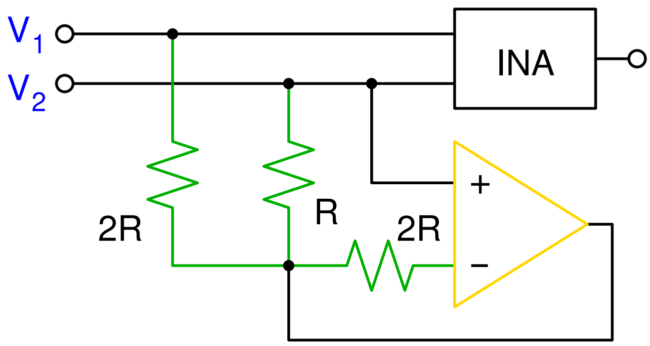
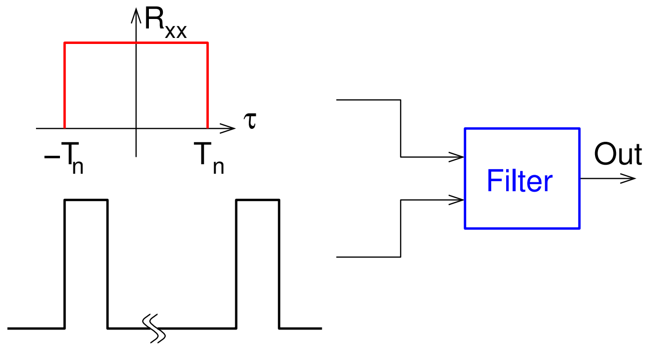
Electronics – February 13 2019 Prof. A. Spinelli





Problem 1

The scheme in the left figure is used for cancellation of the INA bias currents. Consider $R = 10\ \mathrm{M\Omega}$ and a precision OA with $A_o = 106$ dB and $GBWP = 200$ kHz.

1. Consider equal bias currents for the INA and the OA and compute the actual bias currents at the inputs. What is the disadvantage of this solution?
2. Consider a common-mode input and compute the input impedance.
3. Compute the short-circuit noise current PSDs at the inputs due to the equivalent noise source of the OA $\sqrt{S_V} = 30\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$ and the resistors noise ($4k_BT \approx 1.646 \times 10^{-20}$ J).
4. The OA and INA have offset voltage $V_{OS} = 30\ \mu\mathrm{V}$ and bias and offset currents $I_B = 6$ nA, $I_{OS} = 50$ pA. Compute the actual bias and offset currents at the (grounded) inputs.

Problem 2

A pulsed signal is affected by a noise source with nearly rectangular autocorrelation shown Fig. 2.

1. An LPF with time constant T_F is used to reduce the noise. Find the output noise rms value and discuss the cases $T_n \ll T_F$ and $T_n \gg T_F$.
2. A boxcar averager is now used. Evaluate the output noise when the gate opening time $T_O > T_n > T_C$ (gate closing time). Use a rectangular approximation for the BA pulses when computing the time correlation.
3. Repeat the above exercise when $T_n \gg T_O$.
4. Propose a filter to improve S/N with respect to the result obtained in 2.3 and compute the output noise.

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by February 18th **Mark registration: Tuesday, Feb. 19th**

Solution

Problem 1

1.1

If currents I_B flow into the inputs of the INA and the OA, the output of the OA is at

$$V_o = V_2 + 2I_B R,$$

meaning that a current equal to $2I_B$ will flow into resistor R, compensating for the bias currents of INA and OA. No current flows then to/from input V_2. Moreover, a current $I_B + (V_2 - V_1)/2R$ flows into resistor $2R$, again compensating for the bias curent of the INA.
Note however that a current $(V_1 - V_2)/2R$ flows from input V_1, meaning that we have reduced the differential input impedance of the stage (that is also non-symmetric).

1.2

The circuit becomes the one shown in Fig. 1 (left), where $R_1 = (2/3)R$ and resistor $2R$ in series with the non-inverting input does not play any role. The ideal impedance is obviously ∞. We have then:

$$\begin{aligned} Z_{OL} &= R_1 \\ G_{loop} &= -A(s) \end{aligned} \Rightarrow Z_{in} = R_1(1 + A(s)).$$

1.3

The circuit for noise calculations is shown in Fig. 1 (right), where $S_{I_R} = \dfrac{4k_BT}{R} \approx 1.65 \times 10^{-27}$ A^2/Hz, $S_{I_{2R}} = \dfrac{4k_BT}{2R} \approx 8.23 \times 10^{-28}$ A^2/Hz and $S_{V_n} = S_V + 4k_BT(2R) \approx 3.3 \times 10^{-13}$ V^2/Hz. From simple calculations we have then

$$\begin{aligned} S_{I_1} &= S_{I_{2R}} + \frac{S_{V_n}}{(2R)^2} = \frac{4k_BT}{R} + \frac{S_V}{(2R)^2} \approx 1.65 \times 10^{-27} \text{ A}^2/\text{Hz} \\ S_{I_2} &= S_{I_R} + \frac{S_{V_n}}{R^2} \approx 1.65 \times 10^{-27} \text{ A}^2/\text{Hz}. \end{aligned}$$

The current noise (with density ≈ 41 fA/$\sqrt{\text{Hz}}$) is dominated by the resistors.

1.4

Bias currents are compensated as discussed in 1.1, so we do not need to account for them. Also, the offset voltage of the INA does not affect any current. We just need to consider offset currents for OA and INA (I_{OA} and I_{INA} respectively) and the OA offset voltage V_{OS}. The scheme is reported in Fig. 2 (left), where the inputs were grounded for convenience. The results are:

$$\begin{aligned} I_1 &= \frac{I_{INA}}{2} + \frac{I_{OA}}{2} + \frac{V_{OS}}{2R} \\ I_2 &= -\frac{I_{INA}}{2} + \frac{3I_{OA}}{2} + \frac{V_{OS}}{R}, \end{aligned}$$

from which we obtain

$$\begin{aligned} I'_B &= \frac{|I_1 + I_2|}{2} = I_{OA} + \frac{3V_{OS}}{4R} = I_{OS} + \frac{3V_{OS}}{4R} \approx 52 \text{ pA} \\ I'_{OS} = |I_1 - I_2| &= I_{INA} + I_{OS} + \frac{V_{OS}}{2R} = 2I_{OS} + \frac{V_{OS}}{2R} \approx 101.5 \text{ pA}. \end{aligned}$$

Note that the offset voltage contribution is negligible ($V_{OS}/R \approx 3$ pA) and that the offset current is even larger than the bias one (larger or comparable values are a typical outcome of bias current compensation).

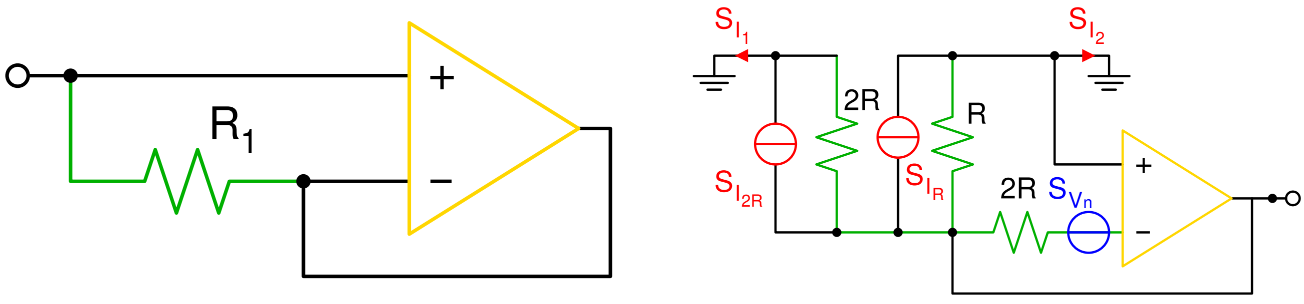


Figure 1: Left = Scheme for calculation of Z_{in}. Right = Scheme for noise calculations.

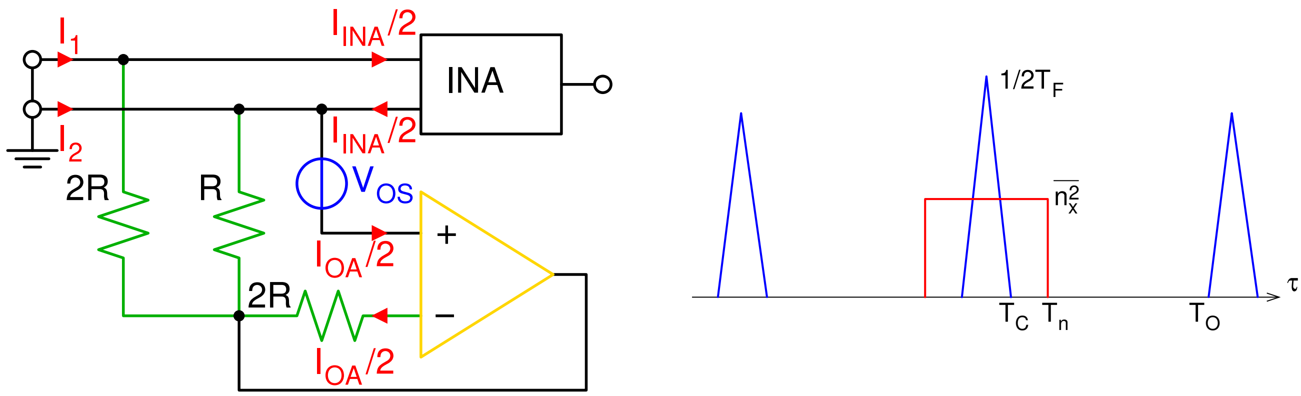


Figure 2: Left = Scheme for calculation of the bias currents. Right = BA weighting function time correlation and input noise autocorrelation.

Problem 2

2.1

The output rms noise is given by

$$\overline{n_y^2} = \int R_{xx}(\tau) k_{hh}(\tau) d\tau,$$

where $k_{hh}(\tau)$ is the time correlation of the LPF weighting function. The result is

$$\overline{n_y^2} = 2\overline{n_x^2} \int_0^{T_n} \frac{1}{2T_F} e^{-\tau/T_F} d\tau = \overline{n_x^2} \left(1 - e^{-T_n/T_F}\right).$$

When $T_n \gg T_F$ the filter does not have any effect, as it is averaging over correlated samples, so we get $\overline{n_y^2} \approx \overline{n_x^2}$. On the contrary, when $T_n \ll T_F$ the noise can be considered as approximatively white and we get (expand the exponential term to first order) $\overline{n_y^2} \approx \overline{n_x^2} T_n/T_F = \lambda/(2T_F)$, because $\lambda = 2T_n\overline{n_x^2}$.

2.2

The situation now is depicted in Fig. 2 (right), where we can approximate the pieces of the BA weighting function time correlation (blue curve) as triangular (this basically means that we are working with $T_C \ll T_F$). We now have (please note that we are considering a unity-gain BA):

$$\overline{n_y^2} = \overline{n_x^2} \int k_{hh}^0(\tau) d\tau,$$

where $k_{hh}^0(\tau)$ is the central pulse of k_{hh}. We get:

$$\overline{n_y^2} = \overline{n_x^2} \frac{T_C}{2T_F} = \frac{\overline{n_x^2}}{N_{eq}},$$

which is not an unexpected result: the BA behaves as an averaging filter with exponential weights and no correlation between successive pulses, reducing the input noise by the equivalent number of pulses.

2.3

If $T_n \gg T_O$ we must account for a large number of BA pulses. Referring again to the time correlation shown in Fig. 2 (right) and recalling that each peak amplitude is reduced by a factor e^{-T_C/T_F}, we get

$$\overline{n_y^2} = \int R_{nn}(\tau) \sum_n k_{hh}^n(\tau) d\tau = \overline{n_x^2} \sum_n \int_{-T_n}^{T_n} k_{hh}^n(\tau) d\tau = \overline{n_x^2} \frac{T_C}{2T_F} \sum_n e^{-|n| T_C/T_F}.$$

If we consider a very large value for n we can use the total sum of the series, obtaining

$$\overline{n_y^2} = \overline{n_x^2} \frac{T_C}{2T_F} \left(\frac{2}{1 - e^{-T_C/T_F}} - 1 \right) \approx \overline{n_x^2}.$$

The result is - once again - not (too) surprising: for a highly-correlated noise, the BA is not effective! In reality, the result will change somewhat if a non-rectangular approximation for R_{xx} is adopted, but the underlying concept remains true.

2.4

For a long noise correlation time, an HPF or a baseline restorer can be used, depending on the pulse repetition rate. For the simpler case of an HPF, we recall that $k_{hh}(\tau) = \delta(\tau) - e^{-|\tau|/T_F}/(2T_F)$ and obtain

$$\overline{n_y^2} = \int R_{xx}(\tau) k_{hh}(\tau) d\tau = \overline{n_x^2} - \frac{\overline{n_x^2}}{2T_F} \int_{-T_n}^{T_n} e^{-|\tau|/T_F} d\tau = \overline{n_x^2} e^{-T_n/T_F}.$$