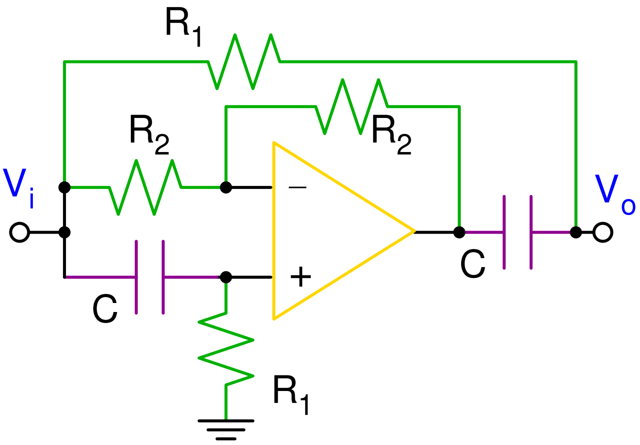
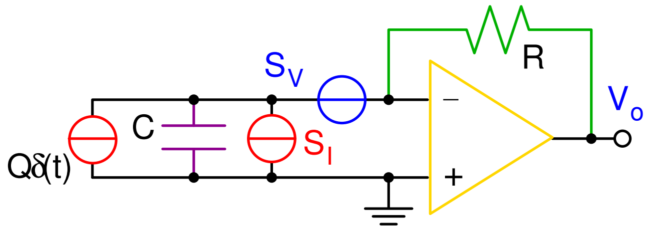
Electronics – September 9 2019 Prof. A. Spinelli





Problem 1

The scheme in the left figure is a filter. Parameter values are $C = 47$ nF, $R_1 = 68$ kΩ, $R_2 = 10$ kΩ. The OA has $GBWP = 1$ MHz.

1. Find the expression of the ideal gain and draw its Bode diagram (hint: evaluate $|T(j\omega)|$). What is the function of the filter?

2. Compute the loop gain and discuss the stability of the circuit.

3. Compute the output rms noise voltage considering the equivalent noise sources of the amplifier $\sqrt{S_V} = 25$ nV/$\sqrt{\text{Hz}}$ and $\sqrt{S_I} = 50$ pA/$\sqrt{\text{Hz}}$. In the calculations, remember that the noise BW of a transfer function $s\tau/(1+s\tau)^2$ is $1/(8\tau)$.

4. Consider the effect of resistor tolerances and quantify its impact on the attenuation of the filter.

Problem 2

The scheme in the right figure represents a detector sending delta-like current pulses to an (ideal) operational amplifier, having white voltage and current noise PSDs S_V and S_I. C is the total input capacitance.

1. Find the optimum S/N (final result, not just the formula, please).

2. Compute the weighting function of the optimum filter and give its expression in the time domain. Comment on the result.

3. The input pulse must be processed within a time $\pm T_P$ from its arrival. Find a suitable approximation for the weighting function and compute the new value of S/N. In the calculations, remember that $\int_0^\infty \text{sinc}^2 x \, dx = \pi/2$, $\int_0^\infty \text{sinc}^4 x \, dx = \pi/3$ and that $\sin^4 x = (4\sin^2 x - \sin^2 2x)/4$.

4. Replace now resistor R with an impedance $Z(s)$ and consider also the finite bandwidth of the amplifier. How does this affect the value of the optimum S/N and its corresponding weighting function?

For a correct evaluation, you are asked to write your answers in a readable way; thank you
Do a good job!

Results will be posted by September 12th **Mark registration: Monday, Sep. 16th**

Solution

Problem 1

1.1

We start with computing the output voltage of the OA as

$$V_o' = V_i\left(-1 + \frac{2sCR_1}{1+sCR_1}\right) = V_i\frac{-1+sCR_1}{1+sCR_1}.$$

The output voltage is now (linear superposition between V_i and V_o'):

$$V_o = V_i\frac{1}{1+sCR_1} + V_o'\frac{sCR_1}{1+sCR_1} = V_i\frac{1+(sCR_1)^2}{(1+sCR_1)^2}.$$

We have then two zeroes along the imaginary axis and two real coincident poles at $f_p = 1/2\pi R_1 C = 50$ Hz. The magnitude of the transfer function becomes:

$$|T(j\omega)| = \frac{|1-(\omega CR_1)^2|}{1+(\omega CR_1)^2},$$

and its Bode diagram is reported in Fig. 1 (left). Note that the asymptotic diagram is constant, but the actual transfer goes to zero at the frequency f_p. This is a so-called *notch* filter, designed to reject a single frequency line.

1.2

We ground the input and notice that the two $C - R_1$ blocks have no effect on the loop calculation, that simply becomes:

$$G_{loop} = -\frac{A(s)}{2}.$$

The circuit is obviously stable, with phase margin of 90° and zero-dB frequency equal to $f_{0dB} = GBWP/2 =$ 500 kHz.

1.3

If we recall once again that the input is grounded during noise calculations, we easily get:

$$V_o = V_n\frac{2sCR_1}{1+sCR_1} + I_n^- R_2\frac{sCR_1}{1+sCR_1} + I_n^+\frac{R_1}{1+sCR_1}\frac{2sCR_1}{1+sCR_1}.$$

The first two transfers must account for the additional pole at f_{0dB}, while the last one has a noise bandwidth equal to $(\pi/4)f_p$, leading to:

$$\begin{aligned}\overline{V_o^2} &= 4S_V\frac{\pi}{2}(f_{0dB}-f_p) + S_I R_2^2\frac{\pi}{2}(f_{0dB}-f_p) + 4S_I R_1^2\frac{\pi}{4}f_p \\ &= 1.96\times10^{-9} + 1.96\times10^{-7} + 2.31\times10^{-9} \approx (0.45\text{ mV})^2.\end{aligned}$$

1.4

We need to recompute the output voltage accounting for the different resistor values. If we label R_1' the upper R_1 resistor and R_2' the feedback resistor, we follow #1.1 and get

$$\frac{V_o'}{V_i} = -\frac{R_2'}{R_2} + \frac{R_2+R_2'}{R_2}\frac{sCR_1}{1+sCR_1} = \frac{-R_2'+sCR_1R_2}{R_2(1+sCR_1)}$$

and

$$\frac{V_o}{V_i} = \frac{1}{1+sCR_1'} + \frac{sCR_1'}{1+sCR_1'}\frac{V_o'}{V_i} = \frac{1+sCR_1\left(1-\frac{R_1'R_2'}{R_1R_2}\right)+s^2C^2R_1R_1'}{(1+sCR_1)(1+sCR_1')}.$$

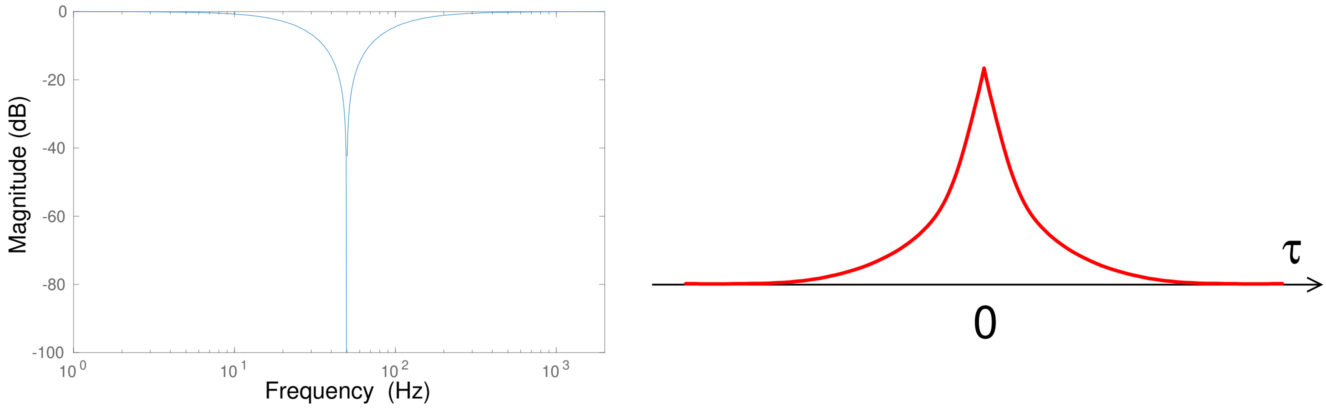


Figure 1: Left = Closed-loop transfer of the filter. Right = optimum weighting function.

Clearly, now the zeroes are no longer on the imaginary axis, meaning that the transfer function will not go to zero. To evaluate the term in parenthesis at the numerator, we consider the worst case:

$$\begin{aligned} R_1 &\to R_1(1+x) \qquad & R_1' &\to R_1(1-x) \\ R_2 &\to R_2(1+x) \qquad & R_2' &\to R_2(1-x) \end{aligned}$$

and obtain (see class slides for $CMRR$ calculations in differential amplifiers):

$$1 - \frac{R_1' R_2'}{R_1 R_2} = 1 - \frac{R_1 R_2 (1-x)^2}{R_1 R_2 (1+x)^2} \approx 1 - (1-2x)^2 \approx 4x.$$

At the notch frequency, $\omega C R_1 = 1$ and the numerator then becomes nearly

$$|1 + j4x\omega C R_1 - (\omega C R_1)^2| = 4x,$$

while each pole term has a magnitude of $1/\sqrt{2}$, leading to a value of $2x$ for the transfer function. As a reference, with $x = 1\%$ we get an attenuation of a factor $-20\log(2x) \approx 34$ dB.

Problem 2

2.1

The output signal is (frequency domain):

$$V_o(f) = QR,$$

while the output noise PSD is:

$$S_{V_o}(s) = S_V|sCR|^2 + S_I R^2 \Rightarrow S_{V_o}(f) = (2\pi C R)^2 S_V f^2 + S_I R^2.$$

The expression for the optimum S/N for non-white noise is then

$$\left(\frac{S}{N}\right)^2_{opt} = \int_{-\infty}^{\infty} \frac{2|V_o(f)|^2}{S_{V_o}(f)} df = \frac{2Q^2}{S_I} \int_{-\infty}^{\infty} \frac{df}{1 + K^2 f^2}, \quad K^2 = \frac{(2\pi C)^2 S_V}{S_I},$$

where the factor of 2 is due to the fact that S_{V_o} is a unilateral noise PSD. This leads to

$$\left(\frac{S}{N}\right)^2_{opt} = \frac{2Q^2}{S_I K}\pi = \frac{Q^2}{C\sqrt{S_V S_I}}.$$

Of course, the same result could have been obtained by considering a whitening filter (LPF) followed by the usual optimum filter for the white noise case.

2.2

The optimum weighting function is given by:

$$|W(f)| = \frac{2|V_o(f)|}{S_{V_o}(f)} = \frac{2Q}{RS_I}\frac{1}{1+K^2f^2},$$

whose inverse Fourier transform is (think of the time correlation of an LPF weighting function):

$$w(0,\tau) \propto e^{-|\tau|/T}, \quad T = \frac{K}{2\pi} = C\sqrt{\frac{S_V}{S_I}}.$$

This is plotted in Fig. 1 (right). Note that the function is *not causal* (i.e., it begins before the signal, which can be solved by adding a delay) and has infinite duration.

2.3

The weighting function can be approximated by a symmetric triangular function with amplitude A extending from $-T_P$ to $+T_P$. The output signal is then

$$y(t) = \int Q\delta(\tau)w(t,\tau)d\tau = QAR.$$

For noise calculation, we consider the frequency domain, where

$$W(f) = AT_P\,\mathrm{sinc}^2(\pi f T_P)$$

and solve for

$$\overline{n_y^2} = \int_0^\infty S_{V_o}(f)|W(f)|^2df = S_IR^2A^2T_p^2\int(1+K^2f^2)\,\mathrm{sinc}^4(\pi f T_p)df.$$

The integral can be expressed as

$$\frac{1}{\pi T_P}\int_0^\infty \mathrm{sinc}^4\,x\;dx + \frac{K^2}{(\pi T_P)^3}\int_0^\infty \frac{\sin^4 x}{x^2}dx = \frac{1}{3T_P} + \frac{K^2}{(\pi T_P)^3}\int(\mathrm{sinc}^2\,x - \mathrm{sinc}^2\,2x)dx,$$

yielding:

$$\left(\frac{S}{N}\right)^2 = \frac{Q^2}{S_I\left(\frac{T_P}{3}+\frac{T^2}{T_P}\right)} < \left(\frac{S}{N}\right)^2_{opt}$$

2.4

The real transfer function of the amplifier stage modifies both signal and noise, but not their ratio as expressed in #2.1. The same holds for R (note in fact that $(S/N)_{opt}$ does not depend on R). This means that these factors will not affect the optimum value of S/N.

The optimum weighting function neded to achieve that value of S/N, instead, will be obviously modified. If we call $H(f)$ the transfer function of the closed-loop amplifier stage, we have then

$$|W(f)| \propto \frac{1}{|H(f)|(1+K^2f^2)}$$