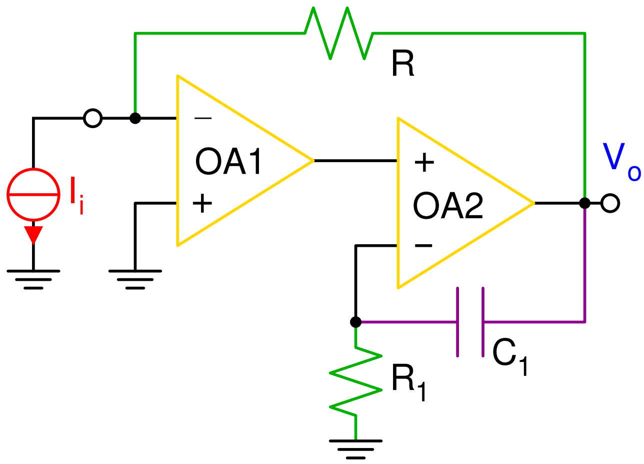
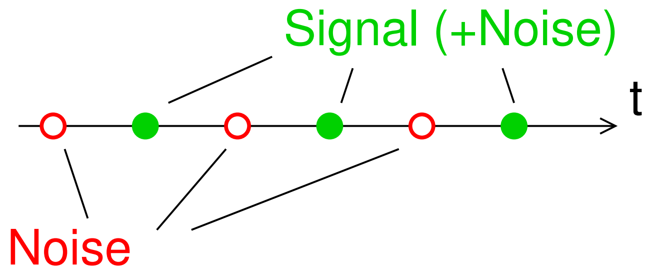
Electronics – June 20 2019 Prof. A. Spinelli





Problem 1

The scheme in the left figure is a transresistance amplifier. Component values are $R = 11$ kΩ, $R_1 = 11.8$ kΩ, $C_1 = 105$ nF. The OAs have $A_o = 80$ dB and $GBWP = 0.5$ MHz.

1. Find the expression of the closed-loop gain.
2. Evaluate the phase margin of the feedback circuit.
3. Compute the output rms noise voltage considering the equivalent noise source of the OA $\sqrt{S_V} = 14\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$ and the resistors noise ($4k_BT \approx 1.646 \times 10^{-20}$ J).
4. Consider the case in which the two OAs have different $GBWP$s and discuss the impact on stability.

Problem 2

A discrete-time filter with sampling time t_s is used to measure the amplitude of a constant signal, taking alternate measurements of noise alone (no signal) and of signal (plus noise), subtracting the former from the latter and averaging over N signal pulses.

1. Compute the weighting function and its time correlation.
2. The input noise has an approximately triangular autocorrelation function with correlation time $T_n \approx 2t_s$. Evaluate the output S/N.
3. Consider now the case in which T_n extends over a few samples $n_s \ll N$ and compute the new value of S/N.
4. Consider a similar filter, but where the N noise samples are collected before the N signal ones. Which one is better for the noise considered in #2.2 and #2.3? Justify your answers.

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by July 2nd — Mark registration: Thursday, July 4th

Solution

Problem 1

1.1

The ideal gain is obviously R. To compute the open-loop gain, we can look at the high-frequency behavior, where C_1 can be approximated as a short-circuit and OA2 becomes a buffer. If we disconnect the outer loop, we obtain

$$G_{OL} = -RA_1(s)A_2(s)$$

The closed-loop gain has then two poles when $|A(s)|^2 = 1$, i.e., at $GBWP$.

1.2

We apply a test voltage V_T at the output of OA2, obtaining

$$V^+ = -A_1(s)V_T; \quad V^- = \frac{sC_1R_1}{1+sC_1R_1}V_T,$$

which means

$$G_{loop} = -A_2(s)\left(\frac{sC_1R_1}{1+sC_1R_1} + A_1(s)\right) \approx -A(s)\frac{A_0 + sC_1R_1A_0 + s^2C_1R_1\tau}{(1+s\tau)(1+sC_1R_1)},$$

where we considered $A(s) = A_0/(1+s\tau)$ and $A_0 \gg 1$. Approximate values of the zeros are

$$f_{z1} \approx \frac{1}{2\pi C_1R_1} \qquad f_{z2} \approx \frac{A_0}{2\pi\tau} = GBWP.$$

The loop gain has then two poles at $1/2\pi\tau = 50$ Hz and one zero at $GBWP = 500$ kHz, meaning that the system is stable, with a phase margin of 45°.
Of course, all the results could have been obtained by replacing the OA2 feedback loop with its real gain and considering the outer loop. Please note that the value of G_{loop} will obviously be different, but the phase margin remains the same.

1.3

It is easy to see that all noise sources of OA2 give no contribution to the output, as they are inside the global feedback loop (in other words, the output is set to zero by OA1). The remaining contributions are

$$S_{V_o} = S_V + 4k_BTR = 1.96\times10^{-16} + 1.81\times10^{-16} \approx \left(19.4\ \text{nV}/\sqrt{\text{Hz}}\right)^2,$$

which gives an rms output noise:

$$\sqrt{\overline{V_o^2}} = 19.4\times10^{-9}\sqrt{\frac{\pi}{2}5\times10^5} \approx 17.2\ \mu\text{V}.$$

1.4

We can start from the expression for G_{loop} already obtained in #1.2, which can be written as (remember the pole-zero cancellation)

$$G_{loop} = -A_2(s)\frac{A_0(1+s\tau_{z1})(1+s\tau_{z2})}{(1+s\tau_1)(1+sC_1R_1)} = -\frac{A_0}{1+s\tau_1}\frac{A_0}{1+s\tau_2}(1+s\tau_{z2}),$$

where the zero is located at $GBWP_1$. After the two low-frequency poles (and before the zero), the gain becomes

$$G_{loop} \approx \frac{A_0^2}{s^2\tau_1\tau_2} \Rightarrow f_{0dB} = \sqrt{GBWP_1\ GBWP_2},$$

which is halfway between the two values on a log scale. To ensure the phase margin remains higher than 45°, it is advised to pick $GBWP_1 < GBWP_2$.

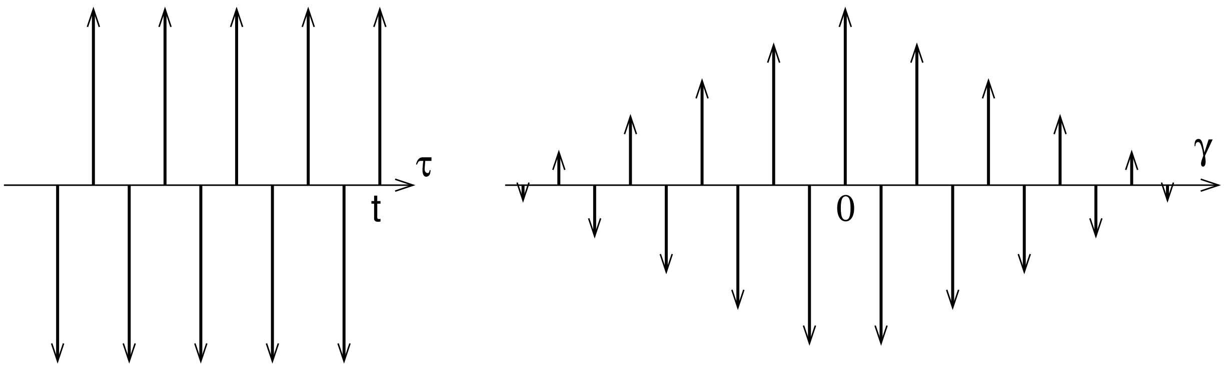


Figure 1: Left = weighting function for the case of $N = 5$. Right = time correlation of the weighting function.

Problem 2

2.1

The weighting function is made up of a series of alternating positive and negative delta functions (Fig. 1, left for the case of $N = 5$), and its time correlation has the shape shown in Fig. 1 (right). If we consider a $1/N$ area of the delta functions (unity gain for signal), the areas of the components of $k_{w_{tt}}(\gamma)$ are

$$k_{w_{tt}}(\gamma) = \sum_{-2N}^{2N}{}_{k} (-1)^k \frac{2N - |k|}{N^2} \delta(\gamma - kt_s).$$

2.2

The output noise is given by

$$\overline{n_y^2} = \int R_{xx}(\gamma) k_{w_{tt}}(\gamma),$$

and in our case only the central and its first neighboring delta functions give a contribution. The result is then

$$\overline{n_y^2} = \overline{n_x^2} \left(\frac{2}{N} - 2\frac{2N-1}{2N^2} \right) = \frac{\overline{n_x^2}}{N^2},$$

leading to

$$\left(\frac{S}{N}\right)_y = \left(\frac{S}{N}\right)_x N$$

The noise is weakly correlated over the averaging time and is hence reduced by the average over N samples.

2.3

If R_{xx} is non-zero over a number of samples n_s much smaller than N, we can approximate the area of each delta function with $2/N$, obtaining:

$$\overline{n_y^2} = \int R_{xx}(\gamma) k_{w_{tt}}(\gamma) \approx \overline{n_x^2} \frac{2}{N} \sum_{-n_s}^{n_s}{}_{k} (-1)^k \left(1 - \frac{|k| t_s}{T_n} \right).$$

Taking advantage of the even symmetry, we can only evaluate non-negative terms twice (see Fig. 2, left), resulting in (remember that $T_N/t_s = n_s$)

$$\overline{n_y^2} = \overline{n_x^2} \frac{4}{N} \sum_{0}^{(n_s-1)/2}{}_{k} 1 - \frac{2k}{n_s} - \left(1 - \frac{2k+1}{n_s} \right) = \overline{n_x^2} \frac{4}{N} \sum_{0}^{(n_s-1)/2}{}_{k} \frac{1}{n_s} = \overline{n_x^2} \frac{4}{N} \frac{n_s+1}{2n_s} = \overline{n_x^2} \frac{2}{N} \left(1 + \frac{1}{n_s} \right).$$

We now remember that in our process we have double-counted also the term in $k = 0$, that amounts to $2/N$. By subtracting this term, we eventually get

$$\overline{n_y^2} = \overline{n_x^2} \frac{2}{N n_s}.$$

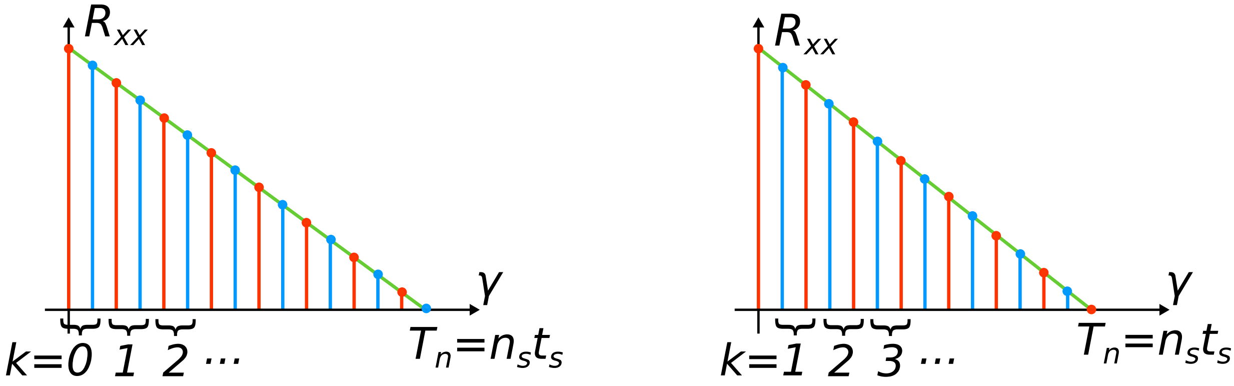


Figure 2: Schemes adopted for noise calculations in #2.3. Red samples = positive ones; blue samples = negative ones. Left side is for odd n_s, right one for even n_s.

2.4

In this case we would have N negative delta functions followed by N positive ones. Clearly, the new weighting function time correlation has the same value in zero, but remains positive until the shift becomes $2Nt_s/3$. This means that all contributions will be positive and there is no cancellation. The new filter has worse performance.

Appendix: Brief discussion on #2.3

The shrewed reader will have noticed that in #2.3 we have implicitly assumed an odd value for n_s. If however we had taken an even value, the result would have been (see Fig. 2, right for the summing scheme)

$$\frac{\overline{n_y^2}}{\overline{n_x^2}} = \frac{2}{N} + \frac{4}{N}\sum_{1}^{n_s/2}{}_k -1 + \frac{2k-1}{n_s} + 1 - \frac{2k}{n_s} = \frac{2}{N} - \frac{4}{N}\sum_{1}^{n_s/2}{}_k \frac{1}{n_s} = 0.$$

This result, though equally correct as the previous one, deserves a short comment: of course, perfect noise cancellation is not possible, and the zero is a consequence of the equal value for the delta areas. Just for fun, we carry on the calculations in this case:

$$\frac{\overline{n_y^2}}{\overline{n_x^2}} = \frac{2}{N} + 2\sum_{1}^{n_s/2}{}_k \left(-1 + \frac{2k-1}{n_s}\right)\frac{2N-(2k-1)}{N^2} + \left(1 - \frac{2k}{n_s}\right)\frac{2N-2k}{N^2}.$$

We know that the terms with weight $2N/N^2$ give zero (this is the previous result) and we drop them, obtaining

$$\frac{\overline{n_y^2}}{\overline{n_x^2}} = 2\sum_{1}^{n_s/2}{}_k \left(1 - \frac{2k-1}{n_s}\right)\frac{2k-1}{N^2} - \left(1 - \frac{2k}{n_s}\right)\frac{2k}{N^2} = \frac{2}{N^2}\sum_{1}^{n_s/2}{}_k 2k - 1 - \frac{(2k-1)^2}{n_s} - 2k + \frac{(2k)^2}{n_s} =$$

$$\frac{2}{N^2 n_s}\sum_{1}^{n_s/2}{}_k 4k - (n_s+1) = \frac{2}{N^2 n_s}\left(4\frac{n_s}{4}\left(\frac{n_s}{2}+1\right) - (n_s+1)\frac{n_s}{2}\right) = \frac{1}{N^2}.$$

It is also interesting to note that the previous result for the odd case is exact even if the correct delta values are used (calculations are left to the reader, if any have reached this point). And, finally, T_n needs not be an exact multiple of t_s: calculations show that in the general case the output noise changes linearly between the minimum (for the even case) and the maximum (for the odd values) values.