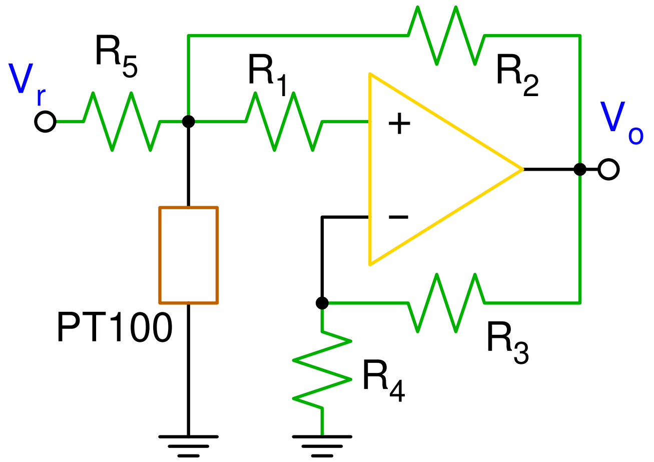
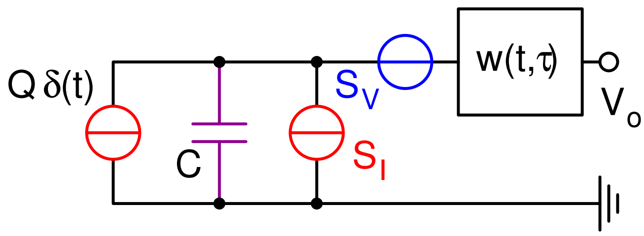
Electronics – January 16 2019   Prof. A. Spinelli





Problem 1

The scheme in the left figure is a PT100 RTD signal conditioner. Component values are $R_1 = 11$ kΩ, $R_2 = 11.8$ kΩ, $R_3 = 105$ kΩ, $R_4 = 12.4$ kΩ, $R_5 = 3.01$ kΩ. The precision CMOS OA has $A_o = 110$ dB and $GBWP = 2$ MHz. Consider the behavior at $T = 0°$C.

1. Find the expression of the output voltage. What is the function of R_1?
2. Compute the loop gain and find the minimum value of R_2 that ensures stability. Consider then a (differential) input capacitance $C_i = 10$ pF and evaluate the phase margin.
3. Compute the output rms noise voltage considering the equivalent noise source of the OA $\sqrt{S_V} = 14$ nV/$\sqrt{\text{Hz}}$ and the resistors noise ($4k_BT \approx 1.646 \times 10^{-20}$ J).
4. The RTD resistance is not perfectly linear with temperature: $R_T = 60.26, 100, 138.5$ and 175.84 Ω for $T = -100, 0, 100$ and $200°$C. Explain the function of resistor R_2 (hint: just evaluate the output. Please, be *quantitative* in your answer).

Problem 2

A charge-based sensor outputs delta-like signals $Q\delta(t)$ onto a capacitor C. The signal is affected by high-frequency noise (with triangular autocorrelation) as well as by a slow baseline. To eliminate the latter, a time-variant filter with a weighting function $w(t, \tau)$ is then applied.

1. The filter subtracts two samples, one with the baseline and one with signal + baseline. Consider the voltage noise only and compute the output S/N.
2. Evaluate the output noise due to S_I (consider for simplicity white current noise and a rectangular approximation of the integral in the frequency domain).
3. The negative delta function is now replaced by a (unit area) gated integrator. Compute the weighting function in the time domain for the input (current) signal.
4. Compute the (exact!) total output noise of the new filter.

For a correct evaluation, you are asked to write your answers in a readable way; thank you
Do a good job!

Results will be posted by January 21st   **Mark registration: Tuesday, Jan. 22nd**

Solution

Problem 1

1.1

The voltage at the inputs of the OA is

$$V^- = V^+ = V_o \frac{R_4}{R_3 + R_4} = kV_o,$$

that is also the voltage at the node before R_1, into which no current flows. Via simple superposition we get:

$$kV_o = V_r \frac{R_2 \parallel R_T}{R_2 \parallel R_T + R_5} + V_o \frac{R_5 \parallel R_T}{R_5 \parallel R_T + R_2},$$

from which we obtain

$$V_o = V_r \frac{\dfrac{R_2 \parallel R_T}{R_2 \parallel R_T + R_5}}{k - \dfrac{R_5 \parallel R_T}{R_5 \parallel R_T + R_2}} = V_r \frac{R_2 R_T}{k(R_2 R_5 + R_2 R_T + R_5 R_T) - R_5 R_T} = V_r \frac{R_2 R_T (R_3 + R_4)}{R_2 R_4 R_T + R_2 R_4 R_5 - R_3 R_5 R_T}$$

R_1 is obviously needed for bias current compensation. In fact, resistances seen from the OA inputs are $R_4 \parallel R_3$ and $R_1 + R_T$, both equal to approximately 11.1 kΩ.

1.2

The loop gain is easily computed:

$$G_{loop} = -A(s) \left(k - \frac{R_5 \parallel R_T}{R_5 \parallel R_T + R_2} \right) \approx -0.1A(s)$$

and remains negative if

$$k > \frac{R_5 \parallel R_T}{R_5 \parallel R_T + R_2} \Rightarrow R_2 > (R_5 \parallel R_T) \left(\frac{1}{k} - 1 \right) = (R_5 \parallel R_T) \frac{R_3}{R_4} \approx R_T \frac{R_3}{R_4} = 847\ \Omega,$$

largely satisfied in our case. To consider the effect of C_i, it is wise to just compute the position of the pole it generates, that is

$$f_p = \frac{1}{2\pi C_i (R_1 + R_T + R_3 \parallel R_4)} \approx 717\ \text{kHz},$$

larger than the 0dB frequency f_0 (= 200 kHz), so stability is not affected. The phase margin is now:

$$\phi_m = 90^\circ - \arctan\left(\frac{f_0}{f_p} \right) \approx 74^\circ.$$

1.3

We use current sources for resistor noise and group them into two sources at the OA inputs, having values

$$S_{I_n}^+ = \frac{4k_B T}{R_2} + \frac{4k_B T}{R_T} + \frac{4k_B T}{R_5} \approx 1.66 \times 10^{-22}\ \text{A}^2/\text{Hz};$$
$$S_{I_n}^- = \frac{4k_B T}{R_3} + \frac{4k_B T}{R_4} \approx 1.48 \times 10^{-24}\ \text{A}^2/\text{Hz},$$

while the voltage noise of R_1 is in series with the OA one ($S_{V_n} = S_V + 4k_B T R_1 \approx 3.77 \times 10^{-16}\ \text{V}^2/\text{Hz}$), as shown in Fig. 1 (left), where we considered $R_5 \parallel R_T \approx R_T$. The output PSD becomes then

$$S_{V_o} = \frac{S_{I_n}^+ (R_T \parallel R_2)^2 + S_{I_n}^- (R_3 \parallel R_4)^2 + S_{V_n}}{(k - R_T/(R_T + R_2))^2} = 1.76 \times 10^{-16} + 1.93 \times 10^{-14} + 3.99 \times 10^{-14} \approx \left(244\ \text{nV}/\sqrt{\text{Hz}} \right)^2,$$

which, considering a closed-loop bandwidth of 200 kHz, translates into an rms output noise:

$$\sqrt{\overline{V_o^2}} = 244 \times 10^{-9} \sqrt{\frac{\pi}{2} 2 \times 10^5} \approx 137\ \mu\text{V}.$$

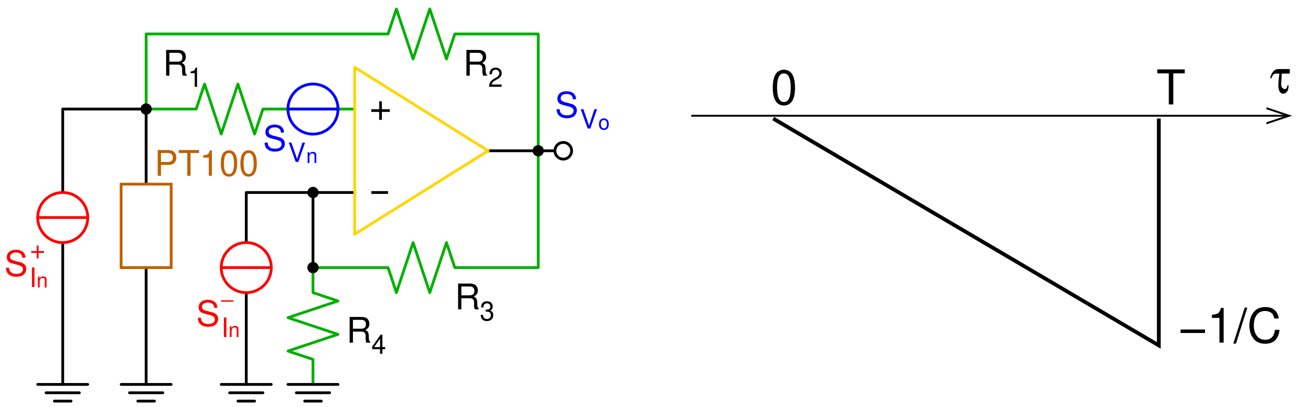


Figure 1: Left = Scheme for noise calculation. Right = Weighting function for the current signal.

1.4

Let's compute the output voltage with and without R_2 for the different temperatures:

T	V_o/V_r (no R_2)	V_o/V_r (with R_2)
$-100°$C	0.1858	0.1940
$0°$C	0.3044	0.3272
$100°$C	0.4165	0.4602
$200°$C	0.5226	0.5933

From these data we can extract the variation in V_o for a $100°$C temperature difference:

T range	$\Delta V_o/V_r$ (no R_2)	$\Delta V_o/V_r$ (with R_2)
$-100 - 0°$C	0.1186	0.1332
$0 - 100°$C	0.1121	0.1330
$100 - 200°$C	0.1061	0.1331

We can see that the scheme with R_2 is much more linear! This happens because R_2 introduces a small positive feedback that provides a slighly higher output for high values of R_T, compensating its nonlinearity.

Problem 2

2.1

The delta-like signal pulse is integrated by the capacitor, resulting in a step voltage that is sampled and gives an output signal

$$V_o = \frac{Q}{C}.$$

while the output noise is obviously two times the input voltage noise, leading to

$$\frac{S}{N} = \frac{Q/C}{\sqrt{2\overline{n_V^2}}} = \frac{Q}{C}\sqrt{\frac{T_n}{S_V}},$$

where we remember that in a triangular autocorrelation model $\overline{n_V^2} = S_V/2T_n$, as S_V is a unilateral PSD.

2.2

In the frequency domain, the current transfer would be

$$V_o = I_n \frac{1}{sC} W(t,s) \Rightarrow S_{V_o} = \frac{S_I}{C^2}\left|\frac{W(t,f)}{2\pi f}\right|^2,$$

where

$$W = 1 - e^{j2\pi fT} \Rightarrow |W|^2 = 2(1 - \cos(2\pi fT)).$$

We must then compute

$$\overline{V_o^2} = \frac{2S_I}{C^2}\int_0^\infty \frac{1-\cos(2\pi fT)}{(2\pi f)^2}df = S_I\frac{T}{\pi C^2}\int_0^\infty \frac{1-\cos x}{x^2}dx = S_I\frac{T}{2C^2},$$

where we have taken as equivalent bandwidth of the function one half of its first zero, i.e., π. The result in ths case is exact, but please do not take this as a general rule.

2.3

We begin with remembering that $w(t,\tau)$ is the response in t to a (current) δ function applied in τ. However, the current is integrated by the capacitor C (see the Laplace transform of the weighting function computed in 2.2, $W(t,s)/sC$), meaning that we have a step voltage at the input of the time-variant filter. The output can easily be computed and results in the triangular weighting function depicted in Fig. 1 (right). Please note that the new weighting function $w'(t,\tau)$ is the integral (up to $t=0$) of $w(t,\tau)$, as apparent from the Laplace analysis.
Note also that this approach can be easily applied to the case in 2.2, resulting in a rectangular weighting function of amplitude $1/C$, which returns the previous result.

2.4

The current noise contribution can easily be computed from

$$\overline{V_o^2} = \frac{S_I}{2}\int w'^2(t,\tau)d\tau = \frac{S_I}{2}\int_0^T \left(\frac{\tau}{T}\right)^2 d\tau = \frac{S_I}{2}\frac{T}{3}.$$

The weighting function for the voltage noise contains a delta function, meaning that the entire input noise is found at the output, and $\overline{V_o^2} = \overline{n_V^2}$. To make this point clear, we can start from

$$\overline{V_o^2} = \int R_{VV}(\gamma)k_{w_{tt}}(\gamma)d\gamma,$$

and note that k must contain a delta at $\gamma = 0$. The rest of the integral will give a result dependent on T_n, much lower than the mean square value. Calculations are left to the reader; the final result for the voltage noise is

$$\overline{V_o^2} = \overline{n_V^2} + \overline{n_V^2}\frac{T_n^2}{3T^2} = \overline{n_V^2} + S_V\frac{T_n}{6T^2}.$$