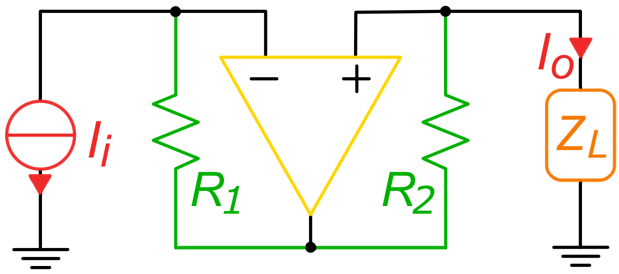
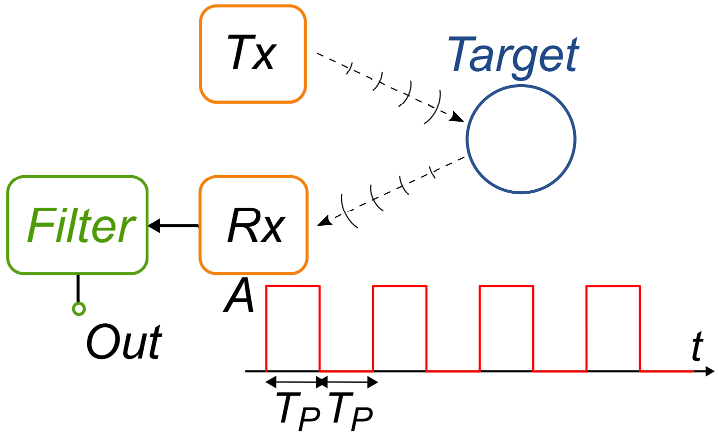
Electronics – September 11 2020 Prof. A. Spinelli





Problem 1

The scheme in the left figure is a current source. Parameter values are $R_1 = 10\ \mathrm{k\Omega}$, $R_2 = 1\ \mathrm{k\Omega}$. The OA has $GBWP = 1$ MHz and $A_0 = 100$ dB.

1. Find the expression of the closed-loop (ideal) gain.
2. The input current source has a parallel capacitance $C_s = 30$ pF. Discuss the stability of the circuit and compensate if necessary (consider for simplicity the case $|Z_L| \ll R_2$).
3. Compute the output rms noise current considering the equivalent noise sources of the amplifier, $\sqrt{S_V} = 10\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$ and $\sqrt{S_I} = 1\ \mathrm{pA}/\sqrt{\mathrm{Hz}}$, and resistors ($4k_BT \approx 1.646 \times 10^{-20}$ J). Consider for simplicity the circuit in #1.1 (i.e., without capacitor C_s).
4. With reference to #1.2, consider the case in which $Z_L = R_L \leq R_2$ and compensate the stage.

Problem 2

In ultrasonic sensing, a train of pulses is sent to a target, and the reflected signal (assumed equal to the transmitted one, for simplicity) is measured to extract the delay and the distance of the target. Consider a signal made up of a series of N rectangular pulses, as in the right figure, with amplitude A and pulse width T_P. A white noise having bilateral PSD λ is also present at the signal recovery stage.

1. A gated integrator is used as a filter. Compute the resulting S/N.
2. What is the best filter that can be used to detect the signal? Sketch the output signal and compute the resulting S/N.
3. Consider an input noise with exponential autocorrelation $R_{xx}(\tau) = \overline{n_x^2}\, e^{-|\tau|/T_n}$ and estimate the output noise (consider the filter in #1.2 and assume large value of N and long correlation time T_n).
4. There is a large low-frequency noise at the input. Is it possible to modify the pulse shape in order to reduce its effect (while still using the best filter option)?

Useful expressions: $\displaystyle\sum_k k\alpha^k = \frac{\alpha}{(1-\alpha)^2}$

For a correct evaluation, you are asked to write your answers in a readable way; thank you
Do a good job!

Results will be posted by September 16th **Mark registration: by Friday, Sept. 18th**

Solution

Problem 1

1.1

As the input pins of the OA must stay at the same potential, the voltage drops across R_1 and R_2 must be equal, which means:

$$I_i R_1 = I_o R_2 \Rightarrow I_o = I_i \frac{R_1}{R_2}.$$

1.2

Breaking the loop at the OA output, we can obtain the following expression for G_{loop}:

$$G_{loop} = -A(s)\left(\frac{1}{1+sC_sR_1} - \frac{Z_L}{Z_L+R_2}\right) \approx -A(s)\frac{1}{1+sC_sR_1},$$

where we assumed $|Z_L| \ll R_2$. The additional pole introduced by C_s is located at $f_p = 1/(2\pi C_s R_1) \approx$ 530 kHz, and reduces the phase margin below 45°. To compensate the stage, we can simply put a capacitor C_c in parallel to R_1, as shown in Fig. 1 (left), obtaining:

$$G_{loop} = -A(s)\frac{1+sC_cR_1}{1+s(C_c+C_s)R_1}.$$

Note that in this case there is no specific requirement on the phase margin, so we can follow two approaches. First, let's set a phase margin of 45°, meaning that we must place the frequency of the zero at f_{0dB}, i.e.

$$f_{0dB} = \sqrt{\frac{GBWP}{2\pi(C_c+C_s)R_1}} = f_z = \frac{1}{2\pi C_c R_1} \Rightarrow C_c^2 - KC_c - KC_s = 0,$$

where $K = 1/(2\pi R_1 GBWP) = 15.9$ pF. This equation leads to $C_c \approx 31$ pF and to $f_p = 261$ kHz, $f_z =$ 513 kHz. Due to the pole-zero proximity, the actual phase margin is $\phi_m = 90 - \arctan(513/261) + 45 \approx 72°$. A more rough-and-ready approach can be as follows: we set the zero at the original pole position, which in this case means $C_c = C_s$. The pole now goes at 265 kHz, where $|G_{loop}| = 1000/265 = 3.77$. The new f_{0dB} is then $265\sqrt{3.77} \approx 515$ kHz, and the phase margin is $\phi_m = 90 - \arctan(515/265) + \arctan(515/530) \approx 71°$.

1.3

The noise transfers are easily computed and result in

$$\begin{aligned} S_{I_o} &= \left(S_I^- + \frac{4k_BT}{R_1}\right)\left(\frac{R_1}{R_2}\right)^2 + S_I^+ + \frac{4k_BT}{R_2} + S_V\frac{1}{R_2^2} = \\ &= (10^{-24} + 1.646\times10^{-24})100 + 10^{-24} + 1.646\times10^{-23} + 10^{-22} = 3.8\times10^{-22}\ \text{A}^2/\text{Hz}, \end{aligned}$$

leading to

$$\overline{I_o^2} \approx S_{I_o}\frac{\pi}{2}GBWP \approx (24\ \text{nA})^2,$$

where we have again considered $|Z_L| \ll R_2$ for simplicity. If we consider $Z_L = R_L = R_2$, we get $f_{0dB} =$ $GBWP/2$ and $\sqrt{\overline{I_o^2}} \approx 17$ nA.

1.4

The solution for the case $R_L = 0$ has been already discussed in #1.2, so we are left with the other extreme, i.e., $R_L = R_2$. If we retain the previous compensation scheme, G_{loop} becomes now

$$G_{loop} = -A(s)\left(\frac{1+sC_cR_1}{1+s(C_c+C_s)R_1} - \frac{1}{2}\right) = -\frac{A(s)}{2}\frac{1+s(C_c-C_s)R_1}{1+s(C_c+C_s)R_1},$$

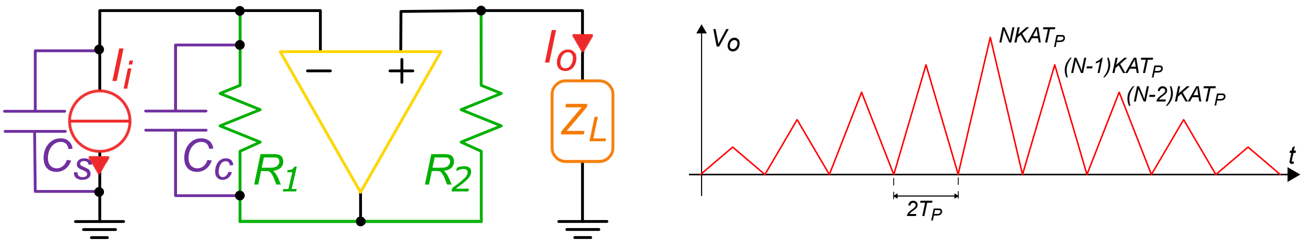


Figure 1: Left = Compensation scheme. Right = Output signal for the case $N = 5$.

where we can see that the position of the zero has changed. In particular, it is a good idea to pick $C_c > C_s$, so that the zero remains in the LHP. In fact, it is quite obvious that any choice of $C_c \gg C_s$ would be fine, as it would lead to a pole-zero cancellation.
Just for the sake of it, we can repeat the calculations in #1.2, leading to

$$C_c^2 - C_c(K + 2C_s) + C_c^2 - KC_s = 0 \Rightarrow C_c \approx 92 \text{ pF}$$

and to $f_p = 130$ kHz, $f_z = 253$ kHz. Again, the close proximity of the pole-zero couplet increases the actual phase margin to $135 - \arctan(253/130) = 72°$.

Problem 2

2.1

The GI should work over the entire signal duration, which is $(2N-1)T_P$. The resulting S/N is then

$$\frac{S}{N} = \frac{KANT_P}{\sqrt{K^2\lambda(2N-1)T_p}} = A\sqrt{\frac{N^2T_P}{(2N-1)\lambda}} \approx A\sqrt{\frac{NT_P}{2\lambda}},$$

where K is the gain of the GI.

2.2

The best option is to use a matched filter, with a weighting function equal to the signal. The output signal is reported in Fig. 1 (right), where K is the amplitude of the weighting function, and we have set $t = 0$ as the pulse arrival time. The output rms noise can be easily computed as that of a GI working over NT_P (remember that this only works if the input noise is white!), and S/N is given by

$$\frac{S}{N} = \frac{KANT_P}{\sqrt{K^2\lambda NT_p}} = A\sqrt{\frac{NT_P}{\lambda}},$$

with an improvement of a factor $\sqrt{2}$ with respect to the previous case.

2.3

To compute the output noise, we need the time correlation of the weighting function that, however, has already been computed: it is in fact equal to the output signal, provided that we set $A = K$ and that the maximum is centered in zero. If we approximate as usual each triangular function with a delta function having the same area, we obtain

$$\overline{n_o^2} \approx \overline{n_x^2}(KT_P)^2\left(N + 2(N-1)e^{-2T_P/T_n} + 2(N-2)e^{-4T_P/T_n} + \ldots\right).$$

We can break the part into parenthesis into two terms, one with the N terms, the other with the remaining. The first one becomes (let $\alpha = e^{-2T_P/T_n}$)

$$N + 2N\alpha + +2N\alpha^2 + \ldots = N\left(2\sum_k \alpha^k - 1\right) = N\frac{1+\alpha}{1-\alpha}.$$

The second term is instead (apart from the minus sign):

$$2\alpha + 4\alpha^2 + 6\alpha^3 + \ldots = 2\sum_k k\alpha^k = \frac{2\alpha}{(1-\alpha)^2}.$$

The total noise becomes then

$$\overline{n_o^2} \approx \overline{n_x^2}(KT_p)^2 \frac{N(1-\alpha^2) - 2\alpha}{(1-\alpha)^2}.$$

2.4

To reduce the LF noise, we need a weighting function with a zero in $f = 0$ (in the fequency domain, of course). In the time domain, this means that the integral of $w(t,\tau)$ must be zero. However, because of the matched filter, $w(t,\tau)$ is equal to the pulse shape, so it suffices to send a pulse with zero average value. In our case, we can simply shift the pulses by a factor equal to $A/2$.