Electronics – July 20 2017

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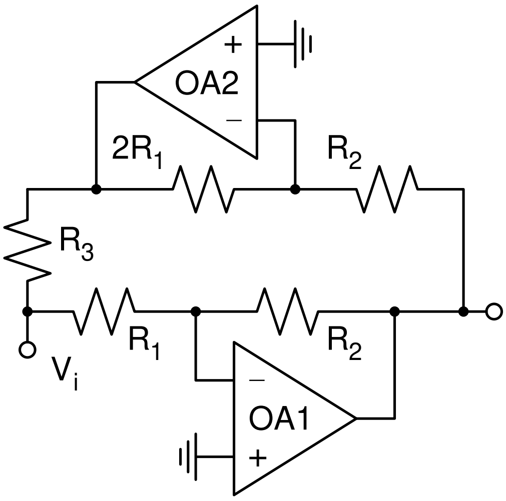

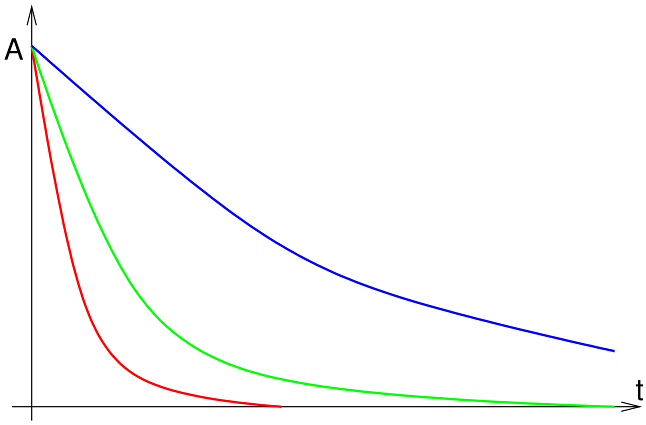

Problem 1

The scheme in the left figure is used to modify the input impedance of the inverting amplifier built with OA1. The amplifiers have $A_o = 120$ dB and pole at 1 Hz.

1. Find the (ideal) input impedance of the stage.
2. Compute the loop gain of OA2 considering OA1 as ideal when the input voltage source has a series resistance R_s. Discuss the condition on R_3 that ensures stability.
3. Compute the output voltage noise PSD due to the OAs voltage noise S_V and discuss the dependence on the source resistance R_s (consider the limiting cases of $R_s = 0$ and $R_s = \infty$).
4. Consider $R_2/R_1 = 10$ and $R_s = 0$. What is the effect of a second pole in $A(s)$ located at 1 MHz?

Problem 2

In an experiment, the time constant T of a decaying exponential signal $Ae^{-t/T}$ (right figure) has to be measured. Such time constant can range from 10 μs to 1 ms. The amplitude of the signal is $A = 1$ mV, superimposed to a white noise with bilateral spectral density of 10^{-12} V^2/Hz.

1. A gated integrator is used. Find a suitable value for the (constant) integration time and compute the resulting S/N.
2. A filter with a fixed exponential weighting function is used. Find a suitable value for the time constant of the filter and compute the new values of S/N.
3. The time constant of the signal has to be measured with a resolution of 10% (i.e., we need to discriminate $T = 1$ ms from $T = 0.9$ ms, but $T = 10$ μs from $T = 11$ μs) using a boxcar averager with repetitive measurements. Find the value of N_{eq}.
4. A CR filter is inserted before the BA stage, to eliminate offset and low-frequency noise. Find a value for its time constant if the pulse repetition rate is 1 s. Do we need a baseline restorer?

Question

Describe the baseline restorer working principle.

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by July 25th

Mark registration: Thursday, July 27th

Solution

Problem 1

1.1

The output of OA1 is obviously $-V_i R_2/R_1$, while the second stage has a gain of $-2R_1/R_2$, meaning that the output of OA2 is $2V_i$. The net current sourced by V_i is then

$$I = V_i\left(\frac{1}{R_1} - \frac{1}{R_3}\right) \Rightarrow R_i = \frac{1}{\frac{1}{R_1} - \frac{1}{R_3}} = \frac{R_1}{1 - R_1/R_3}.$$

Clearly, decreasing R_3 improves the input impedance, up to the point where $R_3 = R_1$ which leads to $R_i = \infty$. Beyond this value we would get a negative input impedance, meaning that we should evaluate the stability.

1.2

We break the loop at the ouput of OA2 and obtain, after a few calculations

$$G_{loop} = -A(s)\left(\frac{R_2}{R_2 + 2R_1} - \frac{R_1 \parallel R_s}{R_1 \parallel R_s + R_3}\frac{R_2}{R_1}\frac{2R_1}{R_2 + 2R_1}\right) = -A(s)\frac{R_2}{R_2 + 2R_1}\left(1 - 2\frac{R_1 \parallel R_s}{R_1 \parallel R_s + R_3}\right).$$

This loop is always stable if the term between the parentheses remains positive. It must therefore be:

$$R_3 > R_1 \parallel R_s.$$

It is interesting to interpret this condition: If we plug the limiting condition into the previous expression for R_i, we obtain

$$R_i = \frac{R_1}{1 - R_1/(R_1 \parallel R_s)} = -R_s,$$

which means that the total impedance seen by the input voltage source is equal to zero. Smaller values of R_3 make the overall impedance negative, bringing the circuit to instability.

1.3

The case of $R_s = 0$ is straightforward, as the upper stage and R_3 are connected between the output and ground and give no contribution to the output noise. We have then

$$S_{V_o} = S_{V_1}\left(1 + \frac{R_2}{R_1}\right)^2.$$

The other case is a bit more complicated, so we should solve the circuit in Fig. 1 (left). The upper block sets:

$$V_A = -V_o\frac{2R_1}{R_2} + V_{n2}\left(1 + \frac{2R_1}{R_2}\right)$$

while the lower gives:

$$V_o = -V_A\frac{R_2}{R_1 + R_3} + V_{n1}\left(1 + \frac{R_2}{R_1 + R_3}\right),$$

leading to

$$S_{V_o} = \frac{S_{V_1}(R_1 + R_2 + R_3)^2 + S_{V_2}(R_2 + 2R_1)^2}{(R_3 - R_1)^2}.$$

The noise is indeed reduced for very large values of R_3, but the circuit does not really work in this situation, as the uper part is isolated from the rest (or, equivalently, the input impedance is not affected, see the solution of 1.1). If we consider, for example, $R_3 = 2R_1$ (i.e., we double the input impedance), we get

$$S_{V_o} = (S_{V_1} + S_{V_2})\left(2 + \frac{R_2}{R_1}\right)^2,$$

larger than the previous one.

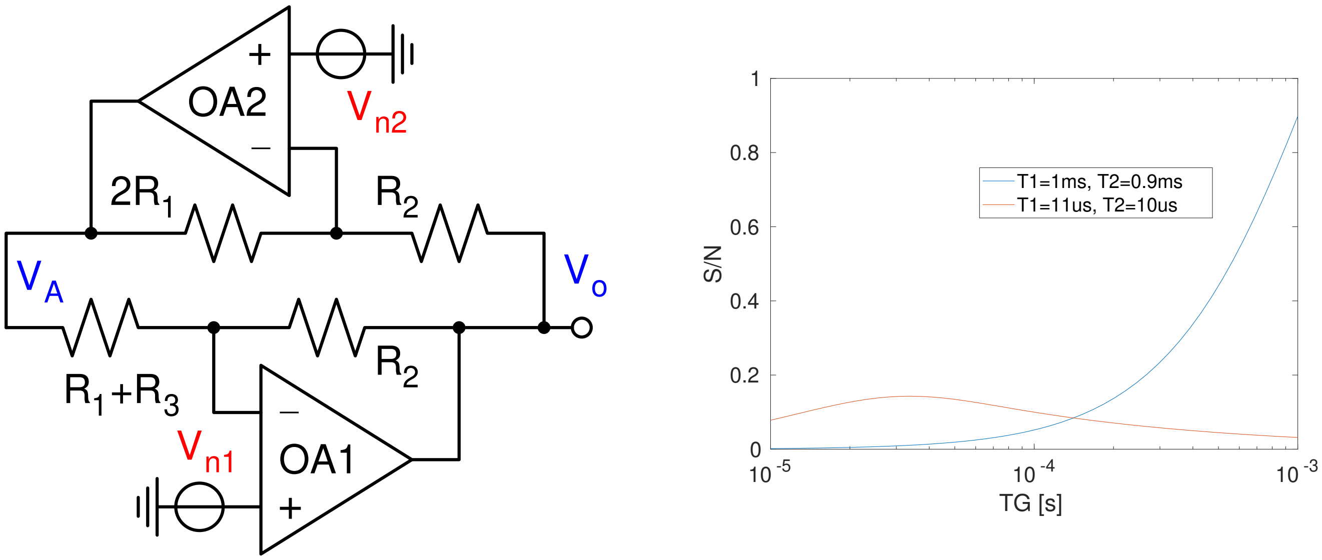

Figure 1: Left: Circuit for noise calculations when $R_s = \infty$. Right = S/N as a function of T_G when a resolution of 10% is required.

1.4

If $R_s = 0$ we just have two OAs connected as inverting amplifiers. OA1 has a gain of -10, hence a closed-loop bandwidth equal to $GBWP/11 \approx 91$ kHz. The second pole at 1 MHz will reduce its phase margin by $\arctan(1/11) \approx 5°$.
OA2 has a gain of -0.2, meaning that its loop gain will be $-(5/6)A(s) = -0.83A(s) \Rightarrow f_{0dB} = 830$ kHz. The second pole will now contribute with $\arctan(0.83) \approx 40°$, leaving a phase margin of about 60°.

Problem 2

2.1

The expression for S/N is

$$\frac{S}{N} = A\frac{T(1-e^{-T_G/T})}{\sqrt{\lambda T_G}},$$

where T_G is the gate time. If we pick $T_G = 1$ ms, we have $S/N \approx 20$ for $T = 1$ ms, but

$$\frac{S}{N} \approx A\frac{T}{\sqrt{\lambda T_G}}$$

for short T, which is nearly 0.3 for $T = 10$ μs. if T_G is reduced to 100 μs, we can raise this value to 1 while getting $S/N = 9.5$ for $T = 1$ ms, so everything seems fine. If T_G is further reduced to 10 μs, the two S/N values become nearly 2 and 3.

2.2

The output signal is now

$$y(t) = \int x(\tau)w(t,\tau)d\tau = AK\int e^{-\tau\left(\frac{1}{T}+\frac{1}{T_F}\right)}d\tau = AK\frac{TT_F}{T+T_F},$$

where K is the initial value of the weighting function. The noise is

$$\overline{n_y^2} = \lambda\int w^2(t,\tau)d\tau = \lambda K^2\frac{T_F}{2},$$

meaning that

$$\frac{S}{N} = A\sqrt{\frac{2}{\lambda}}\frac{T\sqrt{T_F}}{T+T_F}.$$

If we keep $T_F = 100$ μs, the two values of S/N become

$$T = 10\ \mu\text{s} \Rightarrow \frac{S}{N} \approx 1.3$$
$$T = 1\ \text{ms} \Rightarrow \frac{S}{N} \approx 14,$$

while for $T_F = 10$ μs we have

$$T = 10\ \mu\text{s} \Rightarrow \frac{S}{N} \approx 2.2$$
$$T = 1\ \text{ms} \Rightarrow \frac{S}{N} \approx 4.4.$$

S/N has improved somewhat with respect to the GI case as the filter is closer to the optimum.

2.3

For the GI case, the new S/N becomes

$$\frac{S}{N} = A\frac{T_1(1-e^{-T_G/T_1}) - T_2(1-e^{-T_G/T_2})}{\sqrt{\lambda T_G}},$$

where T_1 and T_2 are two time constants separated by 10%. Clearly, the signal is now smaller than before and the shorter values of $T_g = 10$ μs leads to very low values of S/N for large T (1.7×10^{-3} for 1 and 0.9 ms), meaning huge number of samples ($N_{eq} \approx 3.5 \times 10^5$) which is usually unfeasible because of drift, stability and so on.
It is then wiser to bring back T_G to 100 μs, where we get $S/N = 0.052$ and 0.099 for the highest and lowest T values (note that the smaller values are obtained for long T). In the worst case we would need $N_{eq} = (1/0.052)^2 = 370$. If T_G is doubled, we get $S/N = 0.14$ and 0.07, respectively, meaning that we can achieve the desired result with 200 equivalent samples. The two S/N are shown as a function of T_G in Fig. 1 (right).

2.4

The CR time constant must satisfy two requirements: on the one hand, it has to be longer than the gate time $T_G = 0.2$ ms, in order not to filter the individual pulses; on the other hand, it must be significantly shorter than the pulse separation time T_p, so that a baseline restorer is not required. This means:

$$T_G \ll CR \ll T_P \Rightarrow 0.2\ \text{ms} \ll CR \ll 1\ \text{s}.$$

For example, $CR \approx 10$ ms could be a good choice.