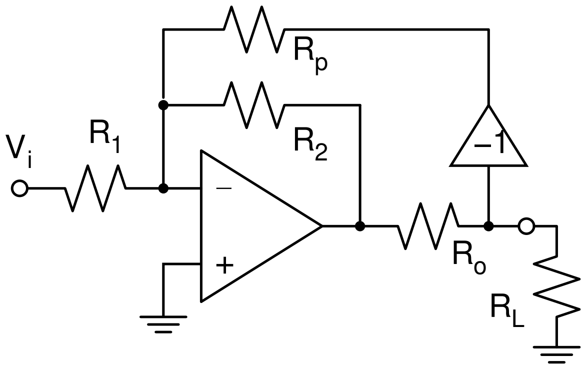
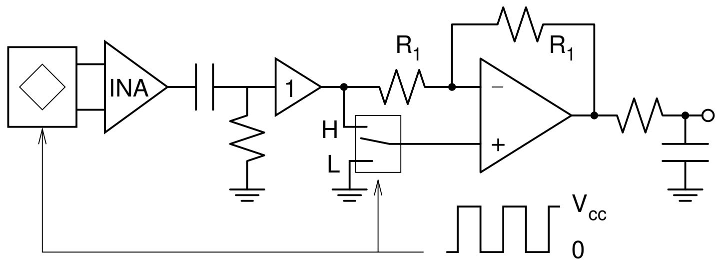
Electronics – September 14 2017 Prof. A. Spinelli





Problem 1

The scheme in the left figure is used to modify the output impedance of the inverting amplifier built with the OA. The amplifier has $A_o = 120$ dB and pole at 1 Hz.

1. Find the (ideal) gain of the stage.
2. Compute the loop gains of the OA and of the external loop (considering the OA as ideal).
3. Compute the output voltage noise PSD considering the equivalent voltage noise S_V of the OA and buffer.
4. Compute the output impedance and comment on the result. Consider the OA as ideal, but remember that there is an external feedback loop.

Problem 2

The right figure shows a simple, low-cost LIA with square-wave modulation at $f_R = 1.01$ kHz. The sensor is a Wheatstone bridge with full-scale sensitivity of 10 mV/V. The output RC filter has a 1 Hz bandwidth. Power supply is $V_{cc} = 5$ V. The output is connected to a 12 bit ADC.

1. Sketch the quoted full-scale voltage waveforms at the INA input, buffer stage input and LIA output. Find the gain of the INA for the ADC input to range between zero and V_{cc}.
2. The INA has input voltage noise $S_V = K/f$ with $K = 10^{-9}$ V^2. Find the output S/N (neglect flicker noise transmission through the high-order windows of the square wave).
3. What is the output noise due to thermal noise of resistors (including the CR filter) and (white) voltage noise of the OA?
4. The OA has offset voltage $V_{OS} = 10$ μV at 25°C. What is the maximum value of its offset voltage drift that grants an error smaller than 10% on the LSB if the operating temperature changes by 50°C at most?

Question

Describe the common-mode rejection problem in OAs: what it is, what are the relevant parameters and their dependences.

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by September 19th **Mark registration: Friday, Sep 22nd**

Solution

Problem 1

1.1

OA1 can be regarded as an adder circuit working with inputs V_i and $-V_o$, so that its output V_{OA} is

$$V_{OA} = -\frac{R_2}{R_1}V_i + \frac{R_2}{R_p}V_o.$$

Considering that

$$V_o = V_{OA}\frac{R_L}{R_L + R_o},$$

we get

$$\frac{V_o}{V_i} = \frac{-R_2/R_1}{1 + \dfrac{R_o}{R_L} - \dfrac{R_2}{R_p}}.$$

1.2

We break the loop at the ouput of the OA and easily obtain

$$G_{loop} = -A(s)\left(\frac{R_1 \parallel R_p}{R_1 \parallel R_p + R_2} - \frac{R_L}{R_L + R_o}\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_p}\right) = -A(s)\frac{R_1 R_2}{R_1 R_2 + R_1 R_p + R_2 R_p}\left(\frac{R_p}{R_2} - \frac{R_L}{R_L + R_o}\right),$$

which is stable if

$$R_p > \frac{R_L R_2}{R_L + R_o}.$$

The gain of the external loop can be computed by breaking it at the inverting stage output, obtaining:

$$G_{loop}^{ext} = \frac{R_2}{R_p}\frac{R_L}{R_L + R_o}.$$

Note that the previous stability condition is equivalent to requiring that the external (positive) feedback loop have a gain smaller than one.

1.3

Here we include also resistor noise for the sake of completeness. This is expressed via current sources, rearranged to end up with the scheme in Fig. 1 (left), where the noise sources are

$$S_{V_1} = S_V \qquad S_{V_2} = S_V \qquad S_{I_1} = \frac{4k_BT}{R_1} + \frac{4k_BT}{R_2} + \frac{4k_BT}{R_p} \qquad S_{I_2} = \frac{4k_BT}{R_o} + \frac{4k_BT}{R_s},$$

The transfers can be obtained by usual circuit inspection but, for the sake of brushing up the feedback circuit theory, we will open the external loop by disconnecting and grounding the inverting buffer stage and compute the open-loop transfers, dividing them by the factor $1 - G_{loop}^{ext}$. The open-loop transfers are

$$V_o^{OL} = \left[V_{n1}\left(1 + \frac{R_2}{R_1 \parallel R_p}\right) + V_{n2}\frac{R_2}{R_p} + I_{n1}R_2\right]\frac{R_L}{R_o + R_L} + I_{n2}(R_o \parallel R_L),$$

leading to

$$S_{V_o} = \left[S_{V_1}\left(1 + \frac{R_2}{R_1 \parallel R_p}\right)^2 + S_{V_2}\left(\frac{R_2}{R_p}\right)^2 + S_{I_1}R_2^2 + S_{I_2}R_o^2\right]\left(\frac{R_L}{R_o + R_L}\right)^2\frac{1}{\left(1 - \dfrac{R_2}{R_p}\dfrac{R_L}{R_o + R_L}\right)^2}.$$

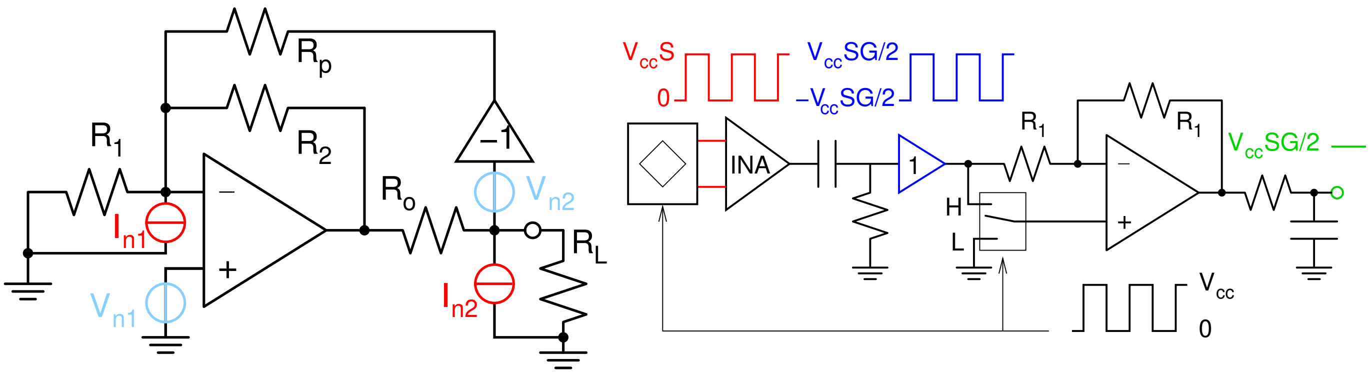


Figure 1: Left: Circuit for noise calculations when $R_s = \infty$. Right = full-scale waveforms in the LIA.

1.4

The open-loop resistance is obviously R_o (the loop here is the external one) and the loop gain in this condition (R_L disconnected) is easily obtained by setting $R_L \to \infty$ in the expression of G_{loop}^{ext}, resulting in R_2/R_p. When the loop gain approaches one, the output resistance increases, leading to the expression

$$R_{out} = \frac{R_o}{1 - \dfrac{R_2}{R_p}}.$$

Obviously, the total resistance $R_{out} + R_L$ must be positive, leading to the same condition discussed for the loop stability (see also the previous exam test).

Problem 2

2.1

The voltage at the output of the bridge is obviously a square-wave signal with full-scale value

$$V_{in} = V_{cc}S = 50 \text{ mV},$$

where $S = 10$ mV/V is the full-scale sensitivity of the bridge (red signal in Fig. 1, right).
The CR filter acts as an AC-coupling stage, removing the DC component of the square wave and leaving a symmetric square wave (blue signal; G is the INA gain). The OA stage switches between gain of $+1$ (switch on H) and -1 (switch on L), providing the synchronous demodulation and resulting into a DC output signal of amplitude $V_{cc}SG/2$ (green curve). The INA gain must then be:

$$V_o^{max} = V_{cc} \Rightarrow G = \frac{2}{S} = 200.$$

2.2

We need to consider the minimum signal, which is obviously

$$V_{min} = \frac{V_{cc}SG/2}{2^{12}} = \frac{5}{2^{12}} = 1.2 \text{ mV}$$

at the demodulator output (at the bridge output it is $0.05/2^{12} = 12.2$ μV). The noise at the demodulator input is

$$S_n = \frac{K}{f}G^2,$$

where we can neglect the effect of the CR filter because of the demodulation (the zero-frequency spectrum will be filtered by the output RC). We perform here a full analysis, considering all frequency contributions:

the demodulation signal (symmetric square wave with unit amplitude) transform can be written as

$$W_R(f) = \frac{2}{\pi} \sum_k \frac{(-1)^k}{2k+1} \left(\delta(f - (2k+1) f_R) + \delta(f + (2k+1) f_R) \right),$$

with time correlation

$$S_{w_R}(f) = \left(\frac{2}{\pi} \right)^2 \sum_k \frac{1}{(2k+1)^2} \left(\delta(f - (2k+1) f_R) + \delta(f + (2k+1) f_R) \right).$$

We then immediately obtain (see equations in lesson slides)

$$\overline{n_{out}^2} = 4\, BW_n\, G^2 \left(\frac{2}{\pi} \right)^2 \sum_k \frac{1}{(2k+1)^2} \frac{K}{(2k+1) 2 f_R} = BW_n\, G^2 \left(\frac{4}{\pi} \right)^2 \frac{K}{2 f_R} 1.052,$$

meaning that flicker noise transmission via higher frequency windows amounts to an excess noise of about 5% and is indeed negligible. Note that there is factor of two at the denominator because the flicker noise of the amplifier is a unilateral PSD. The result for S/N is then

$$\frac{S}{N} \approx \frac{\pi V_{cc} S / 2^{12}}{8 \sqrt{BW_n \frac{K}{2 f_R}}} \approx 5.2.$$

2.3

The noise of the CR filter is located at low frequencies and is pulled off-band by the demodulation, so it gives no contribution. The output noise contributions due to resistors R_1, OA and output resistor R are instead not affected by the demodulation and amount to

$$\overline{n_{out}^2} = (2 S_{V,R_1} + 4 S_V + S_{V,R}) BW_n,$$

meaning that they are negligible when

$$2 S_{V,R_1} + 4 S_V + S_{V,R} \ll G^2 \left(\frac{4}{\pi} \right)^2 \frac{K}{2 f_R} \approx 3.3 \times 10^{-8}\ \mathrm{V^2/Hz}.$$

The condition is easily satisfied. In fact, such noise sources must be compared with the amplified INA noise and are expected to be negligible.

2.4

The LSB value is 1.2 mV, meaning that the error due to the offset cannot exceed one tenth of it, i.e., 120 μV. Noting that the output voltage is two times the offset voltage of the OA, the limit becomes 60 μV. Subtracting the room-temperature value, we have a maximum drift of

$$\frac{dV_{OS}}{dT} = \frac{50}{50} = 1\ \mu\mathrm{V/^\circ C}.$$

This value is small but attainable in precision OAs.