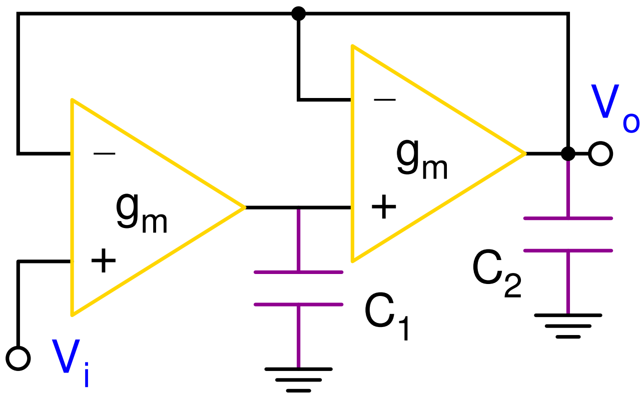
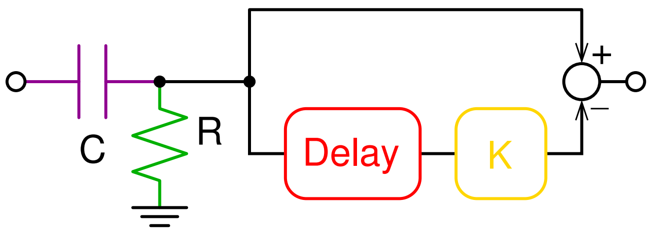
Electronics – July 19 2019 Prof. A. Spinelli





Problem 1

The scheme in the left figure is a filter built using transconductance amplifiers with gain g_m. Capacitor values are $C_1 = 1\ \mu\text{F} = 4C_2$. The amplifiers have $g_m = 32$ mS.

1. Find the expression of the closed-loop gain.
2. Consider the outer loop (i.e., the loop around the first amplifier). What is the minimum value of the amplifiers bandwidth that ensures stability of the circuit?
3. Compute the output rms noise voltage considering the equivalent noise source of the amplifiers $\sqrt{S_V} = 5\ \text{nV}/\sqrt{\text{Hz}}$. In the calculations, remember that the noise BW of a transfer function $s\tau/(1+s\tau)^2$ is $1/(8\tau)$.
4. Compute the delta function response of the circuit.

Problem 2

A baseline restorer stage has the scheme reported in the right figure.

1. Find the relation between delay time T and gain K so that the step voltage response of the filter has a finite length.
2. Compute the delta-function response (in time and frequency domains) and its time correlation. In the latter calculation, neglect for simplicity the exponential decay of the weighting function.
3. The input noise has an exponential autocorrelation $R_{xx} \approx \overline{n_x^2} e^{-|\tau|/T_n}$. Evaluate the output noise (consider for simplicity $K \approx 1$).
4. Consider an input sinusoidal signal. Can we devise a choice for (constant) parameters T and K that gives an output sinusoidal signal with two times the frequency of the input one?

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by July 24th **Mark registration: Thursday, July 25th**

Solution

Problem 1

1.1

If we label V_1 the output of the first stage, we can write:

$$V_1 = \frac{g_m}{sC_1}(V_i - V_o),$$

while the second stage yields

$$V_o = \frac{g_m}{sC_2}(V_1 - V_o).$$

The equations lead to

$$\frac{V_o}{V_i} = \frac{1}{1 + \frac{sC_1}{g_m} + \frac{s^2 C_1 C_2}{g_m^2}} = \frac{1}{\left(1 + \frac{2sC_2}{g_m}\right)^2}.$$

The circuit is a second-order filter with two coincident poles located at $f_p = g_m/\pi C_1 = 10$ kHz.

1.2

Let us start with the infinite-bandwidth case. We break the loop at the input of the second amplifier and apply a test voltage V_T. The output of the second amplifier is given by:

$$V_2 = \frac{g_m}{sC_2}(V_T - V_2) \Rightarrow V_2 = \frac{g_m}{g_m + sC_2} V_T,$$

which leads to

$$G_{loop} = -\frac{g_m}{sC_1} \frac{g_m}{g_m + sC_2}.$$

This function has a pole in the origin and a second one at $g_m/2\pi C_2 = 20$ kHz, and the zero-dB frequency is $f_{0dB} = g_m/2\pi C_1 = 5$ kHz (Fig. 1, left). The phase margin is $\phi_m = 90 - \arctan(1/4) \approx 76°$.

We now consider the finite amplifier bandwidth, setting $g_m = g_0/(1 + s\tau)$. This leads to

$$G_{loop} = -\frac{g_0}{sC_1(1 + s\tau)} \frac{g_0}{g_0 + sC_2 + s^2 \tau C_2},$$

where the second term gives two poles at approximately $f_{p1} = g_0/(2\pi C_2) \approx 20$ kHz (i.e., the previous one) and $f_{p2} = 1/(2\pi\tau)$. We then have two poles at f_{p2}. If we require – say – that the phase margin remains larger than 70°, we must lose no more than 3° per pole, i.e.:

$$\arctan(f_{0dB}/f_{p2}) = 3\pi/180 \Rightarrow f_{p2} \approx 19 f_{0dB} = 95 \text{ kHz}.$$

As a comment, please note that G_{loop} is already apparent from the expression of the gain computed in # 1.1, which can be written as $G_{id}/(1 - 1/G_{loop})$. Noting that the ideal gain (when g_m is infinite) is 1, we directly obtain the expression of the loop gain.

1.3

The voltage noise of the first amplifier goes as the signal, with unit transfer and two coincident poles at 10 kHz. We can then write

$$\overline{V_o^2} \approx S_V \frac{\pi}{2} 10^4 \approx 3.93 \times 10^{-13} \text{ V}^2 = (0.63\ \mu\text{V})^2.$$

Actually, the noise equivalent bandwidth for two coincident poles is slightly smaller ($1.22 f_p$ instead of $1.57 f_p$), but this will not change the result much (0.55 μV rms).

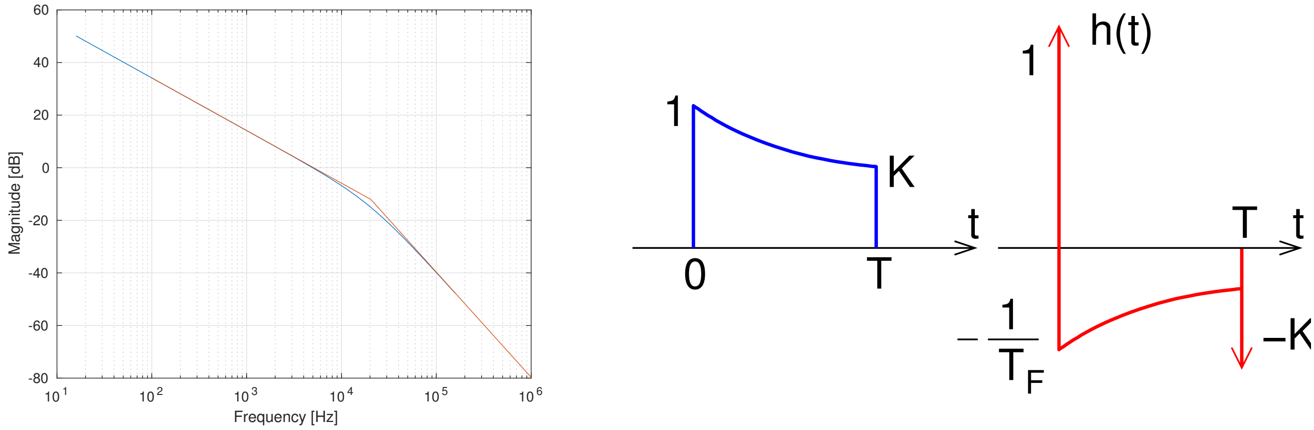


Figure 1: Left = Loop gain with no amplifier BW limitations. Right = step- and delta-function responses.

The transfer of the second amplifier noise becomes instead

$$V_o = V_n \frac{g_m/sC_2}{1+\frac{g_m}{sC_2}+\frac{g_m^2}{s^2C_1C_2}} = V_n \frac{sC_1/g_m}{\left(1+\frac{2sC_2}{g_m}\right)^2} = 2V_n \frac{2sC_2/g_m}{\left(1+\frac{2sC_2}{g_m}\right)^2} = 2V_n \frac{s\tau_p}{(1+s\tau_p)^2},$$

with one zero in the origin and two coincident poles. Taking advantage of the solution provided, we obtain:

$$\overline{V_o^2} \approx 4S_V \frac{1}{8\tau_p} \approx (0.89\ \mu\mathrm{V})^2.$$

Please note that integration of the asymptotic Bode diagram overestimates the noise in this case (about 1.17 μV rms). Total noise becomes then:

$$\overline{V_o^2} \approx (0.63^2 + 0.89^2)\ (\mu\mathrm{V})^2 = (1.09\ \mu\mathrm{V})^2.$$

1.4

One property of the Laplace transform tells that differentiation in the frequency domain correspond to multiplication by time in the time domain (with a minus sign). From the transfer in #1.1 and the single-pole transfer function we can write:

$$\frac{1}{1+s\tau_p} \Leftrightarrow \frac{1}{\tau_p}e^{-t/\tau_p} \Rightarrow \frac{1}{(1+s\tau_p)^2} \Leftrightarrow \frac{t}{\tau_p^2}e^{-t/\tau_p}.$$

The same result could have been obtained as a convolution (product in the Laplace domain) of $e^{-t/\tau_p}/\tau_p$ with itself.

Problem 2

2.1

The step voltage response of the CR filter is

$$V_1 = Ae^{-t/T_F}u(t),$$

$u(t)$ being the step function. The filter output becomes

$$V_o = V_1(t) - KV_1(t-T) = A\left(e^{-t/T_F}u(t) - Ke^{-(t-T)/T_F}u(t-T)\right).$$

For $t > T$ we have then

$$V_0 = A\left(e^{-t/T_F} - Ke^{-(t-T)/T_F}\right) = Ae^{-t/T_F}\left(1 - Ke^{T/T_F}\right) = 0 \Rightarrow K = e^{-T/T_F}.$$

The result is shown in Fig. 1 (left, blue curve).

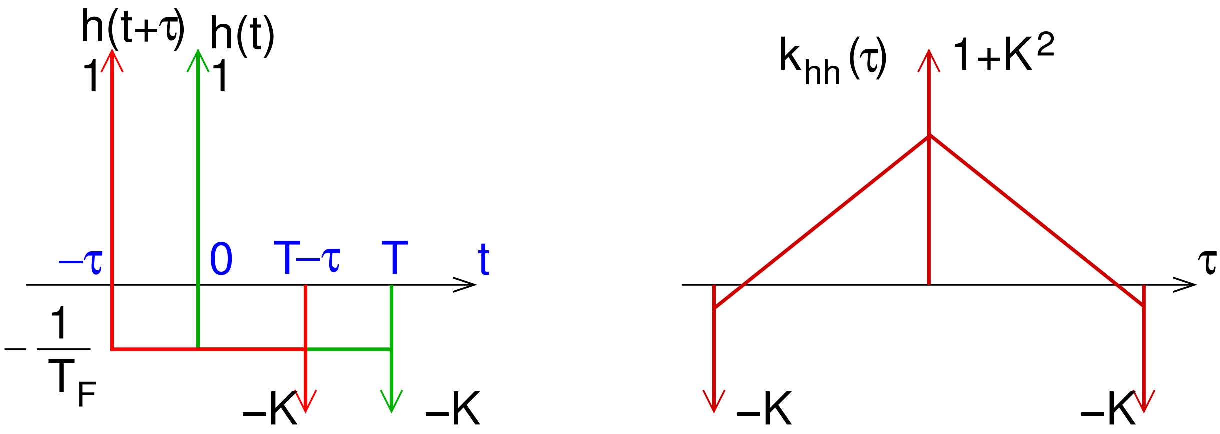


Figure 2: Approximated weighting functions (left) and resulting time correlation (right).

2.2

The delta-function response is the time derivative of what already computed and is also shown in Fig. 1 (right, red curve). In the frequency domain, the symbolic analysis of the scheme yields:

$$H(s) = \frac{sT_F}{1+sT_F}\left(1 - Ke^{-sT}\right) = \frac{sT_F}{1+sT_F}\left(1 - e^{-\frac{T}{T_F}(1+sT_F)}\right).$$

We now neglect the exponential decay and consider a flat behavior at $-1/T_F$. The time correlation (Fig. 2) contains the delta functions in $\tau = 0$ and $\tau = \pm T$ plus the time correlation of the rectangle (with the deltas):

$$k_{hh}(\tau) = (1+K^2)\delta(\tau) - K\delta(|\tau| - T) + \frac{T - |\tau|}{T_F^2} + \frac{K-1}{T_F}.$$

2.3

The output noise is given by

$$\overline{n_y^2} = \int R_{xx}(\tau)k_{hh}(\tau) = \overline{n_x^2}\left(1 + K^2 - 2Ke^{-T/T_n} + 2\int_0^T \left(\frac{T-\tau}{T_F^2} + \frac{K-1}{T_F}\right)e^{-\tau/T_n}d\tau\right),$$

where

$$\int_0^T \frac{T-\tau}{T_F^2}e^{-\tau/T_n}d\tau = \left(\frac{T_n}{T_F}\right)^2\left(\frac{T}{T_n} - 1 + e^{-T/T_n}\right).$$

For $K = 1$ we then have:

$$\overline{n_y^2} = \overline{n_x^2}\left(2 - 2e^{-T/T_n} + 2\left(\frac{T_n}{T_F}\right)^2\left(\frac{T}{T_n} - 1 + e^{-T/T_n}\right)\right)$$

2.4

Obviously no! the circuit is linear and the output will always be a sinusoidal signal with the same frequency as the input. T and K will only affect its amplitude and phase.