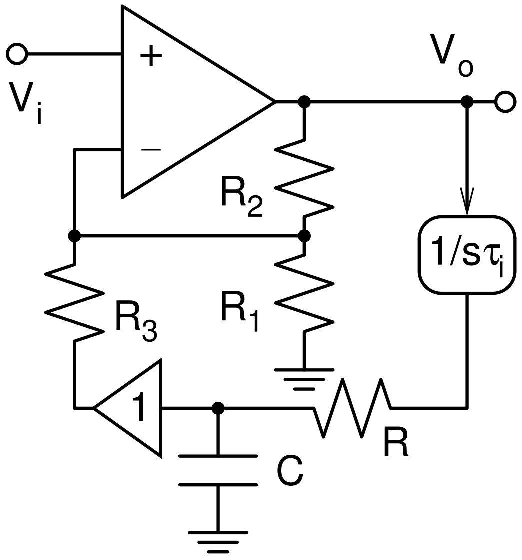
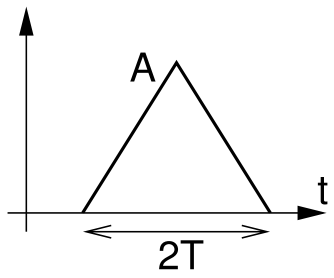
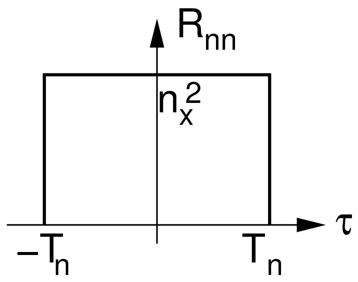
Electronics – July 21 2016 Prof. A. Spinelli







Problem 1

The OA in the left figure is a non-inverting audio amplifier with an offset reduction loop. The amplifier has $A_o = 106$ dB and $GBWP = 10$ MHz. Other parameters are $R_1 = 1$ kΩ, $R_2 = 2$ kΩ, $R_3 = 66$ kΩ, $R = 15$ kΩ, $C = 2.2$ μF, $\tau_i = 60$ ms.

1. Compute the (ideal) closed-loop gain of the stage.
2. Compute the loop gain of the stage and find the maximum input capacitance compatible with a phase margin of 45° at least.
3. Compute the rms output noise due to the input noise voltage of the OA, $\sqrt{S_V} = 6$ nV/$\sqrt{\text{Hz}}$. What are the contributions from the voltage noises of the buffer and integrator stages?
4. What is the maximum DC input offset that can be nulled by the stage if the output dynamics of all blocks is ±10 V? How can it be improved?

Problem 2

A triangular signal is affected by a noise having the nearly rectangular autocorrelation function shown in the figure on the right.

1. Consider $T_n \ll T$ and find the optimum S/N.
2. It is now $T_n \gg T$. Consider a <u>suitable</u> HPF and compute the resulting S/N.
3. Consider a GI. Find the expression for the output S/N and its value when $T_G = T$.
4. Consider now the case of $T_n = T/2$. Compute the optimum filter and <u>comment on the result</u>.

Question

Discuss the instrumentation amplifier architecture, properties and parameters.

For a correct evaluation you are asked to write your answers in a readable way; thank you Do a good job!

Results will be posted by July 25th **Mark registration: Wednesday, July 27th**

Solution

Problem 1

1.1

The voltage at the output of the unity-gain buffer is

$$V_b = \frac{V_o}{s\tau_i}\frac{1}{1+sCR},$$

so that the voltage at the inverting input of the OA becomes (linear superposition)

$$V^- = V_b\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_3} + V_0\frac{R_1 \parallel R_3}{R_1 \parallel R_3 + R_2} \approx \frac{V_b}{100} + \frac{V_o}{3},$$

where we have considered $R_3 \parallel R_1 \approx R_1$. Considering that $V^- = V^+ = V_i$ and substituting for V_b, we eventually get

$$\frac{V_o}{V_i} = \frac{300s\tau_i(1+sCR)}{100s^2\tau_i CR + 100s\tau_i + 3},$$

which is a function with a zero in the origin and one in $f_z = 1/2\pi CR = 4.8$ Hz. The two poles are instead located at $f_{p1} = 0.08$ Hz and $f_{p2} = 4.7$ Hz. There is then a pole-zero cancellation and the amplifier bandwidth extend down to 0.08 Hz, removing only DC and very low frequency components. The midband gain is equal to 3.

1.2

We can solve the second part of the problem very easily, noting that τ_i and RC have pretty large values, meaning that the external loop transfer function is basically zero after a few Hz (the RC pole is at around 5 Hz). For higher frequencies, we are left with a standard non-inverting amplifier with a gain of 3, having bandwidth equal to $GBWP/3 = 3.3$ MHz. As the resistance seen by the input capacitor C_i is $R_1 \parallel R_2 \parallel R_3 = R_1 \parallel R_2$, we immediately get

$$\frac{1}{2\pi C_i\ R_1 \parallel R_2} = 3.3 \times 10^6 \Rightarrow C_i \approx 72 \text{ pF}. \tag{1}$$

As far as the loop gain calculation is concerned, we break the loop at the OA output and apply the test signal V_s. The voltage at the output of the unity-gain buffer is then (see above)

$$V_b = \frac{V_s}{s\tau_i}\frac{1}{1+sCR},$$

and the voltage at the inverting input of the OA is

$$V^- = V_s\frac{R_1 \parallel R_3}{R_1 \parallel R_3 + R_2} + V_b\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_3} \approx \frac{V_s}{3} + \frac{V_b}{100}.$$

Considering that $V_o = -A(s)V^-$, we immediately get

$$G_{loop} = -A(s)\left(\frac{1}{3} + \frac{1}{100s\tau_i(1+sCR)}\right).$$

Obviously, a faster way to reach the result would be to note that the open-loop gain is simply $A(s)$ (just disconnect both loops), so that $G_{loop} = -A(s)/G_{id}$.

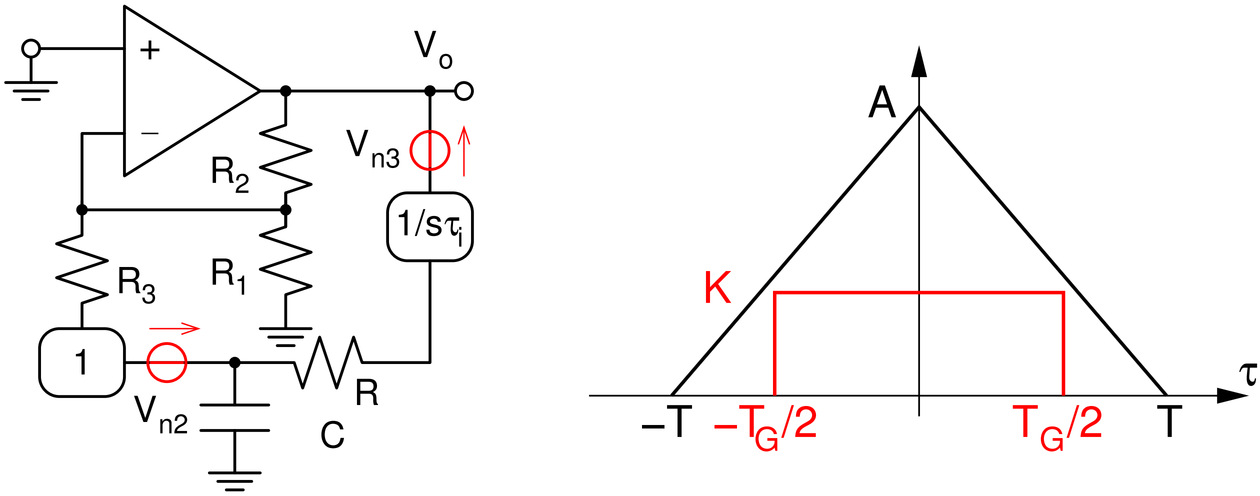


Figure 1: Left: Schematic for the noise calculations. Right: signal and weighting function (red) for the GI.

1.3

The OA contribution is simply trasferrred like the input signal, i.e., with a gain of 3 over the bandwidth 0.08 Hz – 3 MHz. The output contribution is then

$$\overline{V_o^2} = S_v 9 \frac{\pi}{2}\frac{GBWP}{3} = 1.87 \times 10^{-10}\ \mathrm{V}^2 = (41\ \mu\mathrm{V})^2.$$

The scheme for computing the other contributions is shown in Fig. 1 (left). Noting that no current flows in R_1 and that OA is connected as an inverting amplifier with gain $-R_2/R_3$, we can write:

$$(V_o - V_{n3})\frac{1}{s\tau_i}\frac{1}{1+sCR} - V_{n2} = -V_o\frac{R_3}{R_2} \Rightarrow V_o = V_{n2}\frac{s\tau_i(1+sCR)}{1+33s\tau_i(1+sCR)} + \frac{V_{n3}}{1+33s\tau_i(1+sCR)},$$

from which it is clear that these contributions are negligible: V_{n3} sees the first pole at 0.08 Hz (same poles as above), V_{n2} has a bandwidth similar to that of V_i, but with a gain of 1/33. Once again, knowing the pole positions, we could have just short-circuited the capacitor C, working out the simple transfers valid for all frequencies larger than about 5 Hz.

1.4

The result is straightforward from the second equation of #1.1. Setting $V^- = V_i = V_{DC}$ and $V_o = 0$ we get $V_b = 100V_{DC}$, which translates into a maximum input offset of 100 mV. To further improve this value, the best solution is to decrease the value of R_3. This will obviously change somewhat the poles position.

Problem 2

2.1

If $T_n \ll T$ the noise can be approximated as white, with a PSD $\lambda = 2T_n\overline{n_x^2}$, and the optimum weighting function tracks the signal. The value of S/N is then

$$\left(\frac{S}{N}\right) = \frac{A}{\sqrt{\lambda}}\sqrt{\int x^2(t)dt} = \frac{A}{\sqrt{\overline{n_x^2}}}\sqrt{\frac{T}{3T_n}}.$$

2.2

In the time and frequency domains, the HPF is described by

$$h(t) = \delta(t) - e^{-t/T_F}u(t) \leftrightarrow |H(f)| = \frac{\omega T_F}{\sqrt{1+(\omega T_F)^2}}.$$

Since the frequency-domain integral is complicated, we chose the time domain, where

$$k_{hh}(\tau) = \delta(\tau) - \frac{1}{2T_F} e^{-|\tau|/T_F}$$

and (calculations done verbatim in the class)

$$\overline{n_y^2} = \int R_{nn}(\tau) k_{hh}(\tau) d\tau = \overline{n_x^2} - \frac{1}{T_F} \overline{n_x^2} \int_0^{T_n} e^{-|\tau|/T_F} d\tau = \overline{n_x^2} e^{-T_n/T_F}.$$

If the time constant of the filter is much larger than T and there is no baseline problem, the signal is not affected and we have

$$\left(\frac{S}{N}\right) = \frac{A}{\sqrt{\overline{n_x^2}}} \sqrt{e^{T_n/T_F}}.$$

2.3

The output signal is now (see Fig. 1, right)

$$y(t) = \int x(\tau) w(t, \tau) = K \int_{-T_G/2}^{T_G/2} x(\tau) d\tau = KA \left(2 - \frac{T_G}{2T}\right) \frac{T_G}{2},$$

where K is the amplitude of the weighting function and we have centered the integration window around the maximum of the signal. Please note that the result is just the area of the trapezoid and there is no real need to solve (trivial) integrals. As for the noise, assuming $T_n \le T_G$, we have

$$\overline{n_y^2} = \int R_{nn}(\tau) k_{wtt}(\tau) d\tau = K^2 T_G \overline{n_x^2} \left(2 - \frac{T_n}{T_G}\right) T_n,$$

which is easily computed from the previous result, as now $k_{wtt}(\tau)$ is triangular with amplitude $K^2 T_G$ and R_{nn} has a rectangular shape. For $T_G = T$ we have then

$$\left(\frac{S}{N}\right) = \frac{A}{\sqrt{\overline{n_x^2}}} \frac{3}{4} \sqrt{\frac{T}{T_n(2 - T_n/T)}}.$$

Results for $T_n > T_G$ equals those for $T_n = T_G = T$.

2.4

In the general case, the optimum weighting function in the frequency domain is the ratio between the signal transform and the noise PSD, which in this case becomes

$$|W(t, f)| = \frac{X(f)}{S_n(f)} = \frac{AT \operatorname{sinc}^2(\pi f T)}{2\overline{n^2} T_n \operatorname{sinc}(2\pi f T_n)} = K \operatorname{sinc}(\pi f T),$$

meaning that $w(t, \tau)$ has a rectangular shape matching the noise autocorrelation (note: this is **not** a general feature, but a result of this particular case, to be discussed later). The output S/N would then be:

$$\left(\frac{S}{N}\right)^2 = A^2 \int \frac{|X(f)|^2}{S_n(f)} df \propto \int \operatorname{sinc}^3(\pi f T) df,$$

which is not straightforward to evaluate. We must then resort to the time domain, where $w(t, \tau) = K \operatorname{rect}(T)$. The output signal and noise are obtained once again from the results in #2.3 when $T_n = T/2$, i.e.,

$$\left(\frac{S}{N}\right) = \frac{A}{\sqrt{\overline{n_x^2}}} \frac{3}{4} \sqrt{\frac{2}{(2 - 1/2)}}. = \frac{A}{\sqrt{\overline{n_x^2}}} \sqrt{\frac{3}{4}}.$$

Note that the output S/N is *worse* than the input one, given simply by $A/\sqrt{\overline{n_x^2}}$, meaning that the filter cannot be optimal! The twist here is once again in the approximation of the noise autocorrelation and its Fourier transform $\operatorname{sinc}(\pi f T_n)$, which takes negative values in certain frequency range, while real noise PSDs are always positive.