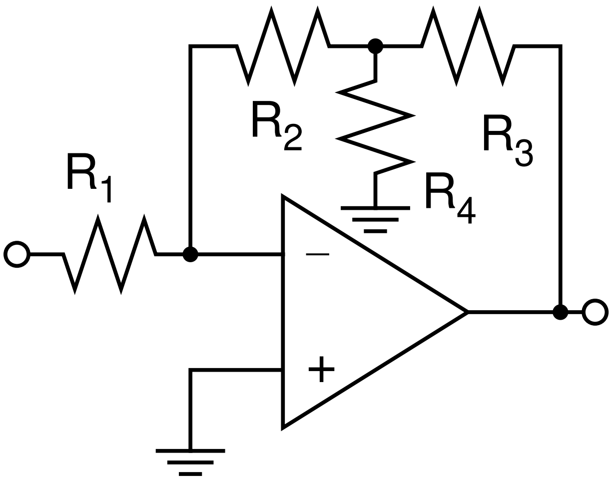
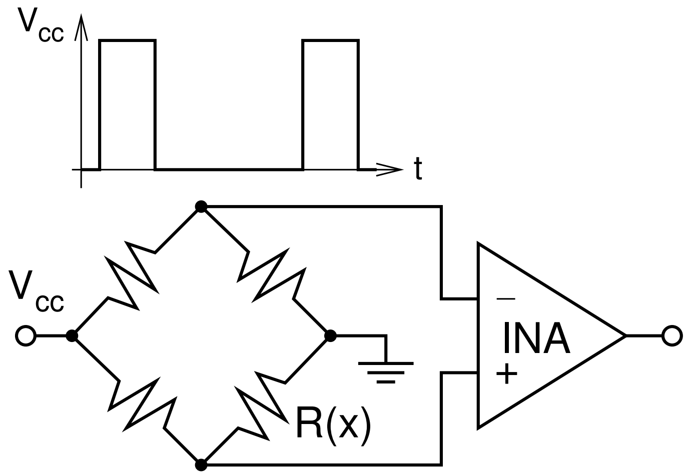
Electronics – July 10 2015 Prof. A. Spinelli





Problem 1

The scheme in the left figure shows an inverting amplifier. OA has $A_0 = 10^5$ and $GBWP = 10$ MHz; component values are $R_1 = R_2 = R_3 = 10\ \mathrm{k\Omega}$, $R_4 = 102\ \Omega$.

1. Compute the (ideal) closed-loop gain of the stage.
2. Compute the maximum value of the input capacitance C_i that guarantees a phase margin $\geq 45°$.
3. Compute the total output rms noise when the input noise source of the OA is $\sqrt{S_V} = 10\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$ ($4k_BT \approx 1.646 \times 10^{-20}$ J).
4. Propose a *simple* modification of the circuit that allows to achieve a gain $G = G_0(1 + s\tau)$ with the zero located at 10 kHz.

Problem 2

The scheme in the right figure is used for a resistive sensor. The Wheatstone bridge has a full-scale sensitivity of 1.5 mV/V and the output of the INA is acquired by a 12 bit ADC. The input noise source of the INA is $\sqrt{S_V} = 100\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$ and its bandwidth is 100 kHz.

1. A constant excitation voltage $V_{cc} = 5$ V is used for the bridge. Design a low-pass filter that allows to match the above requirement.
2. To avoid self-heating, a pulsed V_{cc} is now used (see top of figure), having $T_{ON} = 1$ ms and $T_{OFF} = 4$ ms. Is the previous solution still working? Evaluate S/N when a gated integrator replaces the LPF.
3. Design an improved solution to achieve $S/N = 5$ at minimum signal condition.
4. An HPF at 0.1 Hz has been added at the output of the INA to reject the offset. What are new values of the previous parameters?

Question

Describe the matched filter weighting function and the effect of the finite acquisition time.

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by July 17th **Mark registration: Monday, July 20th**

Solution

Problem 1

1.1

We label V_1 the node between R_2 and R_3 and write the KCL at this node:

$$\frac{V_1}{R_2} + \frac{V_1}{R_4} + \frac{V_1 - V_o}{R_3} = 0,$$

where we have considered that $V^- = 0$. However, for this reason and because the OA draws no current, we also have:

$$V_1 = -V_i \frac{R_2}{R_1}.$$

With simple manipulation we obtain:

$$G_{id} = \frac{V_o}{V_i} = -\frac{R_2 + R_3}{R_1}\left(1 + \frac{R_2 \parallel R_3}{R_4}\right) = -\left(2 + \frac{R}{R_4}\right) = -100,$$

where R is the value of resistors R_{1-3}. It is clear that R_4 boosts the gain allowing – for example – to achieve the same gain with less spread in resistor values, i.e., with higher precision with respect to the classic configuration.

1.2

It is easy to see that the loop gain can be expressed as

$$G_{loop} = -A(s)\frac{(Z_1 + R_2) \parallel R_4}{(Z_1 + R_2) \parallel R_4 + R_3}\frac{Z_1}{Z_1 + R_2},$$

where $Z_1 = R_1 \parallel 1/sC_i = R_1/(1 + sC_iR_1)$. With elementary (though somewhat boring) manipulation we obtain

$$G_{loop} = -A(s)\frac{R_1R_4}{(R_3 + R_4)(R_1 + R_2) + R_3R_4}\frac{1}{1 + sC_i(R_1 \parallel (R_2 + R_3 \parallel R_4))},$$

where the pole position reflects the resistance seen by the input capacitor C_i. Recalling that $R_4 \ll R_1, R_2, R_3$, G_{loop} can be simplified to

$$G_{loop} \approx -A(s)\frac{R_1R_4}{R_3(R_1 + R_2)}\frac{1}{1 + sC_i(R_1 \parallel R_2)} = -A(s)\frac{R_4}{2R}\frac{1}{1 + sC_iR/2}.$$

The loop gain at zero frequency is $A_0R_4/(2R) = 510$ and the OA pole is located at 100 Hz, meaning that the zero-dB crossing will take place at $f_{0dB} \approx 51$ kHz. To achieve a phase margin of 45° we must then have

$$f_p = f_{0dB} = \frac{1}{2\pi C_i(R_1 \parallel R_2)} = \frac{1}{\pi C_i R} \Rightarrow C_i = \frac{10^{-3}}{51\pi R} = 624 \text{ pF}.$$

This value is much larger than the typical values of this parameter, making the input capacitance irrelevant in this case. A Bode plot of the resulting loop gain is shown in Fig. 1 (left).

1.3

The transfer of the input noise voltage can be computed placing a voltage source V_n at the non-inverting input of the OA and writing equations similar to those in 1.1:

$$\begin{cases} \dfrac{V_1 - V_n}{R_2} + \dfrac{V_1}{R_4} + \dfrac{V_1 - V_o}{R_3} = 0 \\ V_1 = V_n \dfrac{R_1 + R_2}{R_1}, \end{cases}$$

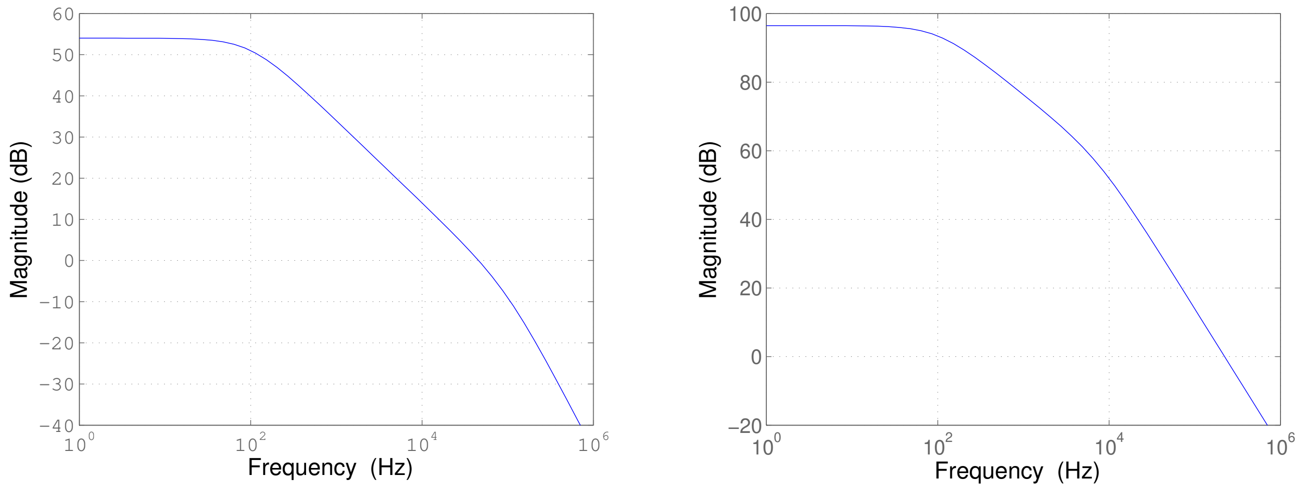


Figure 1: Left: Bode magnitude diagram of the loop gain with $C_i = 318$ pF. Right: same for the modified scheme discussed in 1.4.

leading to

$$V_o = V_n \frac{R_1 + R_2 + R_3}{R_1} \left(1 + \frac{(R_1 + R_2) \parallel R_3}{R_4}\right) = V_n \left(3 + 2\frac{R}{R_4}\right) = 199 V_n.$$

The other contributions are easily calculated, leading to

$$S_{V_o} = 199^2 S_V + 10^4 S_{V_1} + S_{V_2} \left(1 + \frac{R_3}{R_4}\right)^2 + S_{V_3} + S_{I_4} R_3^2 = 199^2 S_V + 1.98 \times 10^4 S_{V_1} + 98^2 S_{V_4} \approx 7.2 \times 10^{-12} \text{ V}^2/\text{Hz}.$$

Since the output noise PSD is white, we must account for the pole in the closed-loop gain due to the OA, which is located at the zero-dB frequency already computed, $f_{0dB} = 51$ kHz. The total rms noise becomes then

$$\overline{V_o^2} = 7.2 \times 10^{-12} \frac{\pi}{2} 51 \times 10^3 \approx (0.76 \text{ mV})^2.$$

Note that the OA contribution amounts to 0.56 mV rms, while resistors count for the remaining 30% or less. This contribution cannot be lowered without compromising the input impedance.

1.4

If we look at the expression for the ideal gain, we can see that the requested result can be easily achieved by replacing R_4 with a capacitor. In fact, substituting $1/sC$ for R_4, we get

$$G_{id} = \frac{V_o}{V_i} = -\frac{R_2 + R_3}{R_1} \left(1 + sC(R_2 \parallel R_3)\right).$$

The capacitor value is now easily determined:

$$f_z = 10^4 = \frac{1}{2\pi C(R_2 \parallel R_3)} \Rightarrow C \approx 3.2 \text{ nF}.$$

For the sake of completeness, though not requested, we note that a further look at this solution would reveal a phase margin issue due to an additional pole introduced in the loop gain by C (Fig. 1, right). This can be easily fixed by a small resistor placed in series to the capacitor, whose value is left to evaluate to the willing reader.

Problem 2

2.1

Recalling that the (full scale) sensitivity of the bridge is the output signal for $V_{cc} = 1$ V, its full-scale output is simply 7.5 mV and 1 LSB corresponds to

$$V_{LSB} = \frac{0.0075}{2^{12}} = 1.8\ \mu\text{V}.$$

If no filter is put in place, $BW_n = (\pi/2)10^5$ Hz and the rms noise becomes (neglecting the INA gain, that has no effect on S/N)

$$V_n^{rms} = \sqrt{S_V BW_n} = 39.63\ \mu\text{V},$$

meaning that an LP filter is indeed necessary. To achieve $S/N = 1$ we must then have

$$BW_n = \frac{V_{LSB}^2}{S_V} = 324\ \text{Hz} \Rightarrow f_p \approx 206\ \text{Hz}.$$

This value is much smaller than the INA bandwidth, meaning that the latter can be neglected.

2.2

Neglecting the INA, the output time constant becomes

$$\tau = \frac{1}{2\pi f_p} = 0.77\ \text{ms},$$

meaning that the output signal will not reach its steady-state value during pulsed operation, but will be limited to $1 - e^{-1/0.77} \approx 72\%$ of such value. S/N will then be degraded accordingly.
If we replace the LPF with a GI operating over $T_G = T_{ON}$ we get

$$\left(\frac{S}{N}\right) = V_{LSB}\sqrt{\frac{T_{ON}}{S_V/2}} \approx 0.81.$$

S/N is somehow improved, but remains insufficient for our purpose.

2.3

The simpler solution is to extend the average over multiple pulses adopting a boxcar averager. To a first approximation, the number of equivalent samples needed to achieve $S/N = 5$ is given by:

$$N_{eq} = \frac{2T_F}{T_{ON}} = \frac{25}{0.81^2} = 38.1 \Rightarrow T_F \approx 19\ \text{ms}.$$

2.4

The HPF does not affect the noise (the lower limit goes from zero to 0.1 Hz), nor it affects the shape of the signal at the amplifier output, but it does *remove the DC component* of the signal, i.e., its average value. The average value of the periodic pulsed signal is

$$\langle V \rangle = \frac{1}{T_{ON} + T_{OFF}} \int_0^{T_{ON}} V(t)dt = \frac{1}{5} V_{LSB},$$

which means that the true signal at the BA input will swing between $0.8V_{LSB}$ and $-0.2V_{LSB}$ rather than between 0 and V_{LSB}. The single-pulse S/N will then be $0.81 \times 0.8 = 0.65$ and the new result for T_F becomes

$$N_{eq} = \frac{2T_F}{T_{ON}} = \frac{25}{0.65^2} = 59.2 \Rightarrow T_F \approx 29.6\ \text{ms}.$$