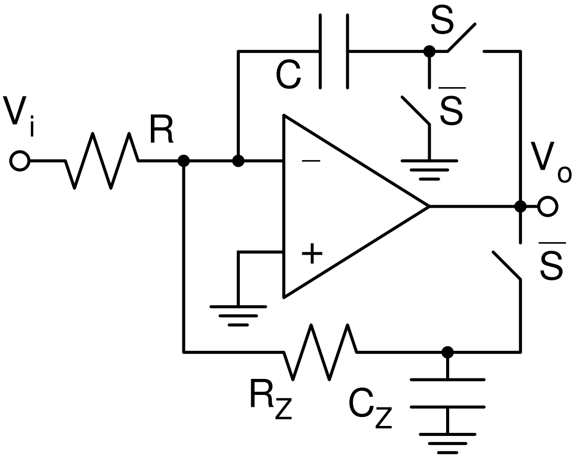
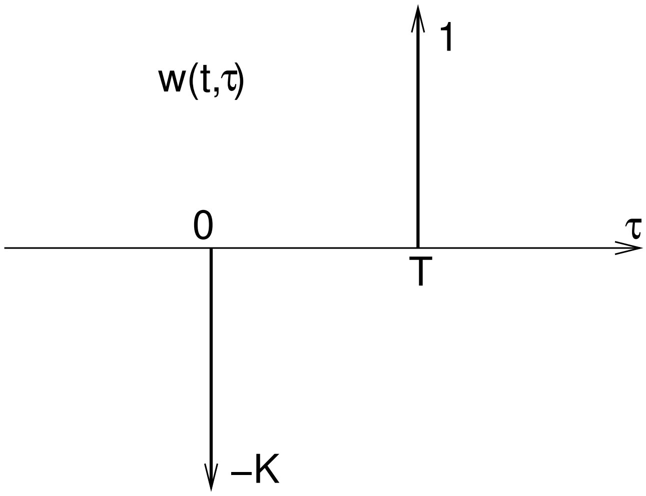
Electronics – September 23 2016 Prof. A. Spinelli





Problem 1

The scheme in the left figure is an integrator with an "autozeroing" loop to cancel the effect of the OA DC non-idealities. The switches are controlled by the digital signal S or by its opposite $\overline{S}$. The amplifier has $A_o = 106$ dB and $GBWP = 10$ MHz. Component values are $R = 1$ kΩ, $R_Z = 10$ kΩ, $C = 1$ μF, $C_Z = 10$ μF.

1. Consider the effect of the offset voltage of the OA. Discuss the behavior when S is low (correcting phase, input grounded) and high (integrating phase, input signal applied), explaining the working principle.
2. Compute the loop gain for one conditions of S at your choice and compensate if needed.
3. Compute the rms output voltage noise when the OA has $\sqrt{S_V} = 10\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$ ($4k_BT \approx 1.646 \times 10^{-20}$ J). Refer to the integration condition and pick a sensible value for the minimum frequency.
4. Propose a solution to avoid/minimize the drawback related to the discharge of C_Z.

Problem 2

To measure the amplitude of a pulse in presence of a noise with known autocorrelation $R_{nn}(\tau)$, we take two weighted samples, as shown in the figure on the right. The first sample acquires the noise only, the second the signal plus the noise.

1. Find the expression of the output noise.
2. Find the value of K which minimizes the output noise.
3. Consider an exponential autorrelation $R_{nn}(\tau) = \overline{n_x^2}\ e^{-|\tau|/T_n}$ and a signal $A(1 - e^{-t/T_s})$. Find the optimum value of T when $T_n \ll T_s$ and $T_n \gg T_s$. Approximated results are fine and encouraged, if justifications are provided.
4. Find the optimum filter and its corresponding S/N for the previous case, considering that the measurement has to be completed in a finite time T_M.

Question

Discuss the bias and offset errors is OAs and their compensation techniques.

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by July 28th **Mark registration: Monday, Oct 3rd**

Solution

Problem 1

1.1

At first glance, the behavior can be explained as follows: when S is low (switch open) and the $\overline{S}$ switches are closed (Fig. 1, left), a voltage equal to V_{OS} is stored onto C. When the switches are reversed (Fig. 1, right), the output goes at $V^- - V_C = 0$, so that the offset does not appear at the output and integration starts from zero.
The actual behavior is a bit more complicated: during the correcting phase, the stage is connected as an amplifier (Fig. 1, left) and the voltage across the zeroing capacitor C_Z is now $V_{OS}(1+R_Z/R) = 11\,V_{OS}$. When the switches are reversed (Fig. 1, right), the circuit works as an integrator. In a standard integrator, a current V_{OS}/R would flow through C, offsetting the output. Here, C_Z itself provides the current $(11\,V_{OS})/(R_Z+R) = V_{OS}/R$, so that no offset current flows through C and no error appears at the output. The circuit will then truly integrate only the input signal.
Of course, as C_Z discharges, cancellation is no longer perfect and errors start to arise, so the operation must be periodically repeated with a time interval smaller than the time constant $C_Z R_Z$.

1.2

We start with the case in Fig. 1, left (S = low). Clearly, there is a pole introduced by C which degrades the phase margin, while C_Z does not come into play as it is short-circuited by the OA output voltage source. We can employ the standard compensation technique used for resistive amplifiers and place a capacitor C_C in parallel to R_Z, chosen such that

$$RC = R_Z C_C \Rightarrow C_C = 100 \text{ nF},$$

leading to $G_{loop} = -R/(R + R_Z)A(s)$. In the other case, we define $Z = R \parallel (R_Z + 1/sC_Z) = R(1 + sC_Z R_Z)/(1 + sC_Z(R + R_Z))$, so that we can write:

$$G_{loop} = -A(s)\frac{Z}{Z + 1/sC} = -A(s)\frac{sCR(1 + sC_Z R_Z)}{1 + s(CR + C_Z R + C_Z R_Z) + s^2 C C_Z R R_Z}.$$

This loop is clearly stable, with poles at around $1/222\pi CR \approx 1.4$ Hz and $111/200\pi CR \approx 176$ Hz (plus the OA one) and zeros in the origin and at 1.6 Hz. It features an asymptotic behavior equal to $-A(s)$ with consequent phase margin of 90°.

1.3

The noise transfer functions are

$$S_{V_o} = S_V \left|\frac{1 + sCZ}{sCZ}\right|^2 + S_{V_R}\left|\frac{1}{sCR}\right|^2 + S_{V_{R_Z}}\left|\frac{C_Z/C}{1 + sC_Z R_Z}\right|^2,$$

where $Z = R \parallel (R_Z + 1/sC_Z) \approx R$ (pole and zero are at 1.45 and 1.59 Hz) and we have a divergence for $f \to 0$. However, the minimum operating frequency is dictated by the offset nulling procedure, which actually holds if C_Z is not significantly discharged. As its discharge time constant is $C_Z R_Z = 100$ ms, we can set the maximum integration time equal to about 10 ms, i.e., a minimum frequency $f_{min} \approx 100$ Hz. We then have:

$$\overline{V_o^2} = S_V \frac{\pi}{2} GBWP + (S_V + S_{V_R})\frac{1}{(2\pi CR)^2}\left(\frac{1}{f_{min}} - \frac{1}{GBWP}\right) + S_{V_{R_Z}}\left(\frac{C_Z}{C}\right)^2 \frac{1}{4C_Z R_Z} \approx (4\ \mu\text{V})^2,$$

dominated by the first term (the additional pole at $GBWP$ has also been accounted for).

1.4

Besides increasing the value of C_Z, we could of course increase the value of R_Z, paying attention to the output dynamic limit, but a better solution is to place a buffer stage between the storage capacitor C_Z and the resistor R_Z. In this way, the discharge is only limited by the leakage current of the buffer.

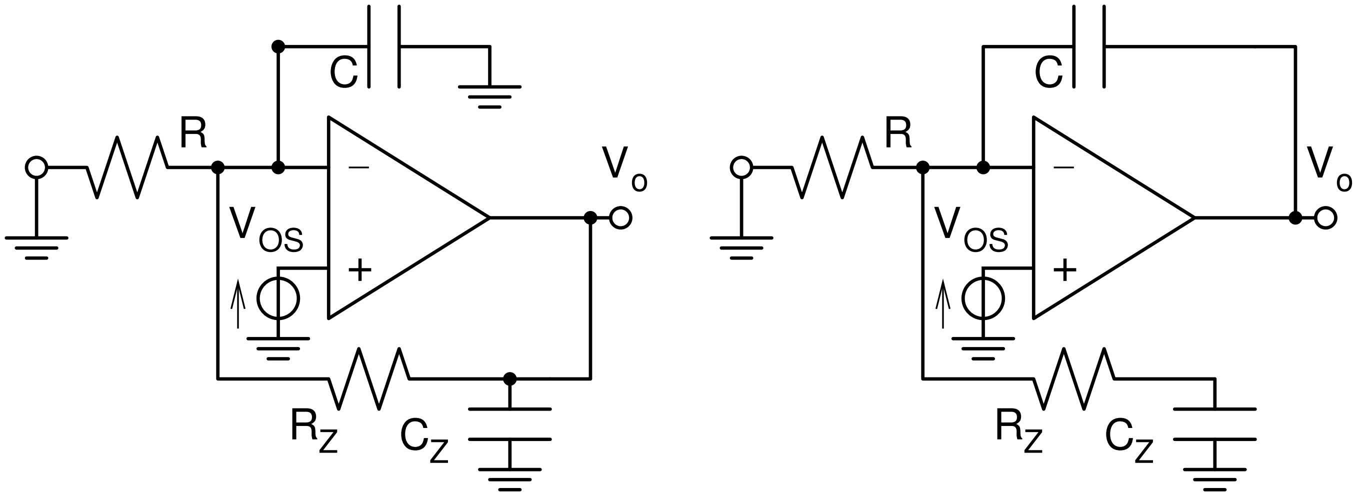


Figure 1: Circuit schemes when S is low (left) or high (right).

Problem 2

2.1

To compute the output rms noise, we need the time correlation of the weighting function, which in this case is

$$k_{ww}(\gamma) = (1 + K^2)\delta(\gamma) - K\delta(\gamma \pm T).$$

The output noise is then

$$\overline{n_y^2} = \int k_{ww}(\gamma)R_{xx}(\gamma)d\gamma = (1 + K^2)\overline{n_x^2} - 2KR_{xx}(T) = \overline{n_x^2}(1 + K^2 - 2K\rho_{xx}(T)),$$

where $\rho_{xx} = R_{xx}/\overline{n_x^2} = R_{xx}/R_{xx}(0)$ is a sort of normalized autocorrelation.

2.2

Minimizing the previous expression with respect to K, we easily obtain $K = \rho_{xx}(T)$, i.e.

$$\overline{n_y^2} = \overline{n_x^2}(1 - K^2) = \overline{n_x^2}(1 - \rho_{xx}^2(T)).$$

2.3

From the above calculations, the expression of the output S/N is

$$\left(\frac{S}{N}\right) = \frac{A}{\sqrt{\overline{n_x^2}}}\frac{1 - e^{-T/T_s}}{\sqrt{1 - e^{-2T/T_n}}}.$$

If $T_n \ll T_s$, we are dealing with quasi-white noise and the filter is ineffective ($K \approx 0$). The best choice is then to sample the signal once it has reached its steady-state, at $T \approx 5\tau$. This leads basically to a single sampling operation and to $S/N = A/\sqrt{\overline{n_x^2}}$.

The above value of S/N can always be attained if $T \gg T_n$. However, if we are dealing with a long noise correlation time, i.e., if $T_n \gg T_S$, we can exploit the filter to improve S/N, whose expression now becomes (expand the exponential at the denominator because of the long T_n):

$$\left(\frac{S}{N}\right) \approx \frac{A}{\sqrt{\overline{n_x^2}}}\frac{1 - e^{-T/T_s}}{\sqrt{2T/T_n}} = \frac{A}{\sqrt{\overline{n_x^2}}}\sqrt{\frac{T_n}{2T_S}}\frac{1 - e^{-T/T_s}}{\sqrt{T/T_S}}.$$

Clearly, the optimum choice of T is only dependent on T_S, while T_n will only determine the value of S/N. T cannot be much larger or much smaller than T_S, as S/N would collapse. A reasonable choice is then $T \approx T_S$ (discussed many times in past exam tests), leading to

$$\left(\frac{S}{N}\right) \approx \frac{A}{\sqrt{\overline{n_x^2}}}\sqrt{\frac{T_n}{2T_S}}(1-e^{-1}) = \frac{A}{\sqrt{\overline{n_x^2}}}\,0.63\,\sqrt{\frac{T_n}{2T_S}}.$$

S/N is improved with respect to the previous case when $T_n > 2T_S/(0.63)^2 \approx 5T_s$.

2.4

The noise PSD is the Fourier transform of a symmetric exponential function and has been computed several times (think of the autocorrelation of an LP-filtered white noise), resulting in

$$S(\omega) = \frac{2\overline{n_x^2}T_n}{1+\omega^2 T_n^2},$$

while the signal in the frequency domain is

$$AX(s) = \frac{A}{s}\frac{1}{1+sT_S} \Rightarrow |X(\omega)| = \frac{A}{\omega}\frac{1}{\sqrt{1+\omega^2 T_S^2}} * \mathrm{sinc}(\omega T_M/2),$$

where the effect of the finite measurement time has been accounted for via the sinc term (neglecting the phase-shift term). The optimum filter weighting funtion is therefore

$$W_{opt}(t,\omega) = \frac{|X(\omega)|}{S(\omega)}$$

and the resulting S/N can be obtained as

$$\left(\frac{S}{N}\right)^2 = A^2\int\frac{|X(f)|^2}{S(f)}df,$$

which unfortunately cannot be directly solved. Let's move to the time domain, then. The whitening filter is

$$H_w(s) = 1 + sT_n,$$

and the signal at its output becomes $AX_w(s)$, where

$$X_w(s) = X(s)H(s) = \frac{1}{s}\frac{1+sT_N}{1+sT_S} = \frac{1}{s} + \frac{T_n - T_S}{1+sT_S} \Rightarrow x_w(t) = \left(1 + \frac{T_n - T_S}{T_S}e^{-t/T_S}\right)u(t),$$

which can also be obtained regarding the filter as the sum of a unity-gain and a differentiator. Since the noise is now white, the final S/N becomes

$$\left(\frac{S}{N}\right)^2 = \frac{A^2}{2\overline{n_x^2}T_n}\int_0^{T_M} x_w^2(t)dt = \frac{A^2}{2\overline{n_x^2}T_n}\left(T_M + \frac{T_n^2 - T_S^2}{2T_s} - \frac{(T_n - T_S)^2}{2T_S}e^{-2T_M/T_S} - 2(T_n - T_S)e^{-T_M/T_S}\right),$$

obviously increasing with T_M.