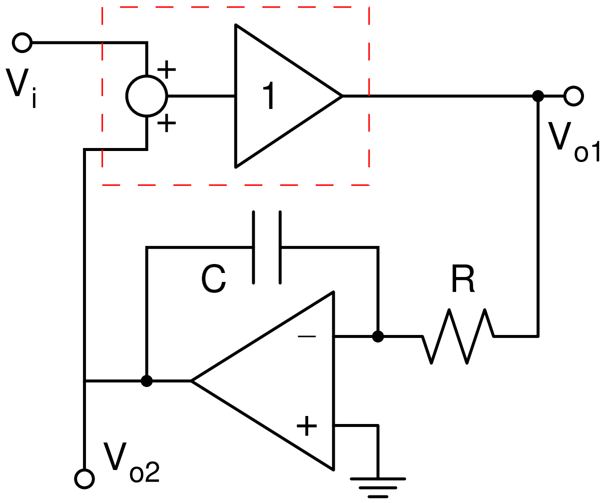
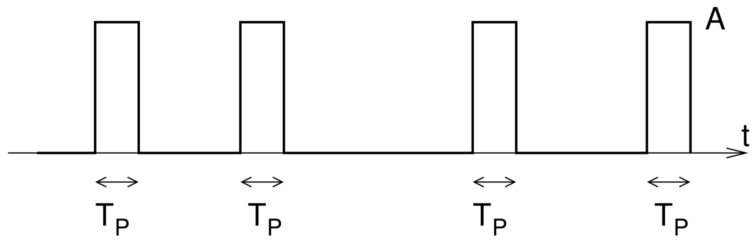
Electronics – July 4 2016 Prof. A. Spinelli





Problem 1

The scheme in the left figure represents a simple crossover filter for audio applications. The JFET amplifier has $A_o = 25$ V/mV and $GBWP = 3$ MHz. Other parameters are $R = 2$ kΩ, $C = 80$ nF.

1. Compute the (ideal) closed-loop gains of the stage.
2. Compute the loop gain of the stage and verify its stability.
3. Compute the rms noise for <u>both</u> outputs when the input noise voltages of the OA <u>and buffer</u> are $\sqrt{S_V} = 18$ nV/$\sqrt{\text{Hz}}$. What are the contributions from the current noise $\sqrt{S_I} = 10$ fA/$\sqrt{\text{Hz}}$?
4. Design a scheme that performs the function in the dashed box using only <u>one</u> OA.

Problem 2

A stream of equal rectangular pulses with amplitude $A \approx 100\ \mu$V, duration $T_P = 10$ ns and average separation $T_S = 1\ \mu$s is affected by a noise having <u>bilateral</u> PSD $\lambda = 10^{-15}$ V^2/Hz and bandwidth $f_n \approx 1$ GHz.

1. Find the parameters of an acquisition system that allows to measure the pulse amplitude with $S/N = 10$.
2. The filter works for <u>one hour</u>. Find the maximum value of the flicker noise corner frequency that does not degrade the total noise performance (hint: set the noise rms values to be equal; use simple approximations to compute the rms value).
3. The gate of the filter in #2.1 closes and opens with a random delay T_D in the $0 - 10$ ns range. Find the new values of the parameters.
4. A discrete-time filter with power-law weighting ($p_k = \alpha^k$) is used in place of the filter in #2.1. Find the value of α (or N_{eq}) that gives $S/N = 10$.

Question

Describe the noise modulation and filtering by an LIA.

For a correct evaluation you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by July 8th **Mark registration: Tuesday, July 12th**

Solution

Problem 1

1.1

The OA is connected as an integrator, so that

$$V_{o2} = -\frac{V_{o1}}{sCR},$$

while the upper part of the circuit imposes:

$$V_{o1} = V_i + V_{o2} = V_i - \frac{V_{o1}}{sCR} \Rightarrow V_{o1} = V_i \frac{sCR}{1+sCR}.$$

From that, we get:

$$V_{o2} = -\frac{V_{o1}}{sCR} = -V_i \frac{1}{1+sCR}.$$

The two outputs provide then a low- and a high-pass filtered version of the input voltage. The pole frequency is $f_p = 1/(2\pi CR) = 995$ Hz ≈ 1 kHz.

1.2

If we break both loops at the output of the OA, we immediately see that there is a unity-gain transfer to the inverting input of the OA, so that it is

$$G_{loop} = -A(s),$$

from which the stage stability is ensured. Outrageously easy!

1.3

The scheme for noise calculation is shown in Fig. 1 (left). From it, we can easily derive the following relationships:

$$\begin{aligned} V_{o2} &= -V_{o1}\frac{1}{sCR} + V_{n2}\frac{1+sCR}{sCR} \\ V_{o1} &= V_{o2} + V_{n1}, \end{aligned}$$

which lead to

$$V_{o1} = V_{n1}\frac{sCR}{1+sCR} + V_{n2} \qquad V_{o2} = V_{n1}\frac{1}{1+sCR} + V_{n2},$$

where a minus sign has been dropped. From here, evaluating separately the noise contributions, we obtain

$$\begin{aligned} \overline{V_{o1}^2} &= S_V \frac{\pi}{2}\left(f_p + 2(GBWP - f_p)\right) \approx (55\ \mu\text{V})^2 \\ \overline{V_{o2}^2} &= S_V \frac{\pi}{2}(2f_p + (GBWP - f_p)) \approx (39\ \mu\text{V})^2, \end{aligned}$$

where the pole at $GBWP$ due to the loop gain evaluated in #1.2 is also included.
As for the current noise, the only one that gives a non-zero contribution is the one on the inverting input of the OA, that can be neglected given its extremely low value (it is much smaller than the current noise of R, which has the same transfer). As a reference, its effect is:

$$S_{Vo1} = S_{Vo2} = S_I \left|\frac{R}{1+sCR}\right|^2,$$

giving a contribution to $\overline{V_{o1}^2}$ and $\overline{V_{o1}^2}$ equal to $(0.8\ \text{nV})^2$.

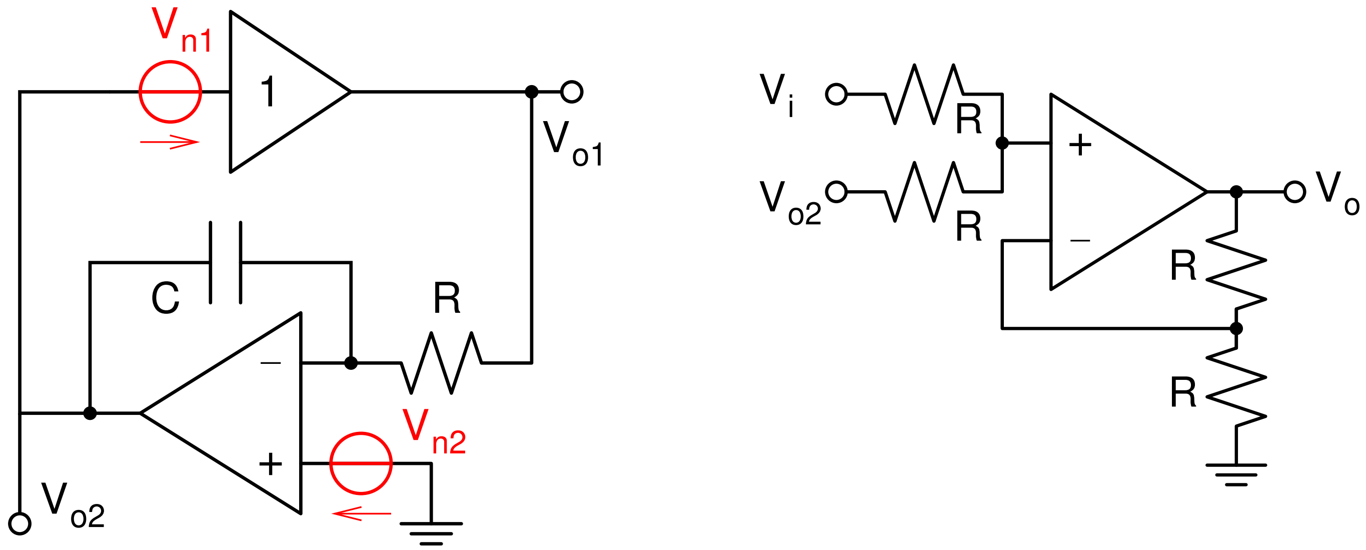


Figure 1: Left: Schematic for the noise calculations. Right: scheme for a two-input adder.

1.4

The standard configuration for the adder is based on an inverting configuration and requires additional inverting stages. However, we can easily exploit the non-inverting amplifier and a voltage divider to achieve the desired result, as shown in Fig. 1 (right).

Problem 2

2.1

As we have a repetitive signal, the best choice is the boxcar averaging filter. The expression of the final S/N is then

$$\left(\frac{S}{N}\right)_{BA} = A\sqrt{\frac{2T_F}{\lambda}} = 10 \Rightarrow T_F = \frac{50\lambda}{A^2} = 5\ \mu\text{s},$$

while we obviously set the gate time equal to the pulse width $T_p = 10$ ns. This corresponds to a number of equivalent samples equal to 1000. Note that we have used a white-noise approximation, which is well justified considering that the noise bandwidth is one order of magnitude larger than the single-pulse equivalent bandwidth.

2.2

We know that the low-frequency part of WF has a pole at a frequency about equal to $f_{max} \approx 1/(2\pi T_{env}) = 318$ Hz, where $T_{env} \approx T_F T_S / T_C = 500\ \mu$s. The white and flicker noise contributions are then

$$\overline{V_o^2} = G^2 \frac{\lambda}{2T_F} + G^2 K \ln\left(\frac{f_{max}}{f_{min}}\right).$$

These are equal for

$$f_{nc} = \frac{1}{4T_F \ln(f_{max}/f_{min})},$$

where we set $K = 2\lambda f_{nc}$ (the factor of 2 is because we are considering a *unilateral* flicker noise). If we now set $f_{min} = 1/3600 \approx 2.8 \times 10^{-4}$ Hz, corresponding to an operating time of one hour, we get $f_{nc} \approx 3.6$ kHz.

2.3

To avoid cutting the signal, we must open the gate 10 ns in advance of the pulse arrival (i.e., delaying the pulses) and close it when the pulse ends. This means that in the worst case (no delay at aperture, maximum delay at closure) we could end up with a gate pulse extending for $T_C = T_p + 2T_D = 3T_p = 30$ ns.

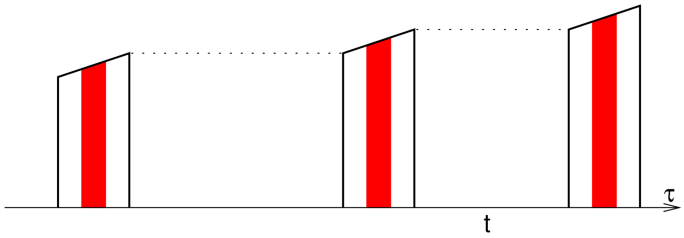


Figure 2: Weighting function of the filter in #2.3 accounting for the gate uncertainty (worst conditions). The signal exists only in correspondence of the red intervals.

Considering that the signal pulse is only 10 ns long, it is clear that we are integrating for 20 ns per pulse without any actual signal (although, to be fair, there is a small probability that two pulses will overlap, as 1 μs is the *average* pulse separation time). However, we are indeed collecting noise during those 20 ns! The new weighting function is shown in Fig. 2, where the red areas represent the intervals when the signal is present (note that the pulse separation time is not constant). We can still regard this filter as a boxcar averager, but we need to reassess the single-pulse S/N, which is now

$$\left(\frac{S}{N}\right)_{sp} \approx \frac{AT_p}{\sqrt{\lambda T_C}} = \frac{AT_p}{\sqrt{3\lambda T_p}}.$$

The final S/N is now

$$\left(\frac{S}{N}\right)_{BA} = \left(\frac{S}{N}\right)_{sp}\sqrt{\frac{2T_F}{T_C}} = \left(\frac{S}{N}\right)_{sp}\sqrt{\frac{2T_F}{3T_p}} = \frac{1}{3}A\sqrt{\frac{2T_F}{\lambda}},$$

from which we obtain the new value $T_F = 45$ μs.
Another way to look at this filter is to notice that *from the signal viewpoint* it is behaving like a ratemeter, for there is a discharge between successive acquisitions. The output signal is then

$$S_{out} = A\frac{1-e^{-T_p/T_F}}{1-e^{-(T_p+T_O)/T_F}} \approx A\frac{T_p}{T_p+T_O} = \frac{A}{3}.$$

From the (white) noise viewpoint, instead, nothing has changed from the previous case, so that we have

$$\overline{n_{out}^2} = \frac{\lambda}{2T_F},$$

meaning that a new value of T_F which is nine times the one computed in #2.1 is needed to achieve the same S/N.

2.4

The output S/N for a discrete-time filter with power-law weighting is

$$\frac{S}{N} = 10 = \frac{A}{\sqrt{\overline{n_{in}^2}}}\sqrt{N_{eq}} \Rightarrow N_{eq} = 100\frac{\overline{n_{in}^2}}{A^2},$$

where $\overline{n_{in}^2} = 2f_n\lambda$. Plugging in the numbers, we get $N_{eq} = 20000$ and $\alpha = (N_{eq}-1)/(N_{eq}+1)$. The difference with respect to the previous case is due to the integration performed by the Boxcar.