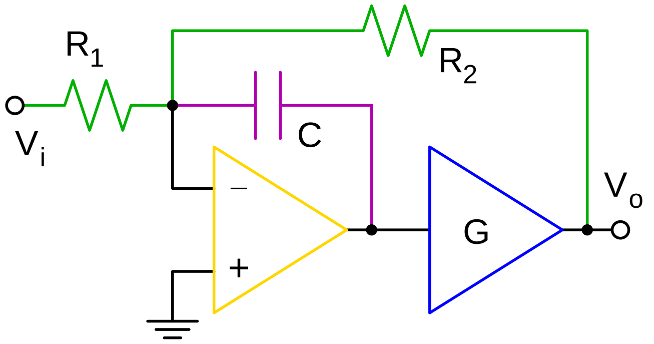
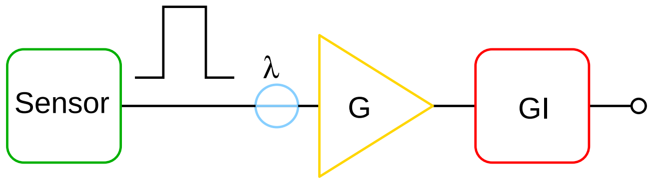
Electronics – June 21 2018 Prof. A. Spinelli





Problem 1

The scheme in the left figure is a composite amplifier. Component values are $R_1 = R_2 = 10\ \mathrm{k\Omega}, C = 680\ \mathrm{pF}$. The OA is a low-power amplifier with $A_o = 100$ dB and $GBWP = 40$ kHz.

1. Find the (ideal) gain of the stage.
2. Discuss the stability of the circuit as a function of the gain G.
3. Compute the output rms voltage noise considering the equivalent voltage noise sources for OA and G, $\sqrt{S_V} = 100\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$. Assume $G = 1$ for simplicity.
4. Discuss the impact of the input parasitic capacitance C_i of the OA on the circuit stability (consider $G = 1$ as ideal).

Problem 2

A sensor produces rectangular pulses with amplitude A and duration $T \approx 10\ \mu$s. The signal is fed into a preamplifier with input white noise with bilateral PSD λ and then to a gated-integrator.

1. Compute the output S/N neglecting the effect of the amplifier bandwidth.
2. The amplifier has a bandwidth $BW \approx 80$ kHz. Write the correct expression for the new S/N, then find approximated values for the integration limits and compute the resulting S/N. Consider the noise as still white.
3. Evaluate the effect of the amplifier BW on noise. To ease your calculation, use $\int_0^K e^{-x}(K-x)dx = K - 1 + e^{-K}$.
4. With reference to the previous point, an optimum filter is put in place of the GI. Find the value of S/N.

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by June 28th **Mark registration: Tuesday, July 3rd**

Solution

Problem 1

1.1

We begin the solution by noting that the circuit is very similar to what proposed in the last exam test, with the input applied to the inverting input of the OA rather than to the non-inverting one.
The voltage at the output of the OA is simply V_o/G, because of the presence of the gain stage G. The current balance at the inverting input of the OA reads then:

$$\frac{V_i}{R_1}+\frac{V_o}{R_2}+sC\frac{V_o}{G}=0 \Rightarrow \frac{V_o}{V_i}=-\frac{R_2}{R_1}\frac{1}{1+sCR_2/G}.$$

We have then an amplifier with a gain of -1 and a bandwidth of $G/2\pi CR_2$.
Before proceeding, it is instructive to interpret this relation in view of feedback theory, considering the outer loop (OA is ideal). Its ideal gain is $-R_2/R_1$, obtained by bringing G to infinity. The outer loop gain is instead $-G/sCR_2$ from which the above equation follows.

1.2

We break the loop at the output of the OA and easily obtain the voltage at the inverting input of the OA as

$$V^-=V_s\frac{R_1\parallel R_2}{R_1\parallel R_2+\dfrac{1}{sC}}+GV_s\frac{R_1\parallel\dfrac{1}{sC}}{R_1\parallel\dfrac{1}{sC}+R_2}=V_s\frac{GR_1}{R_1+R_2}\frac{1+sCR_2/G}{1+sC\ R_1\parallel R_2}.$$

Considering that $R_1=R_2$, we eventually get

$$G_{loop}=-A(s)\frac{G}{2}\frac{1+sCR/G}{1+sCR/2}.$$

For $G=2$ we have pole-zero cancellation and the system is stable. For $G<2$ the zero introduced by the feedback network comes before the pole and stability is also ensured. For $G>2$ the system could have issue as the pole comes before the zero. Also, please note that in this case the 0dB frequency can become larger than $GBWP$, which is to be avoided. It is then wise to maintain $G\leq 2$. Bode diagrams are plotted in Fig. 1 for $G=1$, $G=2$ and $G=10$.

1.3

In the general case, we can write the following current-balance equation for the inverting node of the OA (see Fig. 2, left):

$$\frac{V_{n1}}{R_1}+\frac{V_{n1}-V_o}{R_2}+sC\left(V_{n1}-V_{n2}-\frac{V_o}{G}\right)=0,$$

which, in our case ($G=1; R_1=R_2=R$) leads to

$$V_o=2\frac{1+sCR/2}{1+sCR}V_{n1}+\frac{sCR}{1+sCR}V_{n2},$$

where the pole position is $f_p=1/2\pi CR\approx 23.4$ kHz. We must then consider the effect of the real transfer, that adds a pole at $f_{0dB}\approx GBWP/2=20$ kHz (look at G_{loop} for $G=1$. In reality, pole and zero are very close and the actual crossing will take place somewhere between 20 and 40 kHz). For V_{n1} we have two poles and one zero, all fairly close, and we can just consider the lowest. For V_{n2} the situation is more complex, as a unity transfer holds even beyond f_{0dB} (set to zero the output of the OA and see!). So, this noise is actually limited by the buffer stage. If we ascribe it the same $GBWP$ as the OA, we finally have:

$$\overline{V_o^2}\approx 4S_{V_1}\frac{\pi}{2}f_{0dB}+S_{V_2}\frac{\pi}{2}(GBWP-f_P)\approx(35\ \mu\text{V})^2+(16\ \mu\text{V})^2\approx(39\ \mu\text{V})^2.$$

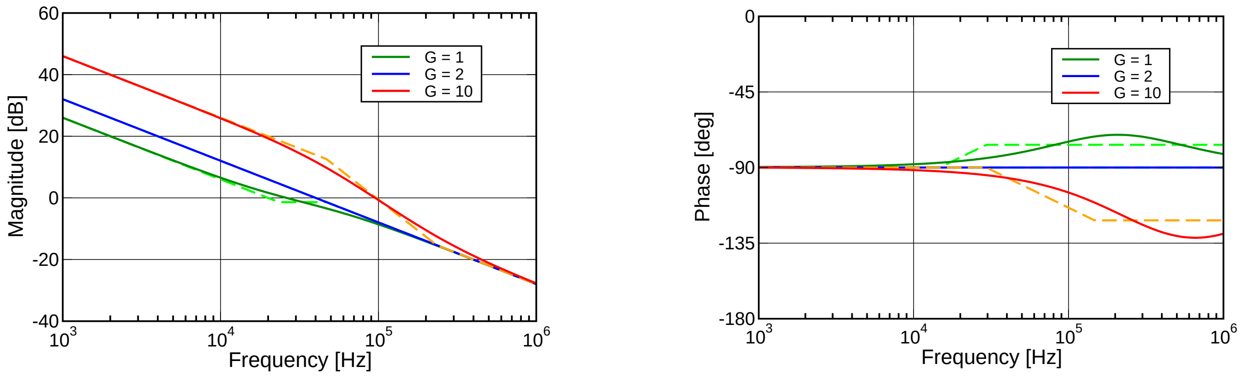


Figure 1: Bode plots of the loop gain for different values of G. Dashed lines = asymptotic diagrams. Solid lines = real diagrams.

1.4

The new value of G_{loop} is easily obtained by replacing R_1 with $R_1/(1 + sC_iR_1)$ in the expression derived in 1.2. Considering then $R_1 = R_2 = R$ and $G = 1$ we obtain:

$$G_{loop} = -\frac{A(s)}{2}\frac{1 + sCR}{1 + s(C + C_i)R/2}.$$

C_i lowers the pole position, but its effect is negligible unless it becomes comparable with $C = 680$ pF. Since typical values for the input capacitances of OAs are of the order of one pF or so, we can safely neglect its effect.

Problem 2

2.1

The obvious choice is to integrate over the entire pulse duration (this is indeed the optimum filter), obtaining:

$$\left(\frac{S}{N}\right) = \frac{A}{\sqrt{\lambda}}\sqrt{T}.$$

2.2

The pole of the amplifier affects both signal and noise. We neglect the effect of noise and consider just the modifications of the signal. The rectangular pulse now has exponential leading and trailing edges (Fig. 2, right) with time constant $\tau = 1/(2\pi BW) \approx 2$ μs, and the integration interval should be modified to account for the new signal shape. With the notation in the figure, we have:

$$\frac{S}{N} = \frac{A}{\sqrt{\lambda}}\frac{\int_{T_{G1}}^{T}\left(1 - e^{-t/\tau}\right)dt + (1 - e^{-T/\tau})\int_0^{T_{G2}} e^{-t/\tau}dt}{\sqrt{T - T_{G1} + T_{G2}}} \approx A\sqrt{\frac{\tau}{\lambda}}\frac{x_T - x_1 - e^{-x_1} + 1 - e^{-x_2}}{\sqrt{x_T - x_1 + x_2}},$$

where $x_1 = T_{G1}/\tau$, $x_2 = T_{G2}/\tau$, $x_T = T/\tau = 5$ and we have neglected the terms e^{-x_T}. Considering that for the case of exponential signals the optimum integration time is usually comparable to the time constant, we can apply the same approximation here and set $T_{G1} \approx T_{G2} \approx \tau$, i.e., $x_1 \approx x_2 \approx 1$ (we are basically shifting the integration time by τ). With these values we eventually get

$$\frac{S}{N} = A\sqrt{\frac{\tau}{\lambda}}\frac{x_T + e^{-x_T} - e^{-1} - e^{-1}}{\sqrt{x_T}} = 1.91\ A\sqrt{\frac{\tau}{\lambda}},$$

which amounts to a reduction of about 15% from the previous result. The optimization is briefly discussed in the Appendix.

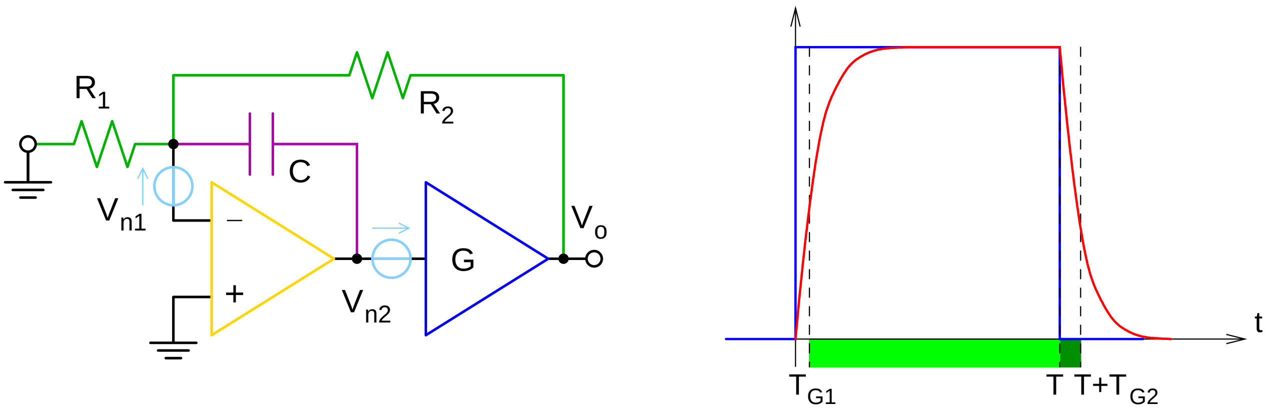


Figure 2: Left: Scheme for noise calculation. Right = filtered pulse and integration time

2.3

The output noise now has an exponential autocorrelation

$$R_{nn}(\gamma) = \frac{\lambda}{2\tau} G^2 e^{-|\gamma|/\tau},$$

and the mean square value of the output noise becomes

$$\overline{n^2} = \int R_{nn}(\gamma) k_{w_{tt}}(\gamma) d\gamma,$$

where $k_{w_{tt}}(\gamma)$ is the time correlation of the weighting function, i.e., a well-known symmetric triangular function. We then have

$$\overline{n^2} = \frac{\lambda G^2}{\tau} \int_0^{T_G} e^{-\gamma/\tau} G_{GI}^2 (T_G - \gamma) d\gamma = \lambda G^2 G_{GI}^2 \tau \int_0^K e^{-x}(K - x) dx,$$

in which $K = T_G/\tau$. The result now becomes

$$\overline{n^2} = \lambda G^2 G_{GI}^2 \left(T_G - \tau + \tau e^{-T_G/\tau} \right).$$

If we still consider $x_1 \approx x_2 \approx 1$, i.e., $T_G \approx T$, the output noise is now reduced with respect to the white noise approximation by a factor of about $1/5$, i.e., 20%.

2.4

Similarly to what discuss in the last exam test, the output of the whitening filter would be a signal plus white noise, which is exactly the condition discussed in 2.1. The result of the optimum filtering is then what already obtained there.

Appendix

By setting $\partial(S/N)/\partial x_1 = \partial(S/N)/\partial x_2 = 0$ to the equation in 2.2, it is easy to obtain the condition

$$e^{-x_1} + e^{-x_2} = 1,$$

from which S/N can be rewritten as

$$\frac{S}{N} = A\sqrt{\frac{\tau}{\lambda}} \frac{x_T - x_1}{\sqrt{x_T - x_1 - \ln(1 - e^{-x_1})}},$$

which unfortunately cannot be optimized analytically. Numerical results yield an optimum value $x_1 \approx 0.54$, i.e., $x_2 \approx 0.87$.