z_i becomes

$$Z_i = (R_i + Z_{out})(1 - G'_{loop}),$$

where G'_{loop} is the new loop gain, accounting for the non-unity transfer through the OA2 stage. Such a transfer is easily given by:

$$\frac{V^-(OA2)}{V^+(OA2)} = \frac{\frac{A_2}{k+1}}{1+\frac{A_2}{k+1}},$$

finally leading to:

$$G'_{loop} = \frac{A_1}{k+1}\frac{\frac{A_2}{k+1}}{1+\frac{A_2}{k+1}}.$$

With some rearrangement, the final expression becomes:

$$Z_i = R_i\left(1 + \frac{A_1}{k+1}\frac{A_2}{k+1+A_2}\right) + R_2\frac{k}{k+1+A_2}.$$

Problem 2

2.1

We note that we do not have a specific limit on the signal bandwidth, so that we can (at least, in principle) select whatever value we would like. A smart approach could be to select $f_R > f_{nc} = 500$ Hz (say, 1 kHz) to minimize noise pick-up and then find BW_n. From the expression for the output S/N of a LIA we would get:

$$\frac{S}{N} = \frac{A}{\sqrt{2S(f_R)BW_n}} = 1 \Rightarrow BW_n = A^2\frac{f_{nc}}{2K} = 250 \text{ Hz}.$$

Note that the factor 2 in the expression of S/N comes from the amplifier noise, which is usually a *unilateral* density. Besides, it is worth pointing out that f_R lies within the bandwidth of the INA, which is not limiting the performances (its DC offset is also unimportant, being modulated at f_R).

2.2

The output signal of the mixer is

$$V_M = A\cos(\omega_R t)\cos(\omega_R t + \phi) = \frac{A}{2}\cos\phi(t) + \frac{A}{2}\cos(2\omega_R t + \phi(t)),$$

which becomes $\frac{A}{2}\cos\phi(t)$ at the output of the LPF, because the second term is filtered. If we assume that the output filter does not affect the (low frequency) random process $\cos\phi(t)$, we then have for the average value:

$$\overline{V_o} = \frac{A}{2}\overline{\cos\phi} = \frac{A}{2\phi_M}\int_0^{\phi_M}\cos\phi\; d\phi = \frac{A}{2}\frac{\sin\phi_M}{\phi_M}.$$

The mean square value is instead:

$$\overline{V_o^2} = \frac{A^2}{4}\overline{\cos^2\phi} = \frac{A^2}{4\phi_M}\int_0^{\phi_M}\cos^2\phi\; d\phi = \frac{A^2}{4}\left(\frac{1}{2} + \frac{\sin 2\phi_M}{4\phi_M}\right).$$

so that

$$\sqrt{\overline{V_o^2} - \overline{V_o}^2} = \frac{A}{2}\sqrt{\frac{1}{2} + \frac{\sin 2\phi_M}{4\phi_M} - \frac{\sin^2\phi_M}{\phi_M^2}}$$

2.3

Since $f_R = 1$ kHz is a multiple of the mains frequency, it will demodulate the 20^{th} harmonics back to DC, degrading S/N. First of all, we need then to move f_R away from such frequencies; then, we'll have to reduce the filter bandwidth to reject the nearby interfering signals.

As harmonics are uniformly spaced by 50 Hz, we could simply set $f_R = 975$ Hz. The nearest harmonics are now at 950 and 1000 Hz, and these are demodulated by the mixer down to $f_i = \pm 25$ Hz, falling within the bandwidth of the output filter; it is then clear that such a filter is no longer effective.

To get an attenuation of a factor of 60 dB at the interfering frequency f_i we must have (single-pole filter):

$$\frac{1}{\sqrt{1 + (2\pi f_i \tau)^2}} = 10^{-3} \Rightarrow \tau \approx \frac{10^3}{2\pi f_i} = 6.4 \text{ s},$$

i.e., a pole frequency $f_p = 1/(2\pi\tau) = 25$ mHz. This is a straightforward result directly stemming from the 20 dB-per-decade attenuation of a single-pole filter.

We just carry out the calculations for the case of second-order filter (assuming two coincident real poles for simplicity):

$$\frac{1}{1 + (2\pi f_i \tau)^2} = 10^{-3} \Rightarrow \tau \approx \frac{10^{3/2}}{2\pi f_i} = 0.2 \text{ s},$$

i.e., a pole frequency $f_p = 1/(2\pi\tau) = 0.8$ Hz, much larger than before.

2.4

If a square wave is used at the mixer input, the LIA spectral response contains windows at the odd harmonics of f_R, which must *not* fall onto the harmonics of f_i. In this case, it is easy to see that the requirement is satisfied, so that no further changes are required.