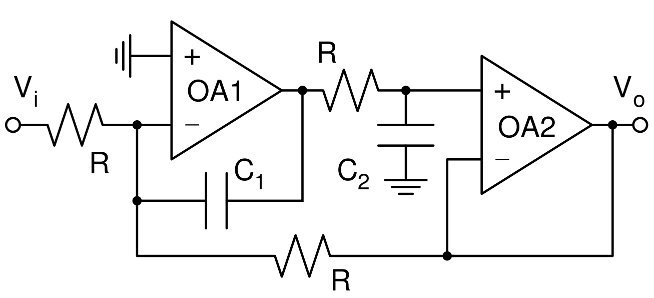
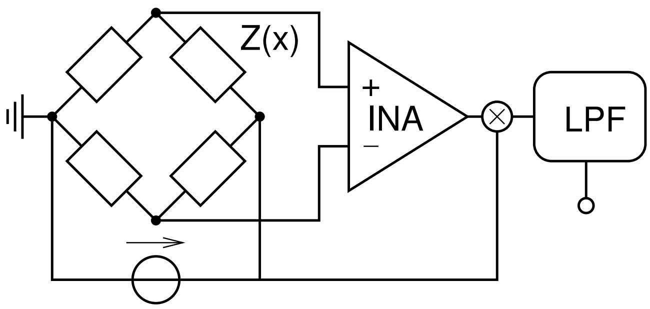
Electronics – September 5 2016 Prof. A. Spinelli





Problem 1

The scheme in the left figure is an active filter. The amplifiers have $A_o = 106$ dB and $GBWP = 10$ MHz. Component values are $R = 10$ kΩ, $C_1 = 0.16$ μF, $C_2 = 20$ nF.

1. Compute the (ideal) closed-loop gain of the stage.
2. Compute the loop gain of one of the OAs while assuming the other to be ideal, and check the stability. Comment on the effect of the real gain of the OA.
3. Compute the rms output noise due to the input noise voltage of the OAs, $\sqrt{S_V} = 10$ nV/$\sqrt{\text{Hz}}$.
4. The OAs output resistances are $R_o = 50$ Ω. What is the maximum attenuation of the signal at high frequencies?

Problem 2

The scheme in the left figure is used to measure small variations in an impedance value by using an LIA. The INA has input noise PSD $S_V = K/f$ with $K = 10^{-9}$ V^2. The bridge AC supply is $V_{cc}\cos(\omega_r t)$ with $V_{cc} = 1$ V.

1. Consider a resistive bridge in which $Z(x) = R + \Delta R$, with $R = 10$ kΩ and $\Delta R \approx 1$ Ω. The value of the impedance remains stable for up to 10 s. Find the LIA parameters to obtain $S/N \geq 10$.
2. What is the minimum bandwidth of the (single-pole) INA if the S/N degradation it introduces must be kept smaller than 1%?
3. The impedance $Z(x)$ can now be schematized as an $R-C$ parallel, in which C may also vary ($C = 1$ nF, $\Delta C \approx 1$ pF) within the same bandwidth as R. Set up a suitable Wheatstone bridge and discuss how you would measure Z, specifying the new LIA parameters and the resulting S/N (hint: perform two measurements).
4. Assuming to have a double-demodulator LIA with phase and quadrature outputs, discuss the new bridge/measurement setup and work out the expressions for the output S/N.

Question

Briefly discuss the deformation sensors operation, parameters and manufacturing technologies.

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by September 8^{th} **Mark registration: Monday, September 12^{th}**

Solution

Problem 1

1.1

The second stage is a voltage follower, so the voltage at its non-inverting input is equal to V_o. Because of the $R - C_2$ LPF, we can then write the following relation between V_{o1} (the output of OA1) and V_o:

$$V_o = V_{o1}\frac{1}{1 + sC_2R} \Rightarrow V_{o1} = V_o(1 + sC_2R).$$

Now, the KCL at the inverting input of OA1 gives us the required solution:

$$\frac{V_i}{R} + \frac{V_o}{R} + sC_1(1 + sC_2R)V_o = 0 \Rightarrow \frac{V_o}{V_i} = -\frac{1}{1 + sC_1R + s^2C_1C_2R^2},$$

which is a second-order filter whose poles are $f_{p1} \approx 1/(2\pi C_1R) \approx 100$ Hz and $f_{p2} \approx 1/(2\pi C_2R) \approx 795$ Hz. The poles are not largely separated, so the approximation is a little shaky. True values are $f_{p1} \approx 117$ Hz and $f_{p2} \approx 679$ Hz.

1.2

We turn off V_i and begins by breaking the loop of OA1 at its output. Applying a test signal V_s at the loop and writing the KCL at the OA inverting input, we get

$$\frac{V^-}{R} = sC_1(V_s - V^-) + \frac{1}{R}\left(\frac{V_s}{1 + sC_2R} - V^-\right),$$

which leads to

$$G_{loop1} = -A(s)\frac{1 + sC_1R + s^2C_1C_2R^2}{(2 + sC_1R)(1 + sC_2R)}.$$

The loop is easily seen to be stable, with an asymptotic behavior equal to $-A(s)$ and consequent phase margin of 90°. OA2, connected as a follower, will introduce a pole at $GBWP$ in its transfer function, but this does not affect the phase margin (just note that OA2 is shortened by C_1; I'll leave the calculations to you).

If the loop is broken at the OA2 output, we see that OA1 now works as an integrator, so that the voltage at the non-inverting input of OA2 is

$$V^+ = -V_s\frac{1}{sC_1R}\frac{1}{1 + sC_2R},$$

leading to

$$G_{loop2} = -A(s)\frac{1 + sC_1R + s^2C_1C_2R^2}{sC_1R(1 + sC_2R)},$$

which is also stable with a phase margin around 90°. OA1 will now add a pole at $GBWP$ in its integrating transfer function, which will not affect the phase margin (again, C_1 will shorten OA1, making its pole irrelevant at high frequencies; do the maths if you don't trust this explanation!). However, at low frequencies the finite gain of OA1 also kicks in, limiting the maximum value of $|G_{loop2}|$ to A_0^2. The loop gains in the ideal case are reported in Fig. 1 (left).

1.3

The noise transfer functions are

$$S_{Vo} = S_{V1}\left|\frac{2 + sC_1R}{1 + sC_1R + s^2C_1C_2R^2}\right|^2 + S_{V2}\left|\frac{sC_1R(1 + sC_2R)}{1 + sC_1R + s^2C_1C_2R^2}\right|^2,$$

where the singularities have already been computed. We have therefore:

$$\overline{V_o^2} = S_{V1}\frac{\pi}{2}\left(4f_{p1} + (f_{p2} - f_{p1})\right) + S_{V2}\frac{\pi}{2}(GBWP - f_{p1}) \approx S_V\frac{\pi}{2}GBWP \approx (39.6\ \mu\text{V})^2,$$

dominated by the noise source of OA2

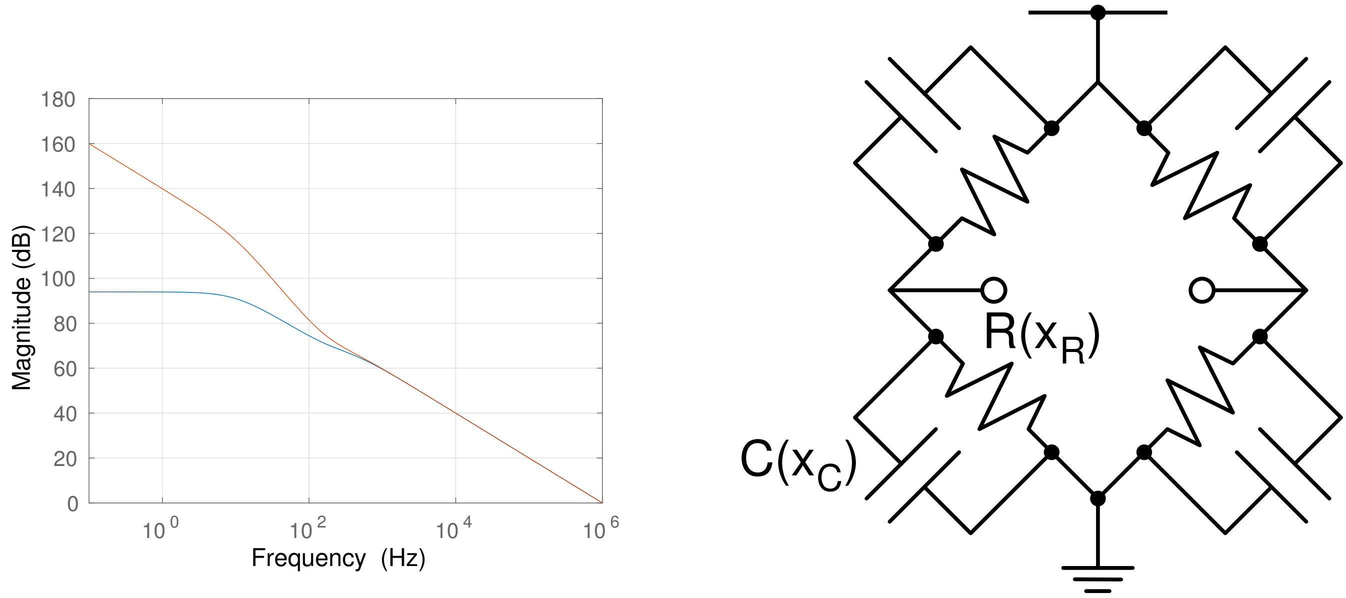


Figure 1: Left: Loop gains (blue = G_{loop1}, red = G_{loop2}). Right: Wheatstone bridge arrangement for $R-C$ parallel impedance measurement.

1.4

The ideal transfer would tend toward zero as the frequency increases. In reality, the OA transfer function goes to zero beyond $GBWP$, meaning that we only see R_o at the OA ouput. Since the capacitors also behaves as short circuits, the transfer now becomes

$$\frac{V_o}{V_i} \approx \left(\frac{R_o}{R+R_o}\right)^2 \approx \left(\frac{R_o}{R}\right)^2 = 25 \times 10^{-6}.$$

Problem 2

2.1

The bandwidth of the signal is approximately 0.1 Hz. Therefore, we pick the output filter bandwidth as 1 Hz. The modulated signal is instead

$$V_s = V_{cc}\frac{x_R}{4} = V_{cc}\frac{\Delta R}{4R} = 25\ \mu\text{V},$$

where obviously $x_R = \Delta R/R$. The modulation frequency f_m can now be obtained from the expression for S/N:

$$\left(\frac{S}{N}\right) = \frac{V_s}{\sqrt{2S_V(f_m)BW_n}} \geq 10 \Rightarrow f_m \geq \frac{200K\ BW_n}{V_s^2} \approx 503\text{ Hz} \Rightarrow 1\text{ kHz}.$$

Note that the total bridge resistor noise is $4k_BTR \approx 1.6 \times 10^{-16}\ \text{V}^2 \ll K/f_m = 10^{-13}\ \text{V}^2$, meaning that the above expression is correct. It is probably needless to specify that the bridge is made up of identical resistors R and that the bandwidrth of the INA must be larger than f_m.

2.2

The INA introduces a (single) pole at f_p in the transfer function, affecting magnitude and phase of the modulated signal. The magnitude is not a problem from the S/N viewpoint, as it affects signal and noise at the same rate, but the phase is: the signal modulated at f_m experiences a phase shift $\Delta\phi = \arctan(f_m/f_p)$, giving an output error term equal to $\cos\Delta\phi$. For small values of $\Delta\phi$ we can then write

$$\cos\Delta\phi \approx 1 - \frac{(\Delta\phi)^2}{2} > 0.99 \Rightarrow \Delta\phi < 0.14 \Rightarrow f_p > \frac{f_m}{0.14} \approx 7\text{ kHz}.$$

A similar value would be obtained if considering the error on the signal originating from the attenuation.

2.3

We need a balanced bridge, with no output when $x_R = x_C = 0$. The best solution is therefore to employ a bridge with $R - C$ elements, as depicted in Fig. 1 (right). The pole of each element is:

$$f_p = \frac{1}{2\pi RC} \approx 16 \text{ kHz},$$

meaning that the bridge can be seen as being essentially resistive at low frequencies and capacitive at high frequencies. So, the best solution is to carry out two measurements, one at low frequency (where x_R is measured) and one at high frequency (where x_C is measured). S/N and parameter values in the first case are just what obtained in #2.1. In the HF case we neglect the resistors, getting:

$$\frac{V_s}{V_{cc}} = \frac{1/sC(x_C)}{1/sC(x_C) + 1/sC} - \frac{1}{2} = \frac{1}{2 + x_C} - \frac{1}{2} = -\frac{x_C}{2(2 + x_C)} \approx -\frac{x_C}{4},$$

leading to a similar expression for S/N:

$$\left(\frac{S}{N}\right) = \frac{V_s}{\sqrt{2S_V(f_H)BW_n}}.$$

The requirement on S/N is easily satisfied as the flicker noise is smaller at higher frequencies and $x_C > x_R$. The new modulation frequency f_H must be larger than f_p, and the INA bandwidth has to be increased accordingly.

Note that a capacitive bridge was last spotted in the Electronics exam of June 29, 2012.

2.4

With a double-demodulator LIA we can use just one frequency and measure phase and quadrature components, relating them to ΔR and ΔC. The sensitive element has an impedance given by

$$Z(x) = \frac{R(x_R)}{1 + sC(x_C)R(x_R)} = \frac{R(1 + x_R)}{1 + sCR(1 + x_R)(1 + x_C)} \approx \frac{R(1 + x_R)}{1 + sCR(1 + x_R + x_C)},$$

where we neglected the higher-order term $x_R x_C$. The other branches have obviously an impedance equal to $Z = R/(1 + sCR)$. The bridge output voltage is now

$$\begin{aligned}\frac{V_x}{V_{cc}} &= \frac{Z(x)}{Z(x) + Z} - \frac{1}{2} = \frac{(1 + x_R)(1 + sCR)}{(1 + x_R)(1 + sCR) + 1 + sCR(1 + x_R + x_C)} - \frac{1}{2} \\ &= \frac{x_R - sCRx_C}{2(2(1 + sCR) + x_R(1 + 2sCR) + sCRx_C)} \approx \frac{x_R - sCRx_C}{4(1 + sCR)}.\end{aligned}$$

It is now preferable to return to $f_m \approx 1$ kHz, so that we can neglect the phase shift and attenuation induced by the $1 + sCR$ term at the denominator. The two demodulators will see signal amplitudes of $V_{cc}x_R/4 = 25\ \mu$V and $(V_{cc}x_C/4)\omega_m CR \approx 15.7\ \mu$V. The latter case leads to $S/N \approx 8.8$, which can be restored to 10 by slightly increaseasing f_m to about 1.3 kHz.