Electronics – July 20 2018

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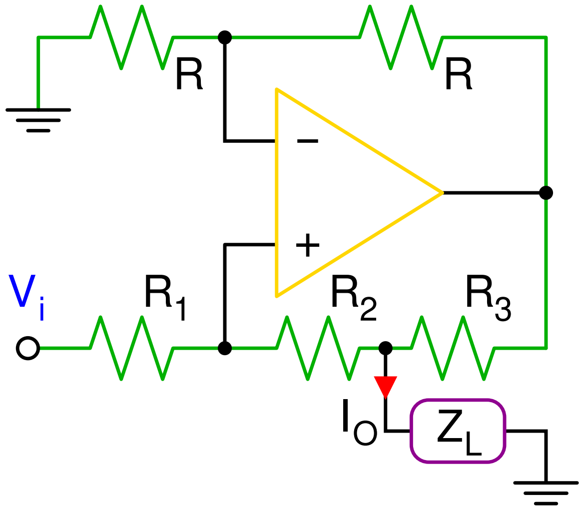

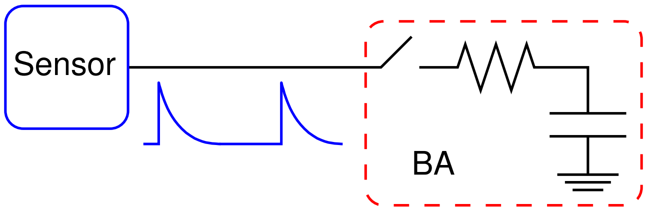

Problem 1

The scheme in the left figure is an improved Howland current source. Component values are $R = 100$ kΩ. The CMOS OA has $A_o = 120$ dB and $GBWP = 1.5$ MHz.

1. Find the condition (and set the values) on R_1, R_2 and R_3 that makes the circuit an <u>ideal</u> current source. Hint: I_O must *not* be dependent on Z_L, so find the expression of I_O and zero the term multiplying Z_L.
2. Compute the DC output impedance of the current source.
3. Compute the output current noise PSD considering the equivalent voltage noise sources $\sqrt{S_V} = 40\ \text{nV}/\sqrt{\text{Hz}}$ and $\sqrt{S_I} = 30\ \text{fA}/\sqrt{\text{Hz}}$.
4. Find the expression of the <u>ideal</u> output impedance when the matching condition of 1.1 is <u>not</u> met. Hint: simply use and make sense of the result of 1.1!

Problem 2

A sensor produces periodic exponential pulses with amplitude $A \approx 10\ \mu$V and time constant $T \approx 1\ \mu$s, affected by white noise with bilateral PSD $\lambda = 10^{-14}\ \text{V}^2/\text{Hz}$. A boxcar averager is used to achieve $S/N \geq 10$.

1. Find a set of the BA parameter values (T_C, T_F, integration window) that satisfies the requirement on S/N.
2. Find the value of the pulse repetition rate f_p that allows to complete the measurement in a time $T_M = 10$ s.
3. With reference to the previous requirement on T_M, find the <u>minimum</u> value of f_p (and corresponding values of the BA parameters). Remember that the signal is not constant and use <u>justified</u> approximations.
4. An input LF noise has exponential autocorrelation $R_{nn}(\tau) = \overline{n^2}e^{-|\tau|/T_n}$, with $T_n \gg 1/f_p$. Find the expression of the output rms noise (use the weighting function time correlation and remember that $T_F \gg T_C$).

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by July 26th

Mark registration: Friday, July 27th

Solution

Problem 1

1.1

If we call V_1 the output voltage of the OA, we have $V^- = V^+ = V_1/2$. Since $I_o Z_L$ is the voltage at the R_2-R_3 midpoint, we write the KCL at such node as

$$\frac{V_1 - I_o Z_L}{R_3} + \frac{V_1/2 - I_o Z_L}{R_2} = I_o \Rightarrow I_o \left(1 + \frac{Z_L}{R_2} + \frac{Z_L}{R_3}\right) = \frac{V_1}{2}\left(\frac{1}{R_2} + \frac{2}{R_3}\right).$$

To eliminate V_1, we write the KCL at the non-inverting input of the OA, that can be reshaped as:

$$V^+ = \frac{V_1}{2} = V_i \frac{R_2}{R_1 + R_2} + I_o Z_L \frac{R_1}{R_1 + R_2}.$$

Substituting into the first equation, we obtain:

$$I_o \left(1 + \frac{Z_L}{R_2} + \frac{Z_L}{R_3} - \frac{Z_L R_1}{R_1 + R_2}\left(\frac{1}{R_2} + \frac{2}{R_3}\right)\right) = V_i \frac{1}{R_1 + R_2}\left(1 + \frac{2R_2}{R_3}\right).$$

The term multiplying Z_L (that ust be zeroed) is

$$\frac{1}{R_1 + R_2} + \frac{1}{R_3}\left(1 - \frac{2R_1}{R_1 + R_2}\right) = 0 \Rightarrow R_1 = R_2 + R_3,$$

giving

$$I_o = \frac{V_i}{R_3}.$$

We can take $R_1 = 101\ \mathrm{k\Omega}$, $R_2 = 100\ \mathrm{k\Omega}$ and $R_3 = 1\ \mathrm{k\Omega}$, to maximize the transconductance.

1.2

The matched circuit is an ideal current source, i.e., with infinite output impedance. We remove Z_L and easily obtain:

$$Z_{OL} = (R_1 + R_2) \parallel R_3 \approx R_3 \qquad G_{loop} = -\frac{A(s)}{2},$$

from which, at low frequencies:

$$Z = Z_{OL}(1 - G_{loop}(0)) = 500\ \mathrm{M\Omega}.$$

1.3

The output current is independent of Z_L, so we pick the easiest case, i.e., $Z_L = 0$. The relative circuit is shown in Fig. 1, left. The resulting transfers are:

$$I_o = \frac{2V_n}{R_3} + I_n^+\left(\frac{R_1}{R_1 + R_2} + \frac{2(R_1 \parallel R_2)}{R_3}\right) + I_n^- \frac{R}{R_3} = V_n \frac{2}{R_3} + I_n^+ \frac{R_1}{R_3} + I_n^- \frac{R}{R_3}.$$

The PSD becomes then

$$\sqrt{S_{I_o}} = \sqrt{6.4 \times 10^{-21} + 9.2 \times 10^{-24} + 9 \times 10^{-24}} \approx 80\ \mathrm{pA}/\sqrt{\mathrm{Hz}}.$$

Note that the OA current noise is negligible, not surprising in a CMOS OA. Furthermore, resistor noise is nearly $40\ \mathrm{pA}/\sqrt{\mathrm{Hz}}$ (for $R = R_2 \approx R_1$), one order of magnitude larger than the OA current noise and with almost the same transfer. However, this will not affect significantly the result, as the toal noise is mainly determined by the OA voltage noise. The current noise of R_3 flows directly into the output and its value is about $8 \times 10^{-24}\ \mathrm{A^2/Hz}$, again negligible.

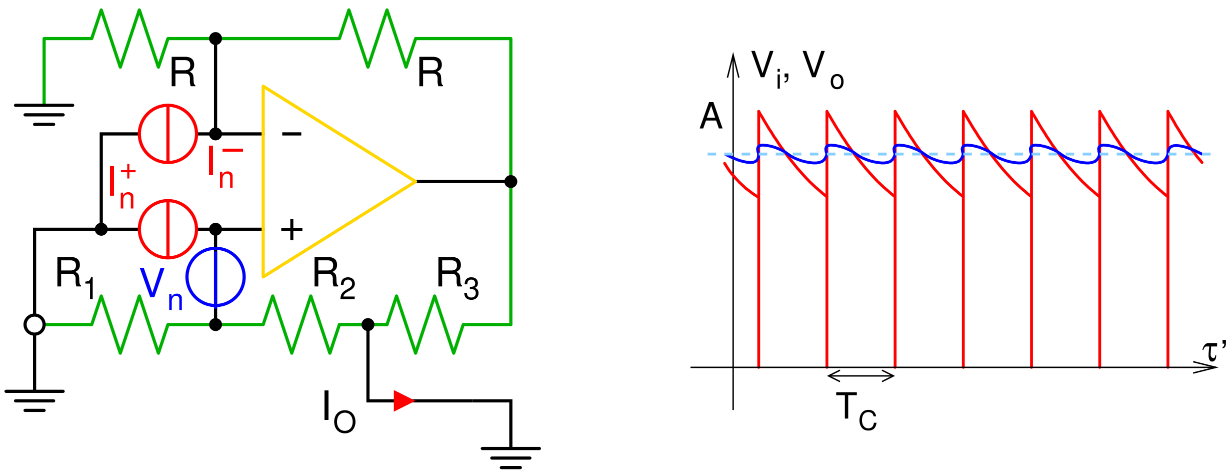

Figure 1: Left = Scheme for noise calculation. Right = Input (red) and output (blue) signals of the BA in the equivalent time.

1.4

The final expression of I_O in 1.1 can be written as:

$$I_o\left(1+\frac{Z_L}{Z_o}\right) = G_m V_i \Rightarrow I_o = G_m V_i \frac{Z_o}{Z_o+Z_L},$$

from which it is obvious that Z_o is indeed the output impedance of the stage (think of the Norton equivalent of the current source). With simple rearrangements, its expression becomes

$$Z_o = \frac{R_3(R_1+R_2)}{R_2+R_3-R_1}.$$

Note that this is the actual value of the impedance that you get, because of tolerances in resistor values.

Problem 2

2.1

The simplest choice is to keep T_C much shorter than T and integrate synchronously with the pulse edge, so that the final boxcar signal will reach the value of A. S/N is hence given by:

$$\frac{S}{N} = \frac{A}{\sqrt{\lambda}}\sqrt{2T_F} \geq 10 \Rightarrow T_F \geq 50\frac{\lambda}{A^2} = 5 \text{ ms}.$$

If we take – say – $T_C = 50$ ns and $T_F = 5$ ms, we have a number of equivalent samples $N_C = 2\times 10^5$.

2.2

The total number of pulses processed over the measurement time T_M is obviously $f_p T_M$. The corresponding "equivalent time" over which the switch is closed is then $f_p T_M T_C$, which must be enough to fully charge the capacitor. Since full charge takes place in about 5 time constants, we then set

$$f_p T_M T_C = 5T_F,$$

which returns $f_p = 50$ kHz.

2.3

The minimum value of f_p corresponds to the maximum single-pulse S/N. This is given by

$$\left(\frac{S}{N}\right)_{sp} = \frac{\displaystyle\int_0^{T_C} Ae^{-t/T} dt}{\sqrt{\lambda T_C}} = A\frac{T\left(1-e^{-T_C/T}\right)}{\sqrt{\lambda T_C}} = A\sqrt{\frac{T}{\lambda}}\frac{1-e^{-x}}{\sqrt{x}},$$

where $x = T_C/T$ cannot be too large or too small. A reasonable choice is hence $x \approx 1$, i.e., $T_C = T = 1$ μs. T_F can then be found from the expression of S/N:

$$\left(\frac{S}{N}\right) = \left(\frac{S}{N}\right)_{sp}\sqrt{\frac{2T_F}{T_C}} \geq 10 \Rightarrow \frac{A(1-e^{-1})}{\sqrt{\lambda}}\sqrt{2T_F} \geq 10 \Rightarrow T_F \geq 12.5 \text{ ms},$$

which gives

$$f_p = \frac{5T_F}{T_M T_C} = 6.25 \text{ kHz}.$$

The actual maximum is achieved for $x \approx 1.26$, leading to $T_F = 15.5$ ms and $f_p = 6.14$ kHz, not far away from the approximated result.

A slightly different approach would be to think in terms of the behavior in the equivalent time, where BA behaves as an $R-C$ filter. Within this reference, the non-constant input signal is shown in Fig. 1, right (red curve). The $R-C$ filter with $T_F \gg T_C$ will reach the average signal value (light-blue curve; in reality there are small oscillations that are negligible, blue curve in the figure), given by:

$$V_C = \frac{A}{T_C}\int_0^{T_C} e^{-t/T} dt = A\frac{T}{T_C}\left(1-e^{-T_C/T}\right).$$

The expression for S/N becomes then:

$$\frac{S}{N} = A\frac{1-e^{-x}}{x}\sqrt{\frac{2T_F}{\lambda}} \geq 10.$$

Since $T_F = f_p T_M T_C/5 = f_p T_M T x/5$, we eventually get

$$\frac{1-e^{-x}}{\sqrt{x}}\sqrt{f_p} \geq \frac{10}{A}\sqrt{\frac{5\lambda}{2T_M T}},$$

which is exactly the same equation obtained previously.

2.4

To compute the output rms noise we need the time correlation of the weighting function $w(t,\tau)$, which is made up of a series of (approximately) triangular pulses with basis $2T_C$ and amplitude decreasing with e^{-T_C/T_F}, leading to

$$\overline{n_{out}^2} = \int R_{nn}(\tau)k_{wtt}(\tau)d\tau = \frac{1}{2T_F}\sum\int R_{nn}(\tau - kT_p)e^{-|k|T_C/T_F}\text{tri}(T_C),$$

where $\text{tri}(T_C)$ is the triangular function of unit amplitude and $\pm T_C$ base. Because of the slow variation of R_{nn} we obtain (assuming unity gain for the BA)

$$\overline{n_{out}^2} \approx \frac{1}{2T_F}\sum_k R_{nn}(kT_p)e^{-|k|T_C/T_F}T_C = \overline{n_{in}^2}\frac{T_C}{2T_F}\sum_{k=-\infty}^{\infty} e^{-|k|\left(\frac{T_p}{T_n}+\frac{T_C}{T_F}\right)}.$$

From the expression for the sum of the geometric series, and recalling that the term in $k=0$ must be counted only once, we get

$$\overline{n_{out}^2} \approx \overline{n_{in}^2}\frac{T_C}{2T_F}\left(\frac{2}{1-e^{-\left(\frac{T_p}{T_n}+\frac{T_C}{T_F}\right)}} - 1\right) \approx \overline{n_{in}^2}\frac{1}{1+\frac{T_P}{T_n}\frac{T_F}{T_C}}.$$

Obviously, significant noise reduction occurs only if there is no strong correlation over the successive pulses.