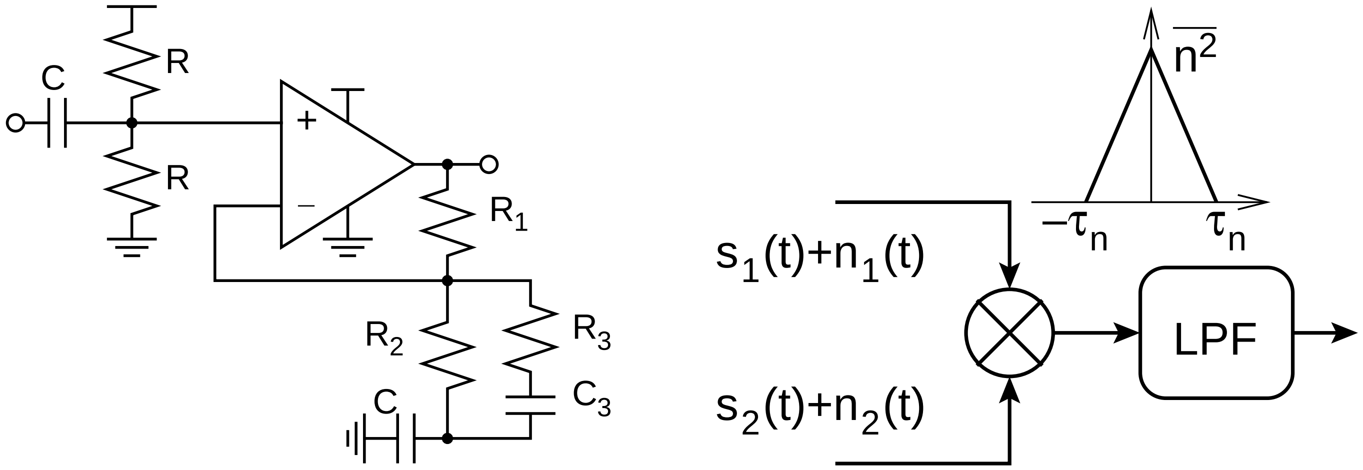
Electronics – February 16, 2015 Prof. A. Spinelli



Problem 1

The scheme in the left figure is an amplifier working with *single supply*, i.e., biased between V_{cc} and ground. OA has $A_0 = 10^4$ and $GBWP = 70$ MHz; component values are $R = 10$ kΩ, $R_1 = 560\ \Omega$, $R_2 = R_3 = 280\ \Omega$, $C = 1$ mF, $C_3 = 560$ pF. To simplify the calculations, approximate the behavior of C where necessary.

1. Compute the ideal and open-loop gain of the circuit and draw the Bode diagram of the losed-loop gain.
2. Verify the circuit stability and evaluate (even approximately) the maximum value of the OA input differential capacitance granting a phase margin of 45°. Discuss the employed approximations.
3. Compute the total output rms noise when the input noise sources of the OAs are $\sqrt{S_V} = 10\ \text{nV}/\sqrt{\text{Hz}}$, $\sqrt{S_I} = 1\ \text{pA}/\sqrt{\text{Hz}}$ (neglect resistor noise).
4. Discuss the function of the capacitor C and resistors R. Can we get rid of them?

Problem 2

The system in the right figure is a multiplier (having gain of 1 V^{-1}) followed by a single-pole filter. Consider $s_1 = A_1 \cos(\omega_R t)$, $s_2 = A_2 \cos(\omega_R t)$ with $f_R = 200$ Hz and $R_{n_1 n_1}$ as shown in the topmost right figure with $\tau_n = 2$ ms and $\overline{n^2} = 4 \times 10^{-9}\ \text{V}^2$.

1. It is $A_1 \approx A_2 \approx 10\ \mu$V and $n_2 = 0$. Size the output filter bandwidth to achieve $S/N = 1$ (referred to signal s_1).
2. The system is used to measure the cross-correlation in 0 between s_1 and s_2. Compute the output S/N knowing that n_2 is a white noise with bilateral PSD $\lambda = 10^{-14}\ \text{V}^2/\text{Hz}$, uncorrelated with n_1 (neglect the term $n_1 n_2$ in the calculations). Comment on the result.
3. With reference to # 2.2, consider now the term $n_1 n_2$ and repeat the calculation of S/N.
4. A delay T is inserted on the s_2 branch, to measure the cross-correlation for different values of T. What is the effect on the output noise?

Question

Describe compensation techniques for OAs under capacitive load.

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by February 18th **Mark registration: Friday, February 20th**

Solution

Problem 1

1.1

We have two signals at the input, V_i and V_{cc}, so we can use the superposition principle. V_{cc} is a DC signal, all capacitors are open circuits and the transfer is equal to 1/2.
We now set V_{cc} to ground and turn on the input voltage, which is connected to a CR group. Note that the value of C is very high, meaning that these elements will only come into play for very low frequencies, close to DC. In fact, the high-pass filter introduces a zero in the origin and a pole at about 30 mHz, i.e., it can be safely neglected for all frequencies of interest (except near DC). As far as the capacitor C in the feedback loop is concerned, its pole is located at

$$f_p = \frac{1}{2\pi C R_2} \approx 0.6 \text{ Hz},$$

and it can be approximated with a short-circuit for all frequencies above this value. The stage gain now becomes:

$$G_{id} = 1 + \frac{R_1}{R_2 \parallel (R_3 + 1/sC_3)} = \frac{R_1 + R_2}{R_2} \frac{1 + sC_3 \left(R_3 + R_1 \parallel R_2\right)}{1 + sC_3 R_3},$$

amounting to 3 at low frequencies and raising to 5 at high frequencies. Singularities are located at

$$f_z = \frac{1}{2\pi C_3 (R_3 + R_1 \parallel R_2)} \approx 609 \text{ kHz};$$
$$f_p = \frac{1}{2\pi C_3 R_3} \approx 1 \text{ MHz}.$$

The open-loop gain is simply $A(s)$, adding a pole to the ideal gain at a frequency of $70/5 = 14$ MHz. Bode diagrams (excluding singularities due to C) are reported in Fig. 1 (left).

1.2

Stability is straightforward from the Bode diagrams in Fig. 1, where G_{loop} is the difference (measured in dB) between G_{open} and G_{id} and crosses the 0dB axis at $f_c = 14$ MHz with a slope of -20 dB/dec.
The maximum value of the input capacitance C_i can then be obtained by imposing that its pole be located exactly at f_c. Considering all others capacitors as short-circuited (we are well beyond any singularity due to C_3), we get:

$$f_p = \frac{1}{2\pi C_i (R_1 \parallel R_2 \parallel R_3)} = f_c \Rightarrow C_i = \frac{1}{2\pi f_c (R_1 \parallel R_2 \parallel R_3)} = 101.5 \text{ pF}.$$

The corresponding loop gain is shown in Fig. 1 (right). Note that the poles of C_i and C_3 are separated by slightly more than a decade, so that a fairly good approximation is expected; exact calculations return in fact a value of about 105 pF. If, however, we consider the real diagram rather than the asymptotic one, the value rises to about 137 pF.

1.3

The PSD of the output noise, still for the frequency range in which we neglect C, is

$$S_{V_o} = S_V \left|G_{id}\right|^2 + S_I R_1^2.$$

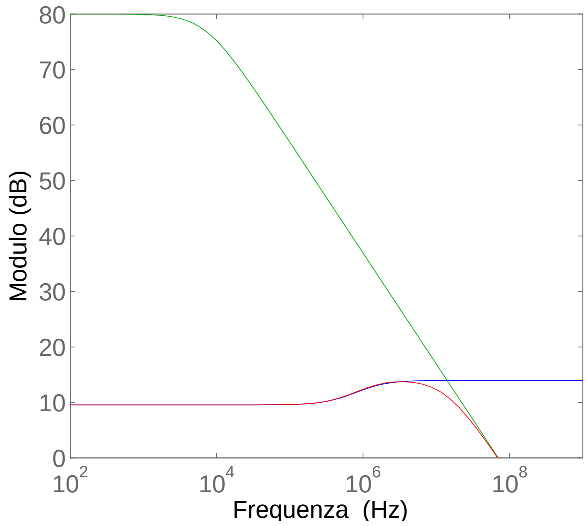
We can now break down the fraction representing $|G_{id}|^2$ as

$$|G_{id}|^2 = \frac{9}{|1 + sCR_3|^2} + 9 \frac{|sC_3(R_3 + R_1 \parallel R_2)|^2}{|1 + sC_3 R_3|^2}.$$

We now add the pole introduced by the OA at frequency f_c to the second transfer, obtaining

$$\overline{V_o^2} = 10^{-16} \left(9 \cdot \frac{\pi}{2} 6 \times 10^5 + 25 (14 \times 10^6 - 6 \times 10^5) \frac{\pi}{2}\right) + 10^{-24} \cdot (560)^2 \cdot 14 \times 10^6 \frac{\pi}{2} \approx (0.23 \text{ mV})^2,$$

largely dominated by the voltage noise of the OA.



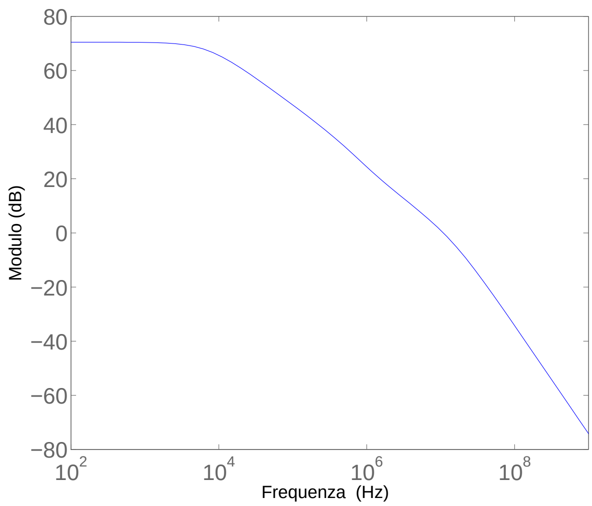


Figura 1: Left: Bode diagram of the ideal (blue), open-loop (green) and closed-loop (red) gain. Right: Bode diagram of the loop gain for the case of $C_i = 101.5$ pF.

1.4

OS is powered by a single-supply, not a dual one as usually assumed. As a consequence, the output of the circuit in absence of input signal cannot be zero, as this is the lower end of te dynamics, but has to be at $V_{cc}/2$, in the middle! This function is actually accomplished by the two resistors R, that bring V^+ to $V_{cc}/2$, plus the capacitor C in the feedback loop, that ensures that the DC gain is unity. We have then $V_o = V^+ = V_{cc}/2$.

Now, note that the input signal is referred to ground, while V^+ has a DC bias, so we cannot simply connect them. We need to decouple the DC component, which is achieved via the capacitor C. Since the resulting CR filter behaves as a HPF, we pick a large value for C, so that signal is not affected (except for its DC component).

Problem 2

2.1

The system is basically an LIA recovering a constant signal A_1 buried into a noise n_1 having bilateral PSD (Fourier transform of the triangle):

$$S_{n1}(f) = \overline{n^2}\tau_n \operatorname{sinc}^2(\pi f \tau_n).$$

The output signal to noise ratio and filter bandwidth are then given by

$$\frac{S}{N} = \frac{A_1}{\sqrt{4S_{n1}(f_R)BW_n}} = 1 \Rightarrow BW_n = \frac{A_1^2}{4S_{n1}(f_R)} \approx 5 \text{ Hz},$$

which means that the single-pole frequency will be placed at $10/\pi \approx 3$ Hz. The filter time constant is $T_F \approx 53$ ms.

2.2

At the multiplier output we have

$$x(t) = s_1 s_2 + s_1 n_2 + s_2 n_1 + n_1 n_2,$$

where the first term represents the useful signal while the other three are noises. The signal at the LPF input is then

$$s_1(t)s_2(t) = A_1 A_2 \cos^2(\omega_R t) = \frac{A_1 A_2}{2}(1 + \cos(2\omega_R t)).$$

The component at $2f_R$ is filterred out, and the output signal becomes $A_1A_2/2$.
We now look at the noise. The two contributions s_1n_2 e s_2n_1 can be rewritten as $s_1(n_1+n_2)$, which is just the expression of a noise source modulated by a sinusoidal waveform, i.e., what is found in the classic LIA filter. The output PSD becomes then:

$$S_y(f) = \frac{A_1^2}{4}\left(S_{n1}(f-f_R)+S_{n2}(f-f_R)+S_{n1}(f+f_R)+S_{n2}(f+f_R)\right)$$

and the rms value of the output noise is

$$\overline{n_y^2} = 2BW_nS_y(0) = A_1^2BW_n\left(S_{n1}(f_R)+S_{n2}(f_R)\right) = A_1^2BW_n\left(\overline{n^2}\tau_n\,\mathrm{sinc}^2(\pi f_R\tau_n)+\lambda\right) = (4.8\times10^{-6}A_1)^2.$$

From them, we compute the output S/N:

$$\frac{S}{N} = \frac{A_1^2}{9.6\times10^{-6}A_1} \approx 1.$$

Its value has not changed from the value in #2.1, except for small differences. In fact, the term that was added to the PSD is more than two orders of magnitude smaller than the one originally present, and doesn't affect the result.

2.3

The autocorrelation of the term $n_x = n_1n_2$ at the multiplier output is given by

$$\begin{aligned}R_{n_xn_x}(t,\tau) = \overline{n_x(t)n_x(t+\tau)} = &\int\!\!\int n_1(t)n_1(t+\tau)n_2(t)n_2(t+\tau)p(n_1,n_2,t,\tau)dn_1dn_2 = \\ &\int\!\!\int n_1(t)n_1(t+\tau)n_2(t)n_2(t+\tau)p(n_1,t,\tau)p(n_2,t,\tau)dn_1dn_2 = \\ &\int n_1(t)n_1(t+\tau)p(n_1,t,\tau)dn_1\int n_2(t)n_2(t+\tau)p(n_2,t,\tau)dn_2 = \\ &\overline{n_1(t)n_1(t+\tau)}\;\overline{n_2(t)n_2(t+\tau)} = R_{n_1n_1}(\tau)R_{n_2n_2}(\tau) = \lambda R_{n_1n_1}(0)\delta(\tau).\end{aligned}$$

This is a white noise with PSD $\lambda_x = \lambda\overline{n^2} = 4\times10^{-23}$ V^2/Hz and output rms value $2\lambda_xBW_n = (2\times10^{-11}\text{ V})^2$, completely negligible with respect to the previous contributions.

2.4

Noises are stationary and uncorrelated among themselves; adding a delay on either channel does not affect the properties of the output noise. This is all the more true if we consider that we are actually computing the *average* PSD of noises s_1n_2 and s_2n_1, which is obviously not affected by delays.
Of course, delays affect the output signal, reducing it (remember that maximum autocorrelation is achieved for zero delay) and degrading S/N.