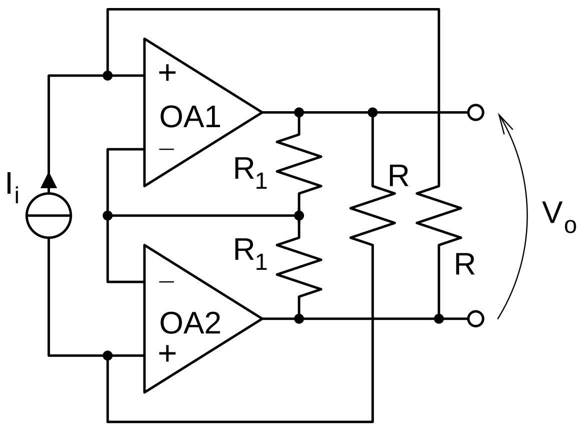
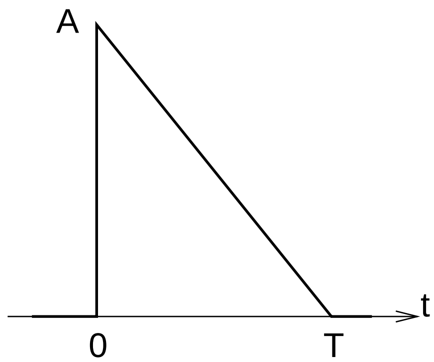
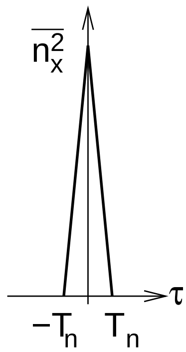
Electronics – January 23 2018 Prof. A. Spinelli







Problem 1

The scheme in the left figure is a transresistance amplifier for a floating current source and differential output. The amplifiers have $A_o = 120$ dB and pole at 1 Hz.

1. Find the (ideal) gain of the stage.
2. Repeat the previous calculation accounting for the finite gain of the OAs.
3. To avoid common-mode problems, the non-inverting inputs of OA2 is now grounded. Compute the output rms voltage noise considering the OA equivalent voltage noise sources $\sqrt{S_V} = 10$ nV/$\sqrt{\text{Hz}}$.
4. The (grounded) circuit is modified by switching the R connections from one output node to the other. Apart from the sign of the output, does this affect any other performance?

Problem 2

A signal approximately triangular in shape (right Figure) is affected by a high-frequency noise $\overline{n_x^2}$ having correlation time $T_n \approx T/20$. To acquire the signal amplitude, a digital filter is employed.

1. Two samples are taken and data are added. Pick the best value of the sampling times and write the expression of the resulting S/N.
2. To improve S/N, $n+1$ samples are now added. Find the expression of the output S/N (hint: write the signal for $n = 0, 1, 2, ...$ and work out the general law).
3. What is the best choice for n in the previous point?
4. A second filter, equal to the first, is now cascaded. Find the new optimum values of n and S/N.

Question

Discuss the discrete-time equivalent of the GI and BA filters and their performance.

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by January 26th **Mark registration: Monday, Jan 29th**

Solution

Problem 1

1.1

It is easy to see that current I_i flows from one resistor R to the other, so that the output voltage is

$$V_o = 2RI_i.$$

Note that the solution could also be reached via the half-circuit approach, grounding the middle node between the two R_1 resistors.

1.2

A simple solution would be to amend the ideal gain by the factor $1/(1 - 1/G_{loop})$, but the calculation of the loop gain does not seem straightforward in this case (both OAs are intertwined in two feedback loops). Therefore, we first proceed with standard calculations and postpone the discussion. Calling V_1 and V_2 the two output nodes, we can easily label the OA input nodes as in Fig. 1 (left), from which we can write:

$$V_1 = A(s)\left(V_2 + I_iR - \frac{V_1}{2} - \frac{V_2}{2}\right)$$
$$V_2 = A(s)\left(V_1 - I_iR - \frac{V_1}{2} - \frac{V_2}{2}\right)$$

which lead to

$$V_1 = -V_2 = \frac{I_iR}{1 + \dfrac{1}{A(s)}}.$$

An interpretation of this expression can be provided if we compute the *differential* loop gain, given the nature of the scheme: we break both loops at the outputs of the OAs and apply a differential signal, computing the differential output. It is easy to see that we get $G_{loop} = -A(s)$. Of course, we could also use the half-circuit approach.

1.3

Considering that the differential input voltage of the OAs must be zero, we immediately get

$$V_{n2} = \frac{V_1 + V_2}{2} = V_2 + V_{n1} \Rightarrow \begin{cases} V_1 = V_{n1} + V_{n2} \\ V_2 = -V_{n1} + V_{n2}, \end{cases}$$

from which $V_o = V_1 - V_2 = 2V_{n1} \Rightarrow S_{V_o} = 4S_V = 4 \times 10^{-16}$ V^2. Note that V_{n2} appears as a common-mode signal and is rejected by the differential output. Just as a reference, we add here the current noise contribution, which is

$$V_o = 2I_{n1}^+R + (I_{n1}^- + I_{n2}^-)R \Rightarrow S_{V_o} = 6S_IR^2 = 6 \times 10^{-16}\ \text{V}^2,$$

where we have taken $\sqrt{S_I} = 1$ pA/$\sqrt{\text{Hz}}$ and $R = 10$ kΩ (this was originally part of the question, but you were eventually spared). Considering that the loop gain is $-A(s)$, as obtained at the previous point, we then get

$$\overline{V_o^2} = S_{V_o}\frac{\pi}{2}GBWP \approx (40\ \mu\text{V})^2$$

in total, of which $(16\ \mu\text{V})^2$ are due to the voltage noise and $(24\ \mu\text{V})^2$ to the current noise.

1.4

Swapping the R connections changes the sign of the feedback, leading to a positive feedback and to instability. This can be seen by – say – computing the loop gain of OA1.

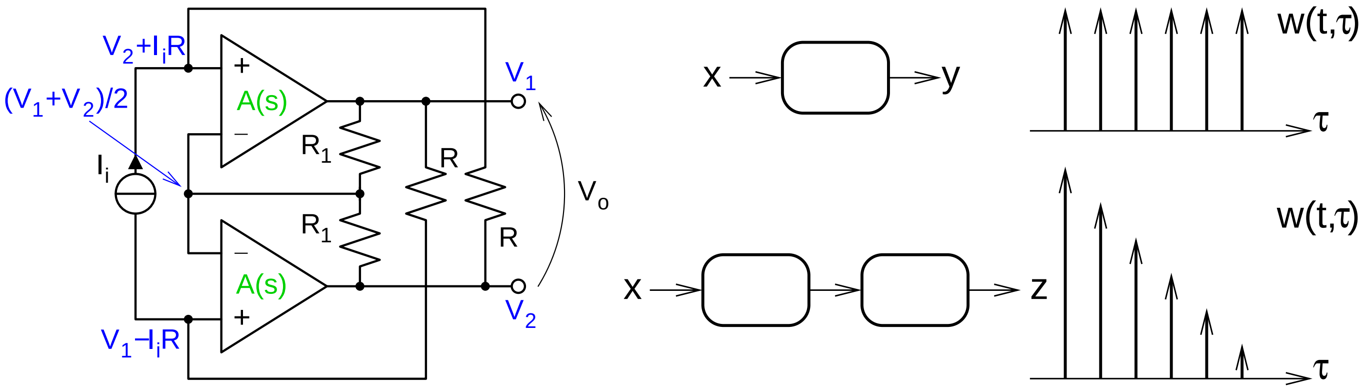


Figure 1: Left: Circuit for noise calculations when $R_s = \infty$. Right = weighting function when one or two cascaded filters are considered.

Problem 2

2.1

It is wise to take the first sample at $t = 0$, when the signal is maximum. The second sample is taken at $t = t_s$ and the output signal becomes

$$y = A + A\left(1 - \frac{t_s}{T}\right) = A\left(2 - \frac{1}{N_s}\right),$$

where we have defined $N_s = T/t_s$. If the output noise samples are uncorrelated, we get $\overline{n_y^2} = 2\overline{n_x^2}$, hence:

$$\left(\frac{S}{N}\right)_y = \frac{A}{\sqrt{\overline{n_x^2}}}\frac{2 - \dfrac{1}{N_s}}{\sqrt{2}}.$$

To maximize S/N, we pick $t_s = T_n$, i.e., $N_s = 20$. The improvement in S/N with respect to a single sample is about 1.38.

2.2

Following the previous line of thought, the $n + 1$ samples taken at $t = 0$, $t = t_s$, $t = 2t_s$ and so on give the following output signal:

$$\begin{aligned}
n = 0 &\Rightarrow A \\
n = 1 &\Rightarrow A + A\left(1 - \frac{t_s}{T}\right) = A\left(2 - \frac{1}{N_s}\right) \\
n = 2 &\Rightarrow A\left(2 - \frac{1}{N_s}\right) + A\left(1 - \frac{2t_s}{T}\right) = A\left(3 - \frac{3}{N_s}\right) \\
n = 3 &\Rightarrow A\left(3 - \frac{3}{N_s}\right) + A\left(1 - \frac{3t_s}{T}\right) = A\left(4 - \frac{6}{N_s}\right) \\
n &\Rightarrow A\left(n + 1 - \frac{n(n+1)}{2N_s}\right).
\end{aligned}$$

The output noise is obviously $\overline{n_y^2} = (n + 1)\overline{n_x^2}$, so that

$$\left(\frac{S}{N}\right)_y = \frac{A}{\sqrt{\overline{n_x^2}}}\frac{n + 1 - \dfrac{n(n+1)}{2N_s}}{\sqrt{n+1}} = \frac{A}{\sqrt{\overline{n_x^2}}}\sqrt{n+1}\left(1 - \frac{n}{2N_s}\right).$$

2.3

The optimum choice can be approximately guessed without calculations, by simply noting that the filter is the discrete-time analogous of a gated integrator, whose optimum gate time is 2/3 of the triangular signal width. This means that we expect to have $n_{opt} \approx 2N_s/3 = 12 - 13$.

Let's carry out the calculations, assuming a continuous range for n. We get:

$$\frac{\partial}{\partial n}\left(\frac{S}{N}\right)_y = 0 \Rightarrow n_{opt} = \frac{2}{3}(N_s - 1).$$

for $N_s = 20$ we have $n_{opt} = 12.67 \rightarrow 13$, which is in good agreement with the expectations. Note that the solution indeed tends to $2N_s/3$ for large values of N_s (i.e., for shorter noise correlation time). The improvement in S/N is now about 2.43.

2.4

We must understand what is the output of the two cascaded filter, so let's look at the output of the first (y) and second (z) filter after the first, second,... n^{th} sample:

$$\begin{aligned}
t = t_s &\Rightarrow y(t_s) = x(t_s) \Rightarrow z(t_s) = x(t_s) \\
t = 2t_s &\Rightarrow y(2t_s) = x(t_s) + x(2t_s) \Rightarrow z(2t_s) = 2x(t_s) + x(2t_s) \\
t = 3t_s &\Rightarrow y(3t_s) = x(t_s) + x(2t_s) + x(3t_s) \Rightarrow z(3t_s) = 3x(t_s) + 2x(2t_s) + x(3t_s) \\
&\dots
\end{aligned}$$

It is clear that the output of the second filter is a linearly-weighted average of the input signal. The combination of the filters amounts then to a weighting function shown in Fig. 1 (right). Clearly, this represents the optimum weights for the white noise case, matching the shape of the signal. In this case, the best number of samples is the maximum that allows to retain the white-noise approximation, i.e., $n = N_s$. To compute the resulting S/N, we can use the continuous approximation:

$$\left(\frac{S}{N}\right)_y \approx \frac{A}{\sqrt{\lambda}}\sqrt{\int_0^T \left(1 - \frac{t}{T}\right)^2 dt} = \frac{A}{\sqrt{\overline{n_x^2}T_n}}\sqrt{\frac{T}{3}} = \frac{A}{\sqrt{\overline{n_x^2}}}\sqrt{\frac{N_s}{3}}.$$

The resulting improvement is now 2.58.