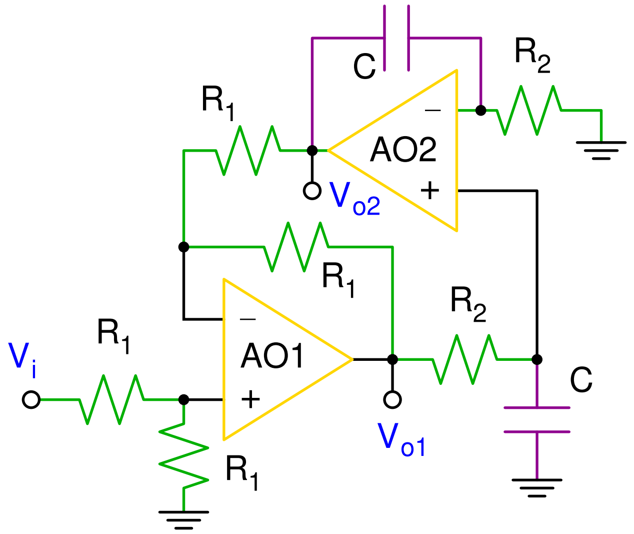
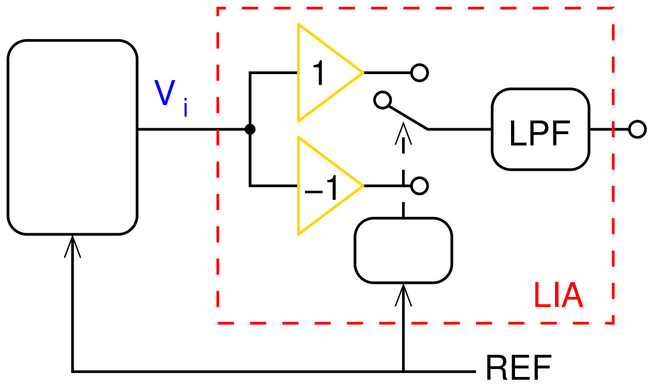
Electronics – January 16 2020 Prof. A. Spinelli





Problem 1

The scheme in the left figure is a filter. Parameter values are $C = 0.16\ \mu$F, $R_1 = 10$ kΩ, $R_2 = 100$ kΩ. The OAs have $GBWP = 1$ MHz.

1. Find the expression of the ideal gains.
2. Compute the loop gain for one OA at your choice, considering the other one as ideal, and discuss the stability.
3. Compute the output rms noise voltages considering the equivalent noise sources of the amplifiers, $\sqrt{S_V} = 20\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$.
4. We want to change both gains, adding one pole to each of them. What is the simplest solution that can be devised? (do not add an LPF at each output but find a better solution).

Problem 2

The right figure schematizes an LIA used for synchronous detection, with a sinusoidal reference signal. The LIA input signal is $V_i = A\cos\omega_R t$ with amplitude $A \approx 10\ \mu$V, submerged in a flicker noise with bilateral PSD $S_V = K/f$, with $K = 10^{-10}$ V^2 and noise corner frequency $f_c = 1$ kHz. Note that the LIA uses a <u>square-wave detection</u>.

1. Find a set of LIA parameters that grants $S/N = 10$.
2. The two amplifiers in the LIA have slightly different gains (apart from the sign). Find the new expression for the output S/N.
3. Because of non-linear response of the sensor, the LIA input signal becomes $V_i = A_1\cos(\omega_R t) + A_3\cos(3\omega_R t)$, with $A_3 \ll A_1$. Can we measure the value of A_3 with the current scheme? If not, add a simple circuit and compute the minimum value of A_3 that can now be measured.
4. Assume that the quantity of interest is the *phase* of the modulated signal $A\cos(\omega_R t + \phi)$, that changes with time. Are the parameters computed in #2.1 still valid? How could we improve the setup?

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by January 20th **Mark registration: Tuesday, Jan. 21st**

Solution

Problem 1

1.1

We know that the input pins of OA1 are both at a voltage equal to $V_i/2$. The relation between V_{o1} and V_{o2} is instead

$$V_{o2} = \frac{1/sC + R_2}{R_2}V^+ = \frac{1+sCR_2}{sCR_2}\frac{1/sC}{R_2+1/sC}V_{o1} = \frac{1}{sCR_2}V_{o1},$$

meaning that the second stage is actually a non-inverting integrator. Considering now the two resistors R_1 between V_{o1} and V_{o2} we can compute V^- of OA1 as

$$\frac{V_{o1}}{2} + \frac{V_{o2}}{2} = \frac{V_i}{2} \Rightarrow V_{o1} = \frac{sCR_2}{1+sCR_2}V_i, \quad V_{o2} = \frac{1}{1+sCR_2}V_i.$$

We then have an LP-HP filering of the input, with pole frequency equal to $f_p = 1/2\pi R_2 C = 10$ Hz.

1.2

We start with OA1, opening its loop at the OA output and applying a test signal V_s. We now have $V_{o2} = V_s/sCR_2$ and

$$V^- = \frac{V_s + V_{o2}}{2} = \frac{1+sCR_2}{2sCR_2}V_s \Rightarrow G_{loop1} = -A(s)\frac{1+sCR_2}{2sCR_2}.$$

As for OA2, we proceed in an analogous manner, noting that OA1 now behaves as an inverting amplifier with gain of -1. We then have:

$$\left.\begin{aligned} V^- &= \frac{R_2}{R_2+1/sC}V_s = \frac{sCR_2}{1+sCR_2}V_s \\ V^+ &= -\frac{1}{1+sCR_2}V_s \end{aligned}\right. \Rightarrow G_{loop2} = -A(s).$$

Both loops are obviously stable, with phase margins of 90°. The first OA has a zero-dB crossing frequency $f_{0dB} = GBWP/2 = 500$ kHz, while the second OA has $f_{0dB} = GBWP = 1$ MHz.

1.3

The contribution of the first OA noise can be immediately computed noticing that the noise voltage source can be placed in series to the non-inverting input of OA1, thus having the same transfer as a signal $V_i/2$. We then have:

$$V_{o1} = \frac{2sCR_2}{1+sCR_2}V_{n1} \qquad V_{o2} = \frac{2}{1+sCR_2}V_{n1}.$$

The scheme for V_{n2} can be found in Fig. 1 (left). Note that AO1 acts as an inverting amplifier, so that $V_{o1} = -V_{o2}$. The non-inverting input of OA2 is then

$$V^+ = \frac{V_{o1}}{1+sCR_2} + V_{n2}$$

from which

$$V_{o2} = V^+\frac{1+sCR_2}{sCR_2} = \frac{V_{o1}}{sCR_2} + V_{n2}\frac{1+sCR_2}{sCR_2}.$$

Recalling that $V_{o1} = -V_{o2}$ we immediately get $V_{o2} = V_{n2} = -V_{o1}$. This leads to:

$$\begin{aligned} \overline{V_{o1}^2} &= 4S_V\frac{\pi}{2}\left(\frac{GBWP}{2} - f_p\right) + S_V\frac{\pi}{2}\frac{GBWP}{2} \approx (40\ \mu\text{V})^2 \\ \overline{V_{o2}^2} &= 4S_V\frac{\pi}{2}f_p + S_V\frac{\pi}{2}GBWP \approx (25\ \mu\text{V})^2 \end{aligned}$$

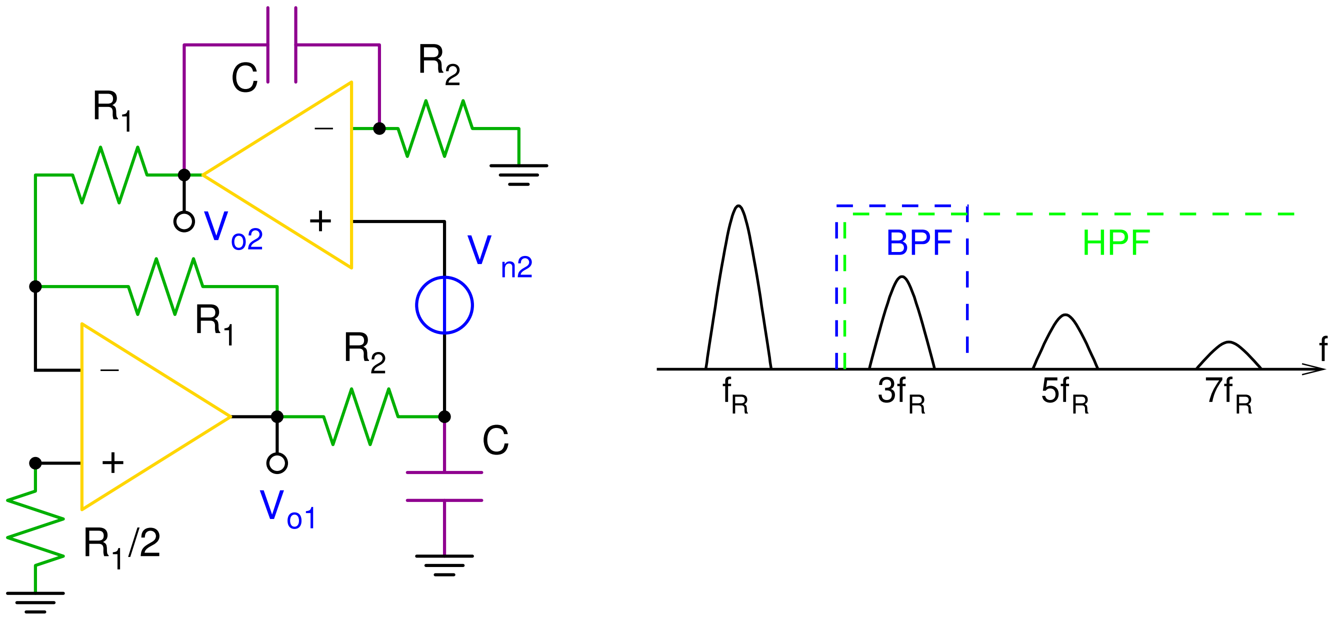


Figure 1: Left = Scheme for OA2 noise calculation. Right = spectral response of the LIA.

1.4

The simplest solution is to just add a capacitor at the input, in parallel to the resistor R_1 connecting the non-inverting input of OA1 to ground.

Problem 2

2.1

It is wise to pick a modulation beyond f_c, so let's say we have $f_R = 2$ kHz. The noise collected by the PSD is then white with a PSD given by $S_{WN} = K/f_c = 10^{-13}$ V^2/Hz, and the expression for the LIA S/N becomes

$$\frac{S}{N} \approx \frac{A}{\sqrt{4S_{WN}BW_n(\pi^2/8)}} = 10 \Rightarrow BW_n = \frac{A^2}{493S_{WN}} \approx 2 \text{ Hz}.$$

If we are using a single-pole output LPF, this amounts to $f_p \approx 1.3$ Hz, becoming about 1.6 for two coincident poles.

2.2

If we call B^+ and B^- the two gains, the mixer multiplies the input signal and noise by a square wave with such high and low values. The amplitude of the wave will differ slightly from 2, but this is not an issue as such amplitude does not enter in the expression for S/N. However, the average value of the square wave is now

$$B_{AV} = \frac{B^+ + B^-}{2} \neq 0,$$

meaning that in the spectral response we have a transmission window around $f = 0$. Such a term brings the input noise directly at the output, and the output S/N becomes:

$$\frac{S}{N} = \frac{2AB/\pi}{\sqrt{\left(\frac{2B}{\pi}\right)^2 4S_{WN}BW_n\frac{\pi^2}{8} + B_{AV}^2 K \log\left(\frac{f_p}{f_{min}}\right)}} = \frac{A}{\sqrt{4S_{WN}BW_n\frac{\pi^2}{8} + \left(\frac{\pi}{2}\right)^2\left(\frac{B_{AV}}{B}\right)^2 K \log\left(\frac{f_p}{f_{min}}\right)}}$$

where f_{min} is the minimum frequency, related to the operating time of the instrument. As a reference, if we take such time to be an hour, meaning that $f_{min} \approx 1/3600 \approx 0.28$ mHz, and a precision of 1%, i.e., $B_{AV}/B = 10^{-2}$, S/N is reduced from 10 to about 9.

2.3

Since the square wave contains all the odd harmonics of the reference frequency, the component at $3\omega_R$ is indeed demodulated and sent to the output. The problem is that the signal at ω_R is demodulated as well, meaning that (in absence of phase errors) the output signal would be proportional to $A_1 + A_3/3$ (the factor of 3 is because the amplitude of the third harmonic is one third that of the fundamental), making it impossible to measure the small quantity A_3.
In order to do this, we need to remove the signal A_1, which can be done with a high- or band-pass filter (with suitable attenuation) inserted at the input of the LIA (see Fig. 1, right). If we place a BPF, we only have the signal and noise components at $3\omega_R$ at the LIA input, and S/N is given by the usual expression for sinusoidal demodulation:

$$\frac{S}{N} = \frac{A_3}{\sqrt{4S_{WN}BW_n}} = 1 \Rightarrow A_{3,min} = 2\sqrt{S_{WN}BW_n} \approx 0.89\ \mu\text{V}.$$

On the other hand, if we place a HPF we get noise from the harmonics at $3\omega_R, 5\omega_R, 7\omega_R$ and so on (basically, all harmonics but not the fundamental). Recalling that the total harmonic contribution (including the fundamental) is $\pi^2/8$, we get:

$$\frac{S}{N} = \frac{A_3/3}{\sqrt{4S_{WN}BW_n\left(\frac{\pi^2}{8} - 1\right)}} = 1 \Rightarrow A_{3,min} = 6\sqrt{\left(\frac{\pi^2}{8} - 1\right) S_{WN}BW_n} \approx 1.3\ \mu\text{V},$$

obviously larger than the previous value.

2.4

The problem with the phase measurement is that the output is proportional to its cosine, which is by no mean linearly related to it. For example, for a phase change from 0 to 10°, the cosine changes from 1 to 0.98, making the input signal very hard to detect. Of course, the solution is to use a double-demodulator.