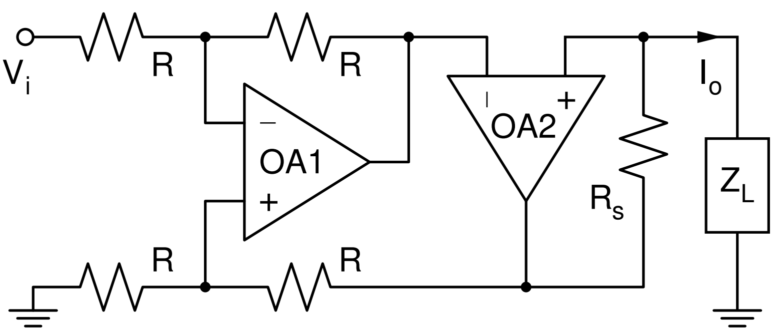
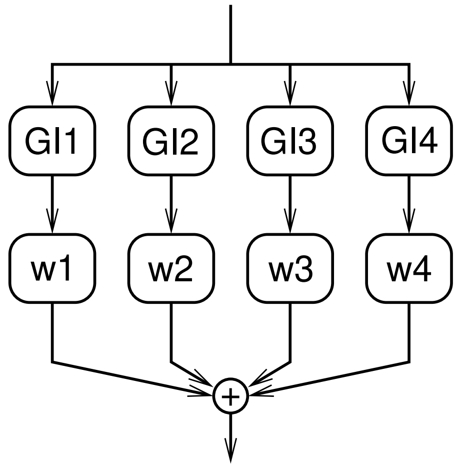
Electronics – July 6 2017

Prof. A. Spinelli





Problem 1

The scheme in the left figure is a voltage-controlled current source. The amplifiers have $A_o = 120$ dB and $GBWP = 1$ MHz.

1. Find the (ideal) closed-loop gain of the stage.
2. Consider $Z_L = R_L$. Compute the loop gain of OA2 considering OA1 as ideal. Discuss then the impact of the pole of OA1 on stability.
3. Compute the output current noise PSD due to the OA equivalent noise voltage S_V.
4. Propose a compensation scheme for OA2.

Problem 2

A filter is made up of four gated integrators, each working sequentially over an interval T. When GI1 finishes, GI2 begins, followed by GI3 and GI4. Their outputs are then multiplied by weight factors $w_1 - w_4$ and added. The input signal is a step voltage.

1. Sketch the weighting function in the time domain and give its expression in the frequency domain.
2. Find the optimum values of the weights w and the resulting expression of S/N for the case of white noise.
3. A very low-frequency offset is now present, in addition to the white noise. Repeat the above optimization.
4. A flicker noise is present at the input of the filter computed at the previous point. Find the output S/N. In the calculations, use $\int_0^\infty \frac{\sin^4 x}{x^3} dx \approx 0.7$.

Question

Describe the instrumentation amplifier architecture and its characteristics.

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by July 13th **Mark registration: Monday, July 17th**

Solution

Problem 1

1.1

We label V_o the output node, which is also the voltage at the input pins of OA2 and the output voltage of OA1. Simple application of the voltage superposition principle leads to

$$V_{OA1}^{-} = V_{OA1}^{+} = \frac{V_i + V_o}{2},$$

i.e.,

$$V_{o2} = V_i + V_o.$$

From this equation, it is straightforward to obtain

$$I_o = \frac{V_i}{R_s}.$$

1.2

We break the loop at the ouput of OA2 and easily obtain

$$G_{loop} = -A(s)\left(1 - \frac{Z_L}{Z_L + R_s}\right) = -A(s)\frac{R_s}{R_L + R_s},$$

where we replaced Z_L with R_L. This is obviously stable for any value of R_L, though high values will degrade the loop gain. OA1 is connected as a non-inverting amplifier with a gain of 2 and (local) loop gain equal to $-A(s)/2$, thereby featuring a closed-loop pole at $GBWP/2 = 0.5$ MHz and time constant τ_c. The new loop gain of OA1 becomes then

$$G_{loop} = -A(s)\left(\frac{1}{1 + s\tau_c} - \frac{R_L}{R_L + R_s}\right) = -A(s)\frac{R_s}{R_s + R_L}\frac{1 - s\tau_c R_L/R_s}{1 + s\tau_c}.$$

Note that the zero, located at $f_z = R_s/(2\pi\tau_c R_L) = GBWP(R_s/2R_L)$, is in the right half-plane, meaning that it *decreases* the phase margin. For small values of R_L the zero is not a problem and we only have the second pole at $GBWP/2$, that degrades the phase margin to $90 - \arctan(\sqrt{2}) \approx 35°$. On the other hand, any condition with $R_L \geq R_s$ is critical. Fig. 1 (left) shows the Bode diagrams of G_{loop} for $R_L = 10R_s$ (blue) and $R_L = 0.1R_s$ (reddish).

1.3

We consider the scheme in Fig. 1 (right) for noise calculation and easily obtain

$$I_o = \frac{V_{n2} - 2V_{n1}}{R_s} \Rightarrow S_{I_o} = \frac{5S_V}{R_s^2}.$$

1.4

We can follow different approaches to compensation. One idea could be to bypass the OA1 stage with a capacitor, short-circuiting it at high frequencies. However, a simple capacitor connected between the outputs of the two OAs would not give any zero or pole. We need then an extra resistor in series with the OA1 stage. Another possibility is looking at the expression for G_{loop} derived in 1.2, noticing that the stability problem mainly arises because of the zero in the right half-plane. In fact, for high frequencies, the term

$$\frac{1}{1 + s\tau_c} - \frac{R_L}{R_L + R_s}$$

becomes negative, providing an undesired phase shift. It is then probably here that we need to intervene. An idea could be to introduce a pole in the second term which keeps the expression positive or, equivalently,

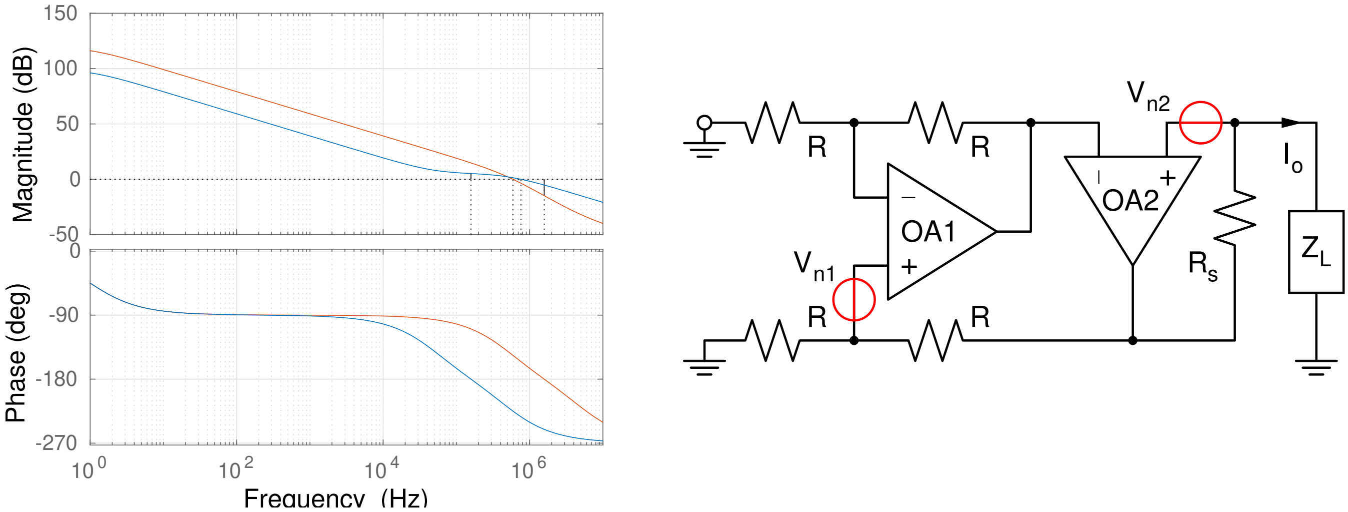


Figure 1: Left: Bode diagrams for $-G_{loop}$ when $R_L = 10R_s$ (blue) and $R_L = 0.1R_s$ (red). Right = circuit for noise calculations.

which loosens the positive feedback on OA2, leaving a truly negative feedback at all frequencies. To this aim, we could place a capacitor C_c in parallel to R_L, obtaining

$$G_{loop} = -\,A(s)\left(\frac{1}{1+s\tau_c} - \frac{R_L}{R_L+R_s}\frac{1}{1+sC_c(R_s \parallel R_L)}\right) = -A(s)\left(\frac{1}{1+s\tau_c} - \frac{k}{1+sC_cR_sk}\right) =$$
$$-\,A(s)\frac{1-k+sk(C_cR_s-\tau_c)}{(1+s\tau_c)(1+sC_cR_sk)},$$

where $k = R_L/(R_s+R_L)$. Clearly, the zero moves to the left half-plane if $C_cR_s > \tau_c$. For large values of R_L, i.e., k close to one, the zero will fall before the poles, increasing the phase margin. Note, however, that the pole at $GBWP/2$ still limits the maximum phase margin, so even OA1 should be compensated. A simple capacitor in parallel to its feedback resistor will move the pole to $GBWP$, increasing the phase margin to $45°$. Do not forget to place an analogous capacitor in parallel to the resistor at the output of OA2, to retain symmetry.

Problem 2

2.1

The generic weighting function is sketched in Fig. 2 (left). if we set $t = 0$ at the beginning of the first pulse, its Fourier transform becomes:

$$W(t,f) = T\,\mathrm{sinc}(\pi f T)\sum_{n=1}^{4} w_n e^{-j2\pi f t_n},$$

where $t_n = (n - .5)T$.

2.2

For a constant signal with white noise, the best choice is to have all weights equal, leading to an equivalent gated integrator working over an interval equal to $4T$:

$$\frac{S}{N} = \frac{A}{\sqrt{\lambda}}\sqrt{4T}.$$

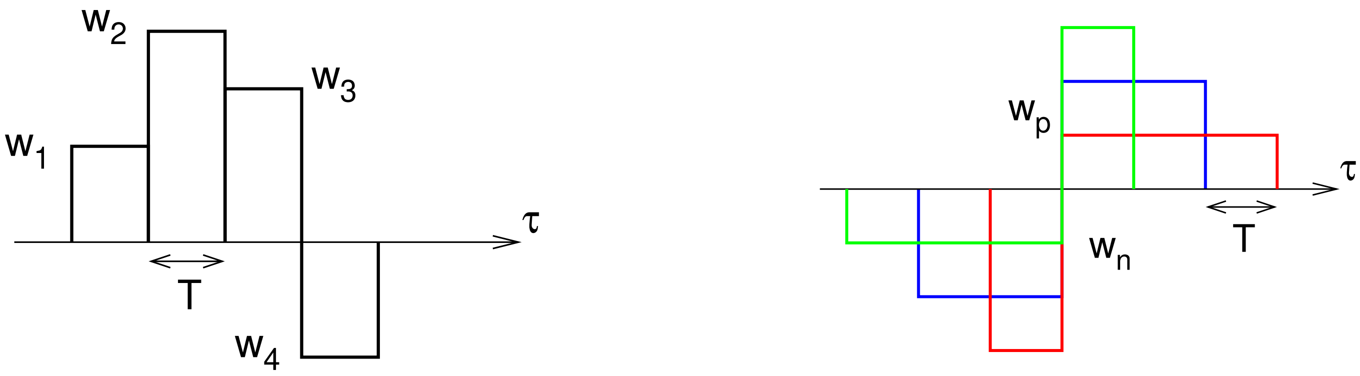


Figure 2: Left = weighting function in the time domain. Right = weigting functions for offset cancellation and white noise filtering.

2.3

We now need to subtract the offset while filtering the white noise. This can be done with the arrangement of Fig. 2 (right), in which we sample the offset before the signal and subtract it later (the step signal begins at the transition from negative weights w_n to positive ones w_p). The negative part can be T, $2T$ or $3T$ long, as shown in Fig. 2 (right). Let's call it nT, where $n = 1, 2, 3$. The positive integration time is then $(4-n)T$ and offset nulling requires that

$$nTw_n = (4-n)Tw_p.$$

The resulting S/N is then easily obtained:

$$\frac{S}{N} = \frac{A(4-n)Tw_p}{\sqrt{\lambda nTw_n^2 + \lambda(4-n)Tw_p^2}} = A\sqrt{\frac{T}{\lambda}}\frac{\sqrt{n(4-n)}}{2},$$

which is maximum for $n = 2$ (blue curve in Fig. 2, right). The result is not strange: we know that uniform weighting is best for white noise.

2.4

To compute the weighting function, we set $t = 0$ at the transition between negative and positive weighs for simplicity, and obtain

$$|W(t,f)| = \left|2Tw\ \mathrm{sinc}(2\pi fT)\left(e^{-j2\pi fT} - e^{j2\pi fT}\right)\right| = |-4jTw\ \mathrm{sinc}(2\pi fT)\sin(2\pi fT)| = 4Tw\frac{\sin^2(2\pi fT)}{2\pi fT},$$

so that the output noise is given by

$$\overline{n^2} = \int_0^\infty |W(t,f)|^2\frac{K}{f}df = K(4Tw)^2\int_0^\infty \frac{\sin^4 x}{x^3}dx \approx 11.2\,K(Tw)^2.$$

The resulting S/N is then:

$$\frac{S}{N} = \frac{2ATw}{\sqrt{11.2K(Tw)^2}} \approx 0.6\frac{A}{\sqrt{K}}.$$