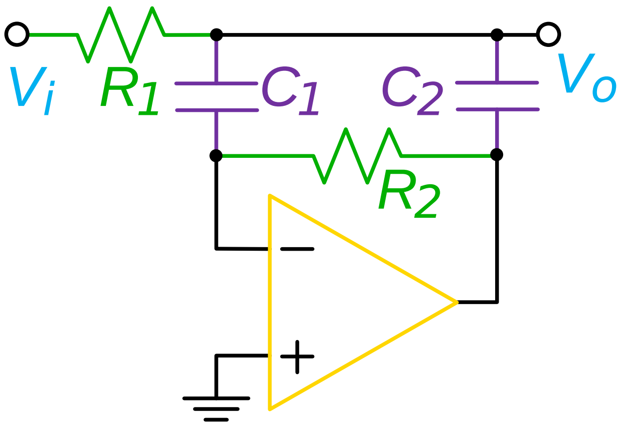
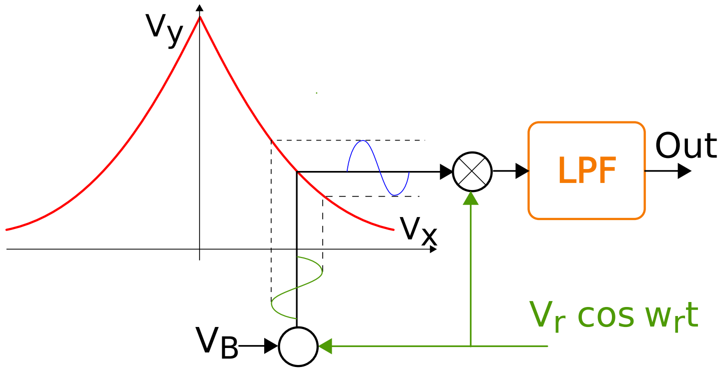
Electronics – July 17 2020 Prof. A. Spinelli





Problem 1

The scheme in the left figure is an active filter. Parameter values are $R_1 = R_2 = 1.5$ kΩ, $C_1 = C_2 = 10$ nF. The OA has $GBWP = 1$ MHz and $A_0 = 100$ dB.

1. Find the expression of the closed-loop (ideal) gain.
2. Discuss the stability of the circuit.
3. Compute the output rms noise voltage considering the equivalent noise source of the amplifier, $\sqrt{S_V} = 10\ \text{nV}/\sqrt{\text{Hz}}$.
4. We want to employ the scheme to build a 4^{th}-order LP filter: sketch the relative scheme. How can we modify it to obtain a band-pass filter?

Problem 2

A system has the I/O characteristic reported in the right figure, where $V_y = V_0 e^{-|V_x|/V_T}$, with $V_0 = 5$ V and $V_T = 2$ V. An LIA is employed to measure the value of the slope of the curve, as schematized in the right figure, where $V_x = V_B + V_r \cos(\omega_r t)$. The maximum reference amplitude is $V_r = 100$ mV. The signal is masked by a unilateral flicker noise $S_V = K/f$ with $K = 10^{-5}$ V^2 and corner frequency $f_{nc} = 1$ kHz.

1. Compute the minimum value of the slope (and the corresponding value of V_B) that can be measured if the measurement time must be shorter than 1 s (hint: linearize the characteristic around V_B).
2. Consider the case in which the input signal is centered around $V_B = 0$. Compute S/N and discuss the case in which a square-wave mixer is used. Would it improve S/N?
3. With reference to the first point ($V_B \neq 0$), consider the case in which the reference signal is a square wave and compute the new S/N.
4. For the case of sinusoidal reference, consider now the full exponential characteristics and provide an approximated expression for the output signal.

Useful expressions:

$$|\cos x| = \frac{2}{\pi} + \frac{4}{\pi}\sum_k \frac{(-1)^k}{1-4k^2}\cos(2kx) \qquad \cos^3 x = \frac{3\cos x + \cos 3x}{4} \qquad \cos^5 x = \frac{10\cos x + 5\cos 3x + \cos 5x}{16}$$

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by July 23rd Mark registration: Monday, July 27th

Solution

Problem 1

1.1

Because of the $C_1 - R_2$ connection, OA acts as a differentiator, and its output voltage V_{OA} is simply equal to $-sC_1R_2V_o$. Applying the KCL at the output node, we get

$$\frac{V_i - V_o}{R_1} = sC_1V_o + sC_2(V_o - V_{OA}),$$

which leads to

$$\frac{V_o}{V_i} = \frac{1}{1 + s(C_1 + C_2)R_1 + s^2C_1C_2R_1R_2} = \frac{1}{(1 + sCR)^2},$$

Where $C_1 = C_2 = C$ and $R_1 = R_2 = R$. We have a 2^{nd} order LPF with coincident poles at 10 kHz.

1.2

We can discuss the stability by simply looking at the high-frequency behavior of G_{loop}, without actually performing the calculation. For frequencies higher than the poles added by the capacitors, we can see that C_1 and C_2 provide a short-circuit connecting the output to the non-inverting input of the OA. This means that at high frequencies we have $G_{loop} \approx -A(s)$ and the stability is ensured. For those wishing to perform the actual calculation, the final expression is

$$G_{loop} = -A(s)\frac{(1 + sCR)^2}{1 + 3sCR + (sCR)^2}$$

1.3

We place the voltage noise source at the OA input and write the following equation for the OA output:

$$V_{OA} = -sCRV_o + V_n(1 + sCR).$$

The KCL at the output node now reads:

$$\frac{V_o}{R} = sC(V_n - V_o) + sC(V_{OA} - V_o),$$

resulting in

$$\frac{V_o}{V_n} = \frac{sCR(2 + sCR)}{(1 + sCR)^2}.$$

The Bode plot is shown in Fig. 1 (left). We can neglect the slight overshoot and pick 5 kHz as the low-frequency cutoff (it is the zero-dB crossing frequency of the asymptotic diagram), obtaining

$$\overline{V_o^2} \approx S_V\frac{\pi}{2}(GBWP - 5 \times 10^3) \approx (12.5\ \mu\text{V})^2,$$

where the additional pole at $GBWP$ added by the OA has been also accounted for.

1.4

To build a 4^{th}-order LP filter we obviously need two stages. However, we cannot simply connect them in series because their input/output impedances are not ideal and the transfer function would be modified. A decoupling buffer stage is then needed.

For a BP filter, we need an LP and an HP filter, again cascaded via a buffer stage. The HP filter can usually be obtained from an LP filter by swapping capacitors and resistors, as shown in Fig. 1 (right). To check if this is the case, we can proceed as in #1.1, where now $V_{OA} = -V_o/sCR$, and write:

$$sC(V_i - V_o) = \frac{V_o}{R} + \frac{V_o - V_{OA}}{R},$$

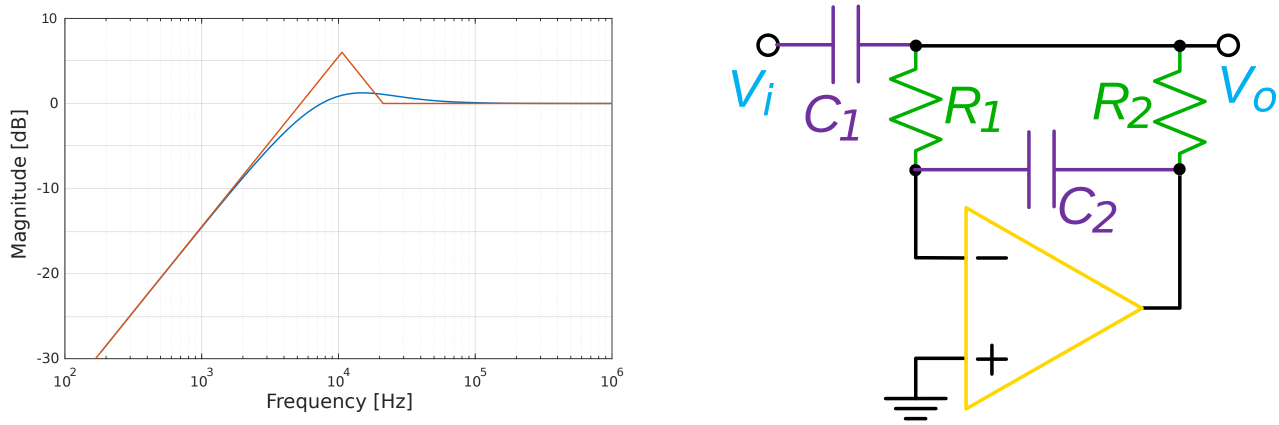


Figure 1: Left = Bode plots for noise transfer (real and asymptotic). Right = HPF scheme.

which leads to

$$\frac{V_o}{V_i} = \frac{(sCR)^2}{(1+sCR)^2},$$

confirming the approach. Note that a faster way to get to this result is to start from the LP expression and apply the substitution $sCR \to 1/sCR$.

Problem 2

2.1

A first-order expansion of the characteristic around V_B leads to:

$$V_y(V_x) \approx V_y(V_B) + \left.\frac{dV_y}{dV_x}\right|_{V_B}(V_x - V_B) = V_y(V_B) + \left.\frac{dV_y}{dV_x}\right|_{V_B} V_r \cos\omega_r t.$$

The amplitude of the sinusoidal signal at the LIA input is then given by:

$$V_i = \left|\left.\frac{dV_y}{dV_x}\right|_{V_B}\right| V_r = \frac{V_0}{V_T} e^{-V_B/V_T} V_r,$$

while the constant signal is obviously rejected. The output S/N becomes then

$$\frac{S}{N} = \frac{V_i}{\sqrt{2S_V(f_r)BW_n}} = 1 \Rightarrow \left|\frac{dV_y}{dV_x}\right| = \frac{\sqrt{2S_V(f_r)BW_n}}{V_r}.$$

To improve the sensitivity, it is better to move the modulation frequency above f_{nc}, where $S_V = K/f_{nc} = 10^{-8}$ $\mathrm{V}^2/\sqrt{\mathrm{Hz}}$. As for the choice of the output LPF pole, we should recall that its time constant controls the LIA output. If the output must reach the final value in 1 s, the time constant τ must be smaller than about 0.2 s. Picking $\tau = 0.1$ s, we have $f_p = 1/(2\pi\tau) \approx 1.6$ Hz, leading to $BW_n = 2.5$ Hz and $|dV_y/dV_x| = 2.2 \times 10^{-3}$. In turn, this means

$$V_B = V_T \log\left(\frac{V_0}{|dV_y/dV_x|V_T}\right) \approx 14 \text{ V}.$$

2.2

Around $V_B = 0$ the characteristic is symmetric, meaning that the input signal to the LIA can be expressed as

$$V_i = V_0 - \frac{V_0}{V_T} V_r |\cos\omega_r t|,$$

where the linear approximation still holds for the two branches of the characteristics. The fundamental frequency of this function is $2f_r$, so no signal is demodulated to the baseband and the output signal is zero. Even a square-wave mixer would not change this situation, as the additional transmission windows are located at the odd harmonics of f_r, while the signal only contains even components.

2.3

If the reference signal is a square wave with amplitude $\pm V_r$, the amplitude of the square wave at the input of the LIA is

$$2A = V_0 \left(e^{-(V_B - V_r)/V_T} - e^{-(V_B + V_r)/V_T} \right) \approx V_0 e^{-V_B/V_T} \frac{2V_r}{V_T},$$

where we have again used a linear approximation of the exponential relation. Considering a square-wave demodulation with amplitude B, we obtain a constant output signal of AB, leading to the expression for S/N:

$$\frac{S}{N} = \frac{AB}{\sqrt{2 \left(\frac{2B}{\pi}\right)^2 S_v \ BW_n \frac{\pi^2}{8}}} = \frac{A}{\sqrt{S_v \ BW_n}}.$$

In the above expression, the factor $\pi^2/8$ accounts for the white-noise transmission through the higher-order windows, while $2B/\pi$ accounts for the amplitude of the fundamental harmonic. Note that S/N is a factor of $\sqrt{2}$ higher than in #2.1.

2.4

From a formal standpoint, the input signal to the LIA is now

$$V_i = V_0 e^{-(V_B + V_r \cos(\omega_r t))/V_T} = V_0 e^{-V_B/V_T} \ e^{-V_r \cos(\omega_r t)/V_T},$$

of whom we should compute the Fourier transform, which leads to Bessel integrals. An alternative is to expand the exponential term, obtaining

$$V_i = V_0 e^{-V_B/V_T} \sum_k \left(-\frac{V_r}{V_T} \right)^k \frac{\cos^k(\omega_r t)}{k!}.$$

Looking at the expressions provided, it is clear that only the odd powers have a contribution at f_r, while even values of k provide even harmonics and give no output contribution. We can therefore write the first terms of the sum as

$$V_i \approx V_0 e^{-V_B/V_T} \left(\frac{V_r}{V_T} \cos(\omega_r t) + \left(\frac{V_r}{V_T} \right)^3 \frac{\cos^3(\omega_r t)}{3!} + \left(\frac{V_r}{V_T} \right)^5 \frac{\cos^5(\omega_r t)}{5!} \right).$$

Expanding the higher frequency terms as suggested in the text, we get, for the only relevant component at f_r:

$$V_i = V_0 e^{-V_B/V_T} \frac{V_r}{V_T} \left(1 + \left(\frac{V_r}{V_T} \right)^2 \frac{3}{24} + \left(\frac{V_r}{V_T} \right)^4 \frac{10}{1920} \right) \cos(\omega_r t).$$

the terms with the powers of $V_r/V_T = 0.05$ represent the non-linearity error, which in our case is nearly 3.13×10^{-4}, dominated by the first term.