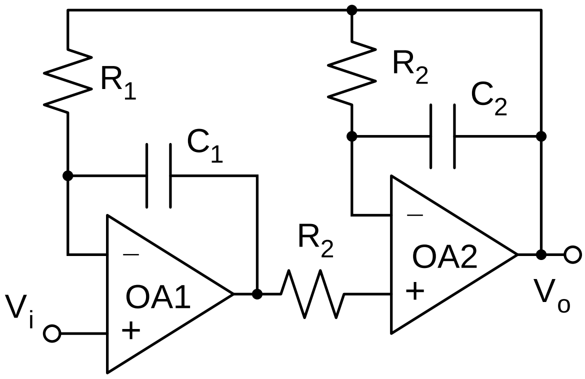
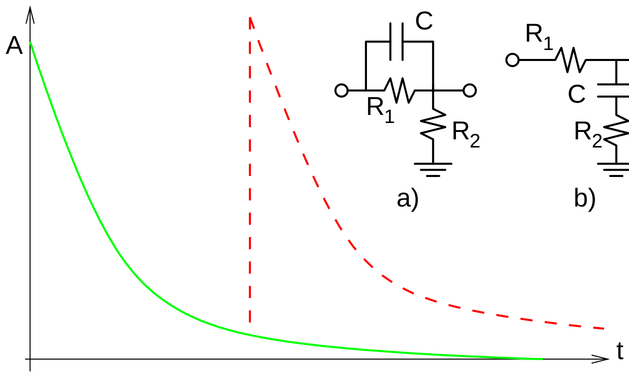
Electronics – February 21 2018 Prof. A. Spinelli





Problem 1

The scheme in the left figure is used as a low-noise reference for ADCs. Component values are $R_1 = 1\ \text{k}\Omega, C_1 = 4.7\ \mu\text{F}$, $R_2 = 750\ \Omega, C_2 = 47$ nF. The amplifiers have $A_o = 120$ dB and $GBWP = 1$ MHz.

1. Find the (ideal) gain of the stage.
2. Find the loop gains of OA1 and OA2 considering the other OA as ideal.
3. Compute the output rms voltage noise considering the equivalent voltage noise sources $\sqrt{S_V} = 10\ \text{nV}/\sqrt{\text{Hz}}$ plus a flicker noise component on S_{V_2}, with noise corner frequency of 1 kHz.
4. The circuit is used to drive a large capacitive load. Discuss the role of R_2 and C_2.

Problem 2

A sensor produces exponential pulses with amplitude A, time constant $\tau = 10\ \mu$s and repetition frequency $f_r \approx 50$ kHz (right figure). A white noise with bilateral PSD λ is superimposed onto the pulses.

1. Consider a single pulse (no repetition). Design a simple LPF and compute the resulting S/N.
2. Consider the pile-up error introduced by the repetitive pulses (dashed line). Insert the correct block – a) or b) – and find the component values to reduce such error below 1‰ (hint: work in the frequency domain). Re-design the LPF and comment on the new S/N.
3. An optimum filter is applied in place of the LPF (after block a or b). Find the resulting value of S/N and discuss the result.
4. The pulse repetition rate is uniformly distributed in time (Poisson random process), with density γ [s^{-1}]. What is the average value of the pile-up error (hint: the pdf of the inter-pulse arrival time is $\gamma e^{-\gamma t}$)?

Question

Describe the compensation techniques for capacitive loading of OAs.

For a correct evaluation, you are asked to write your answers in a readable way; thank you
Do a good job!

Results will be posted by February 23$^{\text{th}}$ **Mark registration: Monday, Feb 26$^{\text{th}}$**

Solution

Problem 1

1.1

We start by computing the output voltage of OA1, V_{o1}. This is given by

$$V_{o1} = -V_o \frac{1}{sC_1R_1} + V_i \left(1 + \frac{1}{sC_1R_1}\right),$$

and this voltage is also present at the non-inverting input of OA2. However, in R_2 and C_2 flows no current, so $V_{o1} = V_o$. It is then immediate to note that this equals to $V_o = V_i$, i.e., the stage behaves as a buffer.

1.2

The calculation is very simple for OA1, as OA2 behaves as a buffer in this case, leading to

$$G_{loop1} = -A(s).$$

When computing G_{loop2}, breaking the loop at the OA output, we have $V^- = V_s$ and $V^+ = -V_s/sC_1R_1$, because OA1 is connected as an integrator. This leads to

$$G_{loop2} = -A(s)\left(1 + \frac{1}{sC_1R_1}\right) = -A(s)\frac{1 + sC_1R_1}{sC_1R_1}.$$

Both loops are obviously stable. A discussion of the signal transfer from the viewpoint of the feedback theory can be found in the Appendix.

1.3

The noise source of OA1 is subjected to the same transfer as the input signal, i.e., $S_{V_o} = S_{V_1}$. To compute the output noise due to OA2, we ground the input and easily obtain:

$$S_{V_o} = S_{V_2}\left|\frac{sC_1R_1}{1 + sC_1R_1}\right|^2.$$

To compute the output rms voltage noise, we need to consider the poles at $GBWP$ introduced by the OAs, getting

$$\overline{V_o^2} = S_{V_1}\frac{\pi}{2}GBWP + S_{V_2}\frac{\pi}{2}(GBWP - f_p) \approx (17.7\ \mu\mathrm{V})^2,$$

where $f_p = 1/2\pi C_1R_1 \approx 34$ Hz is the C_1R_1 pole frequency. The flicker component of the noise of OA2 gives an additional contribution equal to

$$\overline{V_o^2} = K \ln\left(\frac{GBWP}{f_p}\right) = S_{V_2} f_{nc} \ln\left(\frac{GBWP}{f_p}\right) \approx (1\ \mu\mathrm{V})^2.$$

1.4

A large capacitive load can lead to instability due to the pole introduced with the output resistance of the OA. R_2 and C_2 can compensate the scheme, usually with the addition of a small-value (say, a few Ω) resistor R_C in series to the OA output, resulting in the full scheme of Fig. 1 (left). See class notes for further details.

Problem 2

2.1

A simple choice is to pick the time constant of the LPF to be much smaller than τ, say $T_F = 1\ \mu$s. An additional constraint could be to not increase the total noise because of the white resistor noise, i.e., to set

$$4k_BTR \ll 2\lambda.$$

The resulting S/N is then

$$\left(\frac{S}{N}\right) = \frac{A}{\sqrt{\lambda/2T_F}}.$$

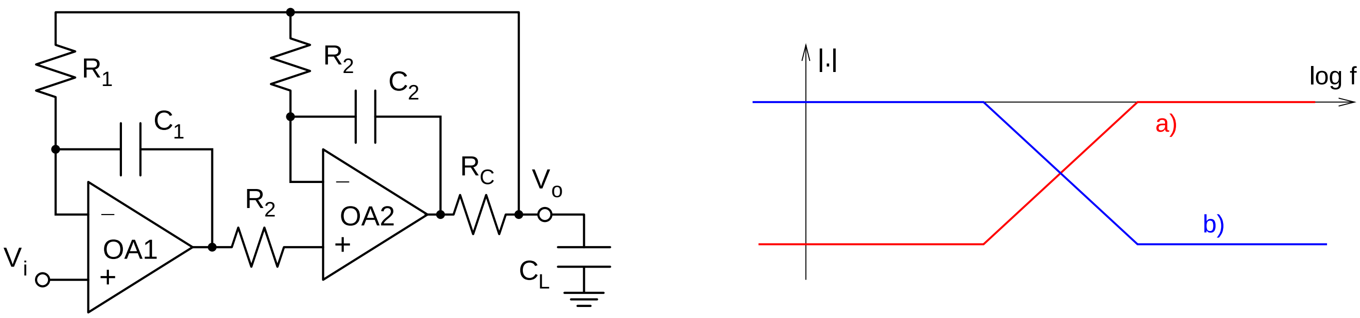


Figure 1: Left: Full circuit for driving large capacitive loads. Right: Bode plot of the transfer functions of schemes a) and b).

2.2

Let's first assess the magnitude of the error in pulse amplitude measurement. This is obviously given by the value of the previous pulse, i.e., $Ae^{-t_r/\tau}$, where $t_r = 1/f_r$ is the pulse separation time. The condition on the pile-up error becomes then

$$e^{-t_r/\tau} < 10^{-3} \Rightarrow t_r > 3\tau \ln 10 \approx 7\tau.$$

In our case, this condition means that τ should be smaller than 2.86 μs, actually being about 10 μs.
We can now look at the two proposed solutions. Their transfer functions are

$$H_a = \frac{R_2}{R_1 + R_2} \frac{1 + sCR_1}{1 + sC(R_1 \parallel R_2)} \qquad H_b = \frac{1 + sCR_2}{1 + sC(R_1 + R_2)},$$

and are basically pole-zero couples where $f_z < f_p$ (a) or $f_z > f_p$ (b), as shown in Fig. 1 (right). The Laplace transform of the input single-pulse signal is a single-pole function

$$V_i(s) = A\frac{\tau}{1 + s\tau},$$

so that we have

$$V_o(s) \propto A\frac{1}{1 + s\tau}\frac{1 + s\tau_z}{1 + s\tau_p}.$$

A good idea is to have a pole-zero cancellation, i.e., set $\tau_z = \tau = 10$ μs, which leaves us with a single-pole function, i.e., an exponential similar to the input signal but with time constant τ_p. To reduce the pile-up, we can then set $\tau_p < t_r/7 = 2.9$ μs $\to 2$ μs, meaning that we need solution a).
The actual output signal becomes now

$$V_o(s) = A\tau\frac{\tau_p}{\tau_z}\frac{1}{1 + s\tau_p} = A\frac{\tau_p}{1 + s\tau_p} \Rightarrow v_o(t) = Ae^{-t/\tau_p},$$

while the output noise is

$$\overline{v_o^2} = \lambda\frac{\pi}{2}\left(\frac{1}{25}f_z + f_F - f_p\right) \approx \lambda\frac{\pi}{2}f_F$$

where f_F is the new LPF pole, say, $f_F = 10f_p$. This means $T_F = \tau_p/10 = 0.2$ μs. S/N is smaller than in 2.1, but the pile-up error is gone.

2.3

Because of block a), the noise is no longer white and a whitening filter should be applied, but this would do nothing but bring us back to the original signal with white noise! So, we can just solve the optimum filter problem for the initial pulse, but first we must note that now we are back to the pile-up issue! This means that we must limit the integration time to $t_r = 20$ μs. We have then

$$\left(\frac{S}{N}\right) \approx \frac{A}{\sqrt{\lambda}}\sqrt{\int_0^{t_r} e^{-2t/\tau}dt} = A\sqrt{\frac{\tau(1 - e^{-2t_r/\tau})}{2\lambda}}.$$

The solution is approximate, as we are neglecting the pile-up contribution.

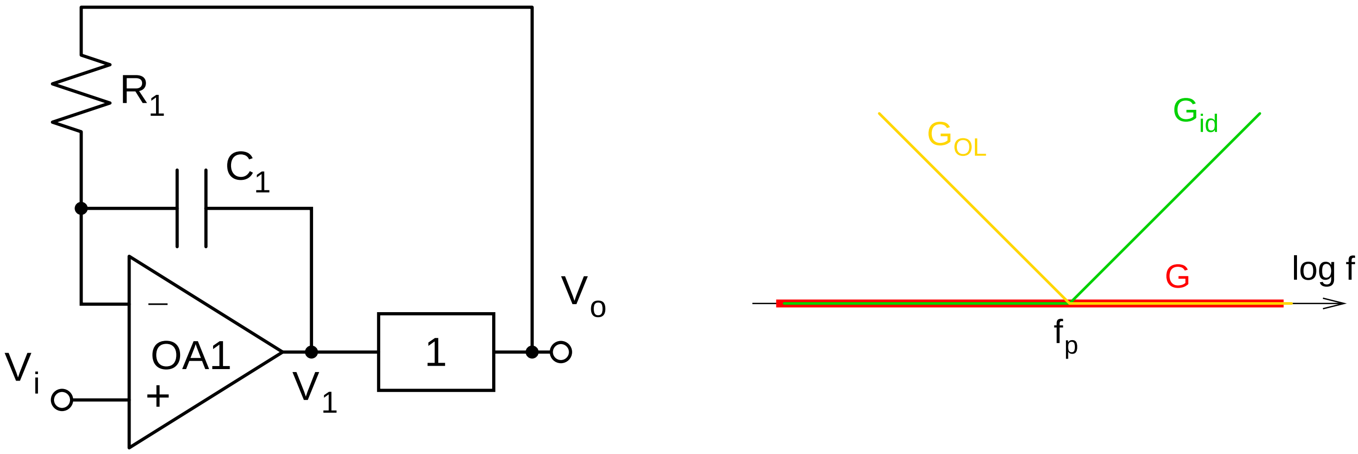


Figure 2: Left: Simplified circuit used in calculations. Right: Bode plot of the gains of the circuit with ideal OAs.

2.4

If a pulse arrives a time t after a previous one, the error in its estimate is $Ae^{-t/\tau}$. Since this event happens with pdf $\gamma e^{-\gamma t}$, the average value of the error is

$$\overline{\epsilon} = \int_0^\infty Ae^{-t/\tau}\gamma e^{-\gamma t}dt = A\gamma\int_0^\infty e^{-(\gamma+1/\tau)}dt = A\frac{\gamma\tau}{1+\gamma\tau}.$$

The relative error is $\frac{\gamma\tau}{1+\gamma\tau}$, conveying the obvious result that to reduce the pile-up error we must have $\gamma\tau \ll 1$, i.e., $\frac{1}{\gamma} \gg \tau$: the average pulse separation time must be much longer than τ.

Appendix

We – briefly – discuss here the complete behavior of the circuit, and particularly the role of the different loops. We begin by considering ideal OAs and note that R_2 and C_2 do not carry any current and can be taken out of the circuit, so that the second stage behaves as a follower with gain of 1. It is now easy to see that the *external* loop has a gain of $-1/s\tau_1$ where $\tau_1 = C_1R_1$, with 0 dB frequency located at $f_p = 1/(2\pi\tau_1)$. So, what is happening beyond this frequency? What is the open-loop and ideal gain? Let's start with computing G_{open}: we open the loop and ground the feedback connection from the output to R_1 and obtain

$$G_{open} = \frac{1+s\tau_1}{s\tau_1}.$$

The ideal gain now poses a problem: its definition is the gain obtained with *infinite* loop gain, while here the loop gain is $-1/s\tau_1$, definitely not infinite! How can we make the loop infinite? An idea is to look at Fig. 2 and replace the buffer stage with a gain stage with infinite gain. This means that V_1 must be always zero and the output becomes (keep in mind that the input pins of the OA must have the same potential)

$$G_{id} = 1 + s\tau_1.$$

The resulting loop gain (minimum between G_{id} and G_{open}) is plotted in Fig. 2.
The same exercise can be done for the input noise of OA2, obtaining (calculations are left to the reader, if any):

$$G_{open} = 1 \qquad G_{id} = s\tau_1.$$

We can now discuss the impact of the gain of OA1. This affects of course the loop gain and the open-loop gain, which now becomes

$$G_{OL} = G_1,$$

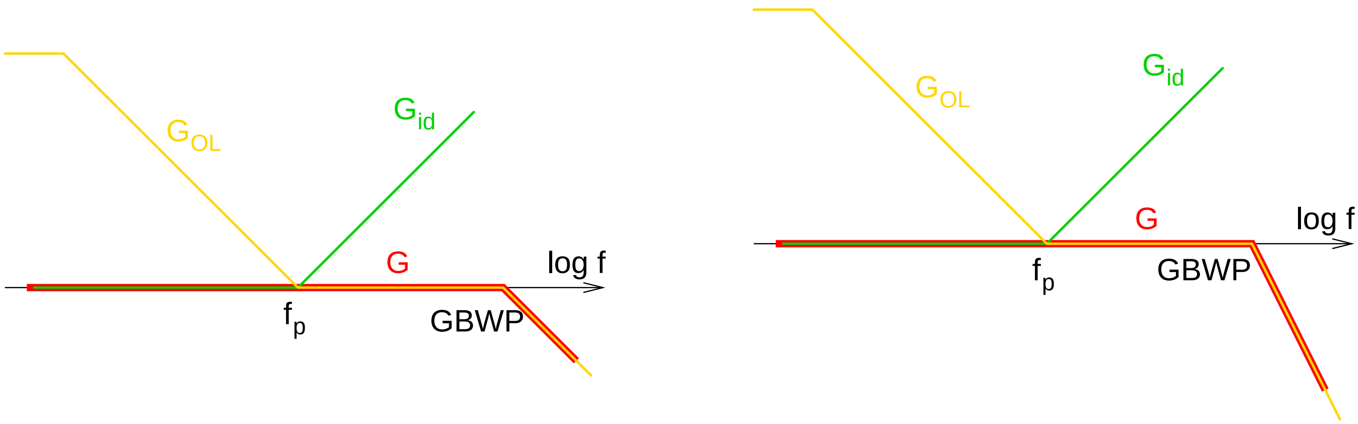


Figure 3: Left: Bode plot of the gains considering the finite gain of OA1. Right: same, but accounting also for OA2.

where G_1 is the real transfer of the integrator stage, given by

$$G_1 = G_{OL}^1 \frac{1}{1 - G_{loop}^1} = A_1 \frac{1}{1 + A_1 \dfrac{s\tau_1}{1 + s\tau_1}}.$$

Please note that these are *local* quantities, related to the first stage. Considering single-pole OAs with $A(s) = A_0/(1 + s\tau)$ we obtain:

$$G_{OL} = \frac{A_0}{(1 + s\tau)\left(1 + \dfrac{s\tau_1 A_0}{(1 + s\tau)(1 + s\tau_1)}\right)} = \frac{A_0(1 + s\tau_1)}{(1 + s\tau)(1 + s\tau_1) + s\tau_1 A_0} \approx A_0 \frac{1 + s\tau_1}{1 + s\tau_1 A_0 + s^2 \tau\tau_1}.$$

We can find approximate pole positions with the usual techniques seen in the class, obtaining:

$$f_{p1} = \frac{1}{2\pi A_0 \tau_1} = \frac{f_p}{A_0} \qquad f_{p2} = \frac{A_0}{2\pi\tau} = GBWP.$$

The new Bode plot is reported in Fig. 3 (left). It should be mentioned that OA1 does not affect G_{id} in this case (not shown).

Finally, we can assess the role of OA2. The connection as a buffer leads to a transfer given by $A_2/(1 + A_2) = 1/(1 + s\tau_0)$, where the pole due to τ_0 is at GBWP. This of course shows up in G_{OL}, leading to a second pole at $GBWP$ (and not affecting G_{id}). The very final Bode plot is in Fig. 3 (right).