Electronics – February 13 2020

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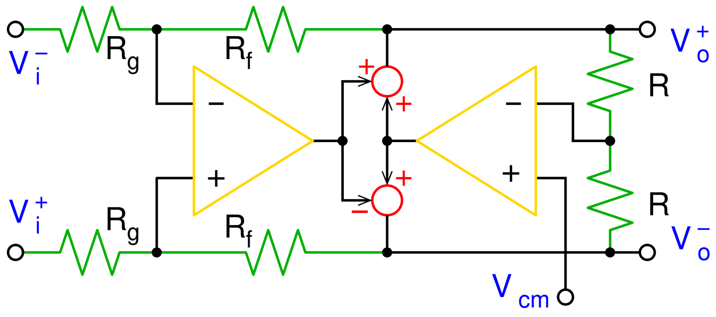

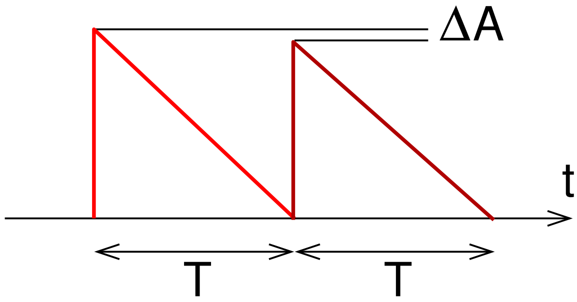

Problem 1

The scheme in the left figure is a differential amplifier (circles are ideal summing nodes). Parameter values are $R_g = 15$ kΩ, $R_f = 150$ kΩ, $R = 30$ kΩ. The OAs have $GBWP = 4$ MHz. Consider for simplicity $V_{cm} = 0$.

1. Find the expression of the closed-loop (ideal) gain.
2. Compute the loop gain for OA1 (consider OA2 as ideal for simplicity) and discuss the stability as a function of the gain.
3. Compute the (differential) output rms noise voltage considering the equivalent noise sources of the amplifiers, $\sqrt{S_V} = 20\ \text{nV}/\sqrt{\text{Hz}}$.
4. The OAs (and summing stages) are powered from a 5 V single-supply source and the input signal is fully differential with dynamics of ±100 mV (i.e., $V_i^+, V_i^- \leq 100$ mV). Find the maximum value for the gain (and the corresponding value of V_{cm}).

Problem 2

A signal is made up of a series of two consecutive pulses having approximatively triangular shape and width $T \approx 10\ \mu$s (right figure). We want to measure the difference in pulse amplitude $\Delta A \approx 100\ \mu$V in presence of a white noise with bilateral PSD $\lambda = 10^{-12}\ \text{V}^2/\text{Hz}$.

1. We use two gated integrators to sample the amplitudes, which are then subtracted. Compute the resulting S/N.
2. To improve S/N we replace the GIs with boxcar averagers. Compute the BA parameters that allow to reach $S/N = 1$.
3. The pulses width T is not constant, but can fluctuate between 7 and 13 μs with uniform probability. Evaluate the impact on the BA S/N.
4. The input noise has nearly-rectangular autocorrelation with correlation time $T_n \approx 2T$. Estimate the output noise.

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by February 17th

Mark registration: Tuesday, Feb. 18th

Solution

Problem 1

1.1

If $V_{cm} = 0$, the inverting input of OA2 is also at zero, meaning that we have $V_o^- = -V_o^+$; the output is fully differential. To compute its value, we can evaluate the input voltage of OA1, V_{OA}, from the upper and lower resistive dividers:

$$\left.\begin{aligned} V_{OA} &= V_i^- \frac{R_f}{R_f + R_g} + V_o^+ \frac{R_g}{R_f + R_g} \\ V_{OA} &= V_i^+ \frac{R_f}{R_f + R_g} + V_o^- \frac{R_g}{R_f + R_g} \end{aligned}\right\} \Rightarrow V_o^+ = \frac{R_f}{2R_g}(V_i^+ - V_i^-),$$

which means that the differential gain (differential output divided by differential input) is $R_f/R_g = 10$.

1.2

We break the loop at the output of OA1, applying a test signal V_s. If we label V_{o2} the output of OA2, the circuit outputs are $V_o^+ = V_{o2} + V_s$ and $V_o^- = V_{o2} - V_s$. However, as in the previous case, we have $V_o^- = -V_o^+$, i.e., $V_{o2} = 0$. The voltage at the input pins of OA1 are then

$$V^- = \frac{R_g}{R_f + R_g} V_s = -V^+ \Rightarrow G_{loop} = -A(s)\frac{2R_g}{R_f + R_g}.$$

Note that the circuit is stable with a closed-loop bandwidth equal to $f_{0dB} = GBWP/5.5 \approx 0.7$ MHz. The circuit is in fact stable for any (differential) gain larger than 1. Gains smaller than one (if needed) would result in a loop gain larger than $A(s)$, which is better to avoid.

For the sake of discussion, we compute the loop gain for OA2. We break the loop at the output and apply the test voltage. The two outputs become $V_o^+ = V_s + V_{o1}$ and $V_o^- = V_s - V_{o1}$. Please note that now $V_o^- \neq -V_o^+$, as the loop around OA2 is broken! To compute V_{01}, we set $V^+ = V^-$ for OA1, i.e.:

$$(V_s + V_{o1})\frac{R_g}{R_f + R_g} = (V_s - V_{o1})\frac{R_g}{R_f + R_g} \Rightarrow V_{o1} = 0.$$

This means that the inverting input of OA2 is at bias V_s and its loop gain is equal to $-A(s)$.

1.3

The contribution of the second OA can be immediately computed noticing that the V_{cm} input sets the common-mode voltage of the ouput (we have in fact $V_{cm} = (V_o^+ + V_o^-)/2$). This means that the differential noise is zero.
The contribution from the first OA must be computed thoroughly (see scheme in Fig. 1, left). Note that AO2 acts as an inverting amplifier, so that $V_o^- = -V_o^+$. We can then equal the voltages at the input pins of OA1, obtaining:

$$V_o^+ \frac{R_g}{R_f + R_g} = -V_o^+ \frac{R_g}{R_f + R_g} + V_n \Rightarrow V_o^+ = V_n \frac{R_f + R_g}{2R_g} \Rightarrow V_o^+ - V_o^- = V_n \frac{R_f + R_g}{R_g} = 11V_n.$$

The output rms noise becomes then

$$\overline{(V_o^+ - V_o^-)^2} = 121\ S_V \frac{\pi}{2} f_{0dB} \approx 235\ \mu\text{V}.$$

1.4

The output voltages must swing in the $0 - 5$ V range, meaning that the best value for their common mode is 2.5 V. It is then best to pick $V_{cm} = 2.5$ V. The maximum output differential signal is then equal to 5 V (a bit smaller in reality, because of the output stage limitation). Given that the maximum input differential signal is 200 mV, the maximum achievable gain is 25.

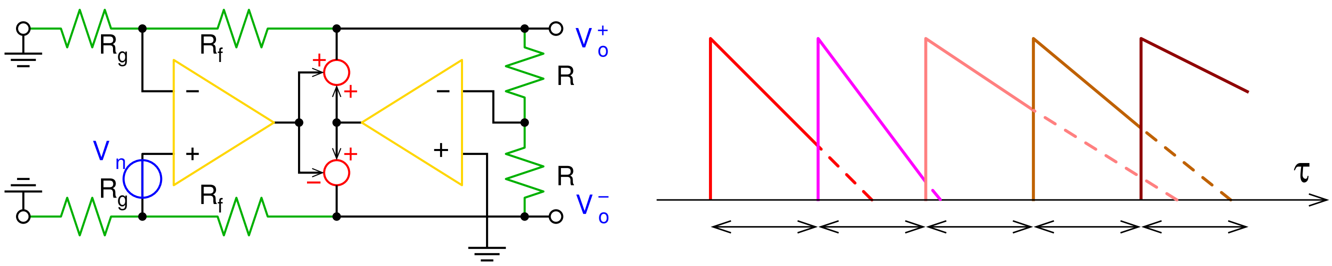

Figure 1: Left = Scheme for OA1 noise calculation. Right = Input signal to the BA in the equivalent time.

Problem 2

2.1

We can use an integration time $T_G = 2T/3$, which is the optimum for a triangular signal. In this case signal and noise at the GI outputs become

$$V_o = \left(A + \frac{A}{3}\right)\frac{T_G}{2} = \frac{2}{3}AT_G \qquad \overline{n_o^2} = \lambda T_G.$$

When we take the difference, we double the mean square noise, leading to the final expression

$$\frac{S}{N} = \Delta A \frac{2}{3}\sqrt{\frac{T_G}{\lambda}}\frac{1}{\sqrt{2}} \approx 0.122.$$

2.2

If we are using a BA to improve S/N, we can retain the same integration time as before, i.e., $T_C = T_G$. We then need a number of equivalent pulses given by:

$$0.122\sqrt{N_{eq}} = 1 \Rightarrow N_{eq} \approx 67.1 \Rightarrow 68,$$

meaning a time constant of the filter equal to

$$\frac{2T_F}{T_C} = 68 \Rightarrow T_F \approx 227\ \mu\text{s}.$$

Note that we could have achieved this result also by looking at the behavior in the "equivalent time", where the BA (that now behaves like an LPF) sees a sawtooth wave with average value $2A/3$.

2.3

Since the integration time remains constant, a change in T has no effect on the noise, but only affects the output signal. As said previously, if we look in the "equivalent time" τ frame, we see an LPF with an input signal given by a sawtooth wave having different "teeth" (Fig. 1, right). Since the time constant is very large compared to T_G, the output is the average value of the input signal, which is

$$\overline{V_o} = A\overline{\left(2 - \frac{T_C}{T}\right)}\frac{T_C}{2} = \frac{AT_C}{2}\left(2 - T_C\overline{\left(\frac{1}{T}\right)}\right),$$

where (labelling $T_M = 13\ \mu$s, $T_m = 7\ \mu$s)

$$T_C\overline{\left(\frac{1}{T}\right)} = T_C\int\frac{p(T)}{T}dt = \frac{T_C}{T_M - T_m}\int_{T_m}^{T_M}\frac{dT}{T} = \frac{T_C}{T_M - T_m}\log\frac{T_M}{T_m} \approx 0.688.$$

Please note that in this case the result is not much different from the previous one (equal to $2/3$), obtained for constant $T = \overline{T}$ (but this is not a general result!). Parameters of the BA will be barely affected (new $N_{eq} = 64$).

2.4

We can start with an approximate solution, considering each BA separately and then summing the mean square values. Given the extent of the noise autocorrelation R_{nn}, only the central part of the weighting function time correlation matters. This can be approximated by a triangular function with basis equal to $2T_C$ and value $1/2T_F$ in zero (unity-gain BA). We then have:

$$\overline{n_y^2} = \int k_{w_{tt}}(\gamma) R_{nn}(\gamma) d\gamma = \overline{n_x^2} \frac{T_C}{2T_F},$$

meaning that a value $\overline{n_x^2} T_C/T_F$ is obtained after the difference operation. Note that this simple solution is approximate, because it neglects the correlation between the noises collected by the two BAs. Since in this case we are taking the difference between the two BA outputs, having correlated noise results in a smaller total noise and the above expression is an overestimate of the output noise.