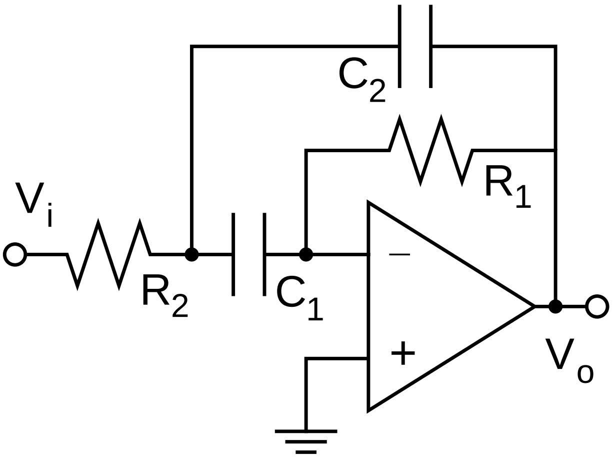
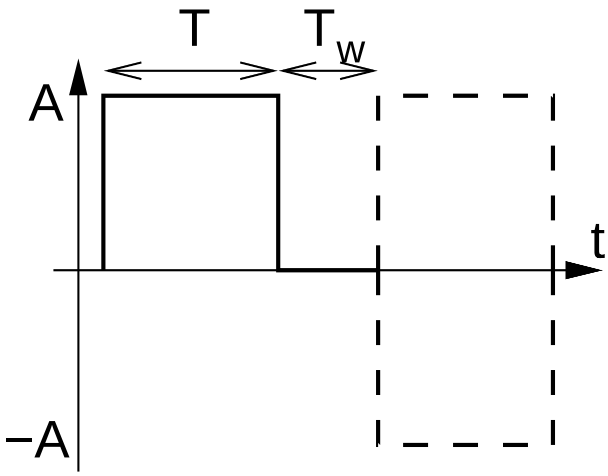
Electronics – July 23 2015 Prof. A. Spinelli





Problem 1

The scheme in the left figure shows a 2^{nd} order filter. OA has $A_0 = 10^4$ and $GBWP = 70$ MHz; component values are $R_1 = R_2 = 8$ kΩ, $C_1 = C_2 = 20$ pF.

1. Compute the closed-loop gain of the stage.
2. Verify the stability of the scheme.
3. The input noise PSDs of the OA are equal to 10 nV/$\sqrt{\text{Hz}}$ and 1 pA/$\sqrt{\text{Hz}}$. Compute the low- and high-frequency output noise PSD and find an approximate value for the total rms noise.
4. Sketch the step voltage response of the stage.

Problem 2

A signal approximately rectangular in shape, having amplitude A and duration T, is buried into a white noise having bilateral spectral density λ.

1. A matched filter is used to detect the pulse. Find the optimal S/N ratio.
2. The signal is now made of a stream of rectangular pulses, each having amplitude of $\pm A$ and separated by a wait time T_w (Fig. 2). Sketch the shape of the output signal and find the *minimum* value of T_w that allows a proper detection of the pulses (i.e., without any error due to the previous ones).
3. The noise PSD is now $\lambda + K/f^2$. Find the new expression of S/N for the previous filter and a suitable whitening filter.
4. An optimum filter is used after the whitening stage just computed. What is now the minimum value of T_w? Is the optimum filter solution convenient? Compute then the new value of the optimum S/N.

Question

Discuss the AC parameters of operational amplifiers.

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by July 28th **Mark registration: Friday, July 31**st

Solution

Problem 1

1.1

We label V_1 the node between R_2 and C_1 and note that the output can easily be expressed as

$$V_o = -sC_1R_1V_1$$

because the inner stage made up of the OA plus R_1 and C_1 works as a differentiator. To find V_1 we then write the KCL at node V_1:

$$\frac{V_i - V_1}{R_2} = sC_1V_1 + sC_2(V_1 - V_0).$$

With simple manipulation we obtain:

$$G_{id} = \frac{V_o}{V_i} = -\frac{sC_1R_1}{1 + s(C_1 + C_2)R_2 + s^2C_1C_2R_1R_2} = -\frac{sCR}{(1 + sCR)^2},$$

i.e., a transfer function with a zero in the origin and two coincident poles at a frequency of 1 MHz. The ideal gain is shown in Fig. 1 (left).

1.2

If we call V_2 the voltage between R_2 and C_1, we can write:

$$V_2 = \frac{R_2}{R_2 + 1/sC_2 \parallel (R_1 + 1/sC_1)}V_s = \frac{sCR(2 + sCR)}{1 + 3sCR + (sCR)^2}V_s.$$

Considering now the $R_1 - C_1$ branch we have

$$V^- = V_2\frac{sCR}{1 + sCR} + V_s\frac{1}{1 + sCR}$$

which apparently returns a third order network. Since we have only two reactive elements, there must be a pole-zero simplification (the numerator is actually a perfect cube). We eventually get

$$G_{loop} = -A(s)\frac{(1 + sCR)^2}{1 + 3sCR + (sCR)^2},$$

with two zeros at 1 MHz and two poles at 0.38 and 2.62 MHz. The singularities are so close that they do not give any significant contribution to the phase margin. Note also that the loop gain coincides with $-A(s)$ outside this small frequency interval.
The conclusion could have been reached via a simplified argument: if we consider the capacitors beyond their pole frequencies, their impedance is much lower than that of the corresponding resistances. In this range of frequencies, therefore, C_2 and C_1 provide a low-impedance path from the OA output to its inverting input, short-circuiting R_1. This means that the high-frequency loop gain will tend to $-A(s)$, thereby ensuring stability.

1.3

At low frequency the stage behaves as a follower, meaning that the output PSD becomes:

$$S_{Vo} = S_V + S_IR^2.$$

At high frequencies R_1 is short-circuited and we get $S_{Vo} = S_V$. We then conclude that the input noise voltage experiences a unity-gain transfer over (almost) the entire frequency range; the rms output noise will then be

$$\overline{V_o^2} \approx S_V\frac{\pi}{2}GBWP \approx (0.1\ \text{mV})^2.$$

The noise current is instead filtered by the capacitor poles, located around 1 MHz; its contribution will then approximately be

$$\overline{V_o^2} \approx S_IR^2\frac{\pi}{2}10^6 \approx (0.01\ \text{mV})^2.$$

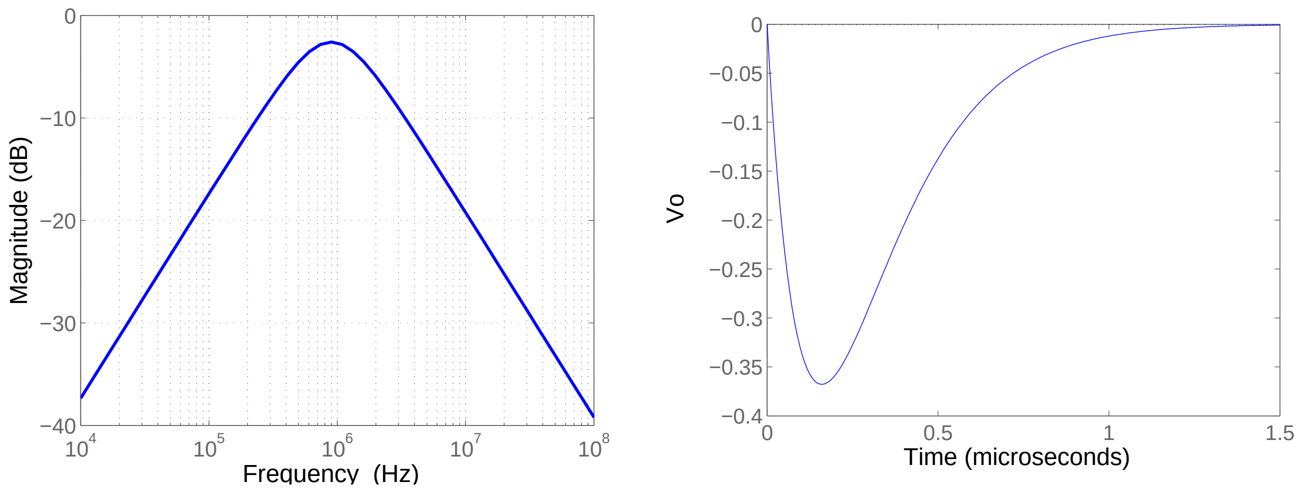


Figure 1: Left: Bode magnitude diagram of the closed-loop gain. Right: output step voltage response.

1.4

In the Laplace domain, the output voltage would be

$$V_o(s) = G_{id}V_i(s) = -\frac{sCR}{(1+sCR)^2}\frac{V_i}{s},$$

whose initial and final values can be easily computed:

$$V_o(0) = \lim_{s\to\infty} = sV_o(s) = 0$$
$$V_o(\infty) = \lim_{s\to 0} = sV_o(s) = 0.$$

We expect then a negative pulse with zero initial and final value that can be approximately drawn. For those longing for the analytical expression, we recall that multiplication by $-t$ equals derivation in the s domain, so that

$$\frac{1}{\tau}e^{-t/\tau} \Rightarrow \frac{1}{1+s\tau}$$
$$\frac{t}{\tau}e^{-t/\tau} \Rightarrow \frac{\tau}{(1+s\tau)^2},$$

which means

$$V_o(t) = -V_i\frac{t}{CR}e^{-t/CR},$$

shown in Fig. 1 (right) for $V_i = 1$ V.

Problem 2

2.1

The matched filter for this case has a rectangular weighting function and the output S/N becomes:

$$\frac{S}{N} = \frac{GAT}{\sqrt{G^2\lambda T}} = A\sqrt{\frac{T}{\lambda}}.$$

The output of the matched filter is a symmetric (mathematically: isosceles) triangular signal (autocorrelation of the input signal), reaching the maximum value GAT at $t = T$ (Fig. 2, left).

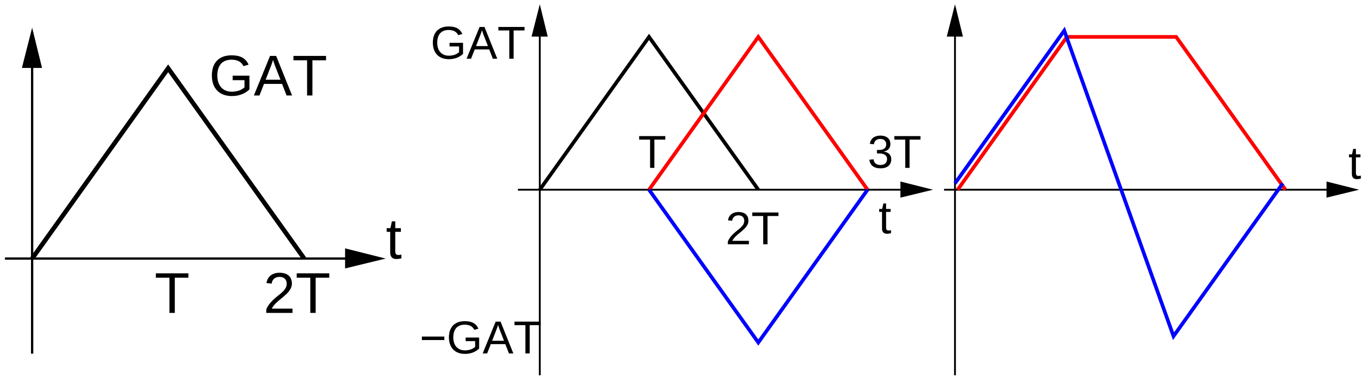


Figure 2: Left: Output signal to a single positive input pulse. Center: output contribution for two pulses, the second being either positive (red) or negative (blue). Right: Superposition of the pulses in Fig. 2 (center). Note that the values at T, $2T$, and so on are always $\pm GAT$.

2.2

Since the filter is linear, the output to a series of pulses is the superposition of the single-pulse response just highlighted. It is then clear that the output pulses do not interfere and can be easily detected if the pulses are separated by a time interval longer than T. If the separation becomes smaller, we have a pile-up of the responses, what in communication is called the *inter-symbol interference*.
However, it is easy to see that even in the case of $T_w = 0$ (see output pulses in Fig. 2, center) the output after a duration T are always correct, i.e., are the same that would be measured with a single pulse (Fig. 2, right). This means that in principle we could have $T_w = 0$. Of course, in this case we'd have to have a synchronization signal for sampling the output at the right time, but this goes beyond our treatment.

2.3

The new S/N becomes:

$$\frac{S}{N} = \frac{GAT}{\sqrt{G^2\lambda T + 2(GT)^2\left(\frac{K}{f_{min}} - \frac{K}{f_{max}}\right)}} \approx A\sqrt{\frac{T^2}{\lambda T + 2KT^2/f_{min}}}$$

and is of course no longer the optimum one. As the total noise PSD is now $\lambda + K/f^2$, the whitening filter transfer function H must be such that

$$\left(\lambda + \frac{K}{f^2}\right)|H|^2 = \text{const} \Rightarrow |H|^2 = \text{const}\frac{f^2}{K + \lambda f^2} \Rightarrow H = \text{const}\frac{s\tau}{1 + s\tau},$$

which is a simple high-pass filter having $\tau = \sqrt{\lambda/K}/2\pi$.

2.4

The HPF response to a rectangular pulse is well-known and won't be discussed here once again. Clearly, we have now a baseline problem which affects the pulse stream detection, meaning that T_w must now be larger than $4 - 5\tau$. This solution may not be very effective, in particular when τ is much larger than T and we want to maintain a high pulse rate.
To put the icing on the cake, we get the new value of the optimum S/N with simple calculations:

$$\frac{S}{N} = A\sqrt{\frac{\tau(1 - e^{-T/\tau})}{\lambda}}$$