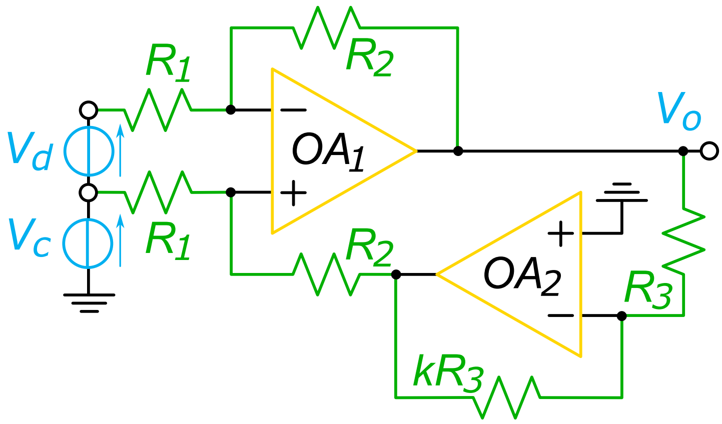
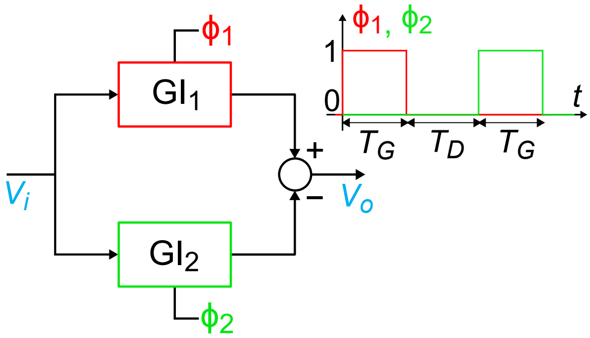
Electronics – June 18 2020 Prof. A. Spinelli





Problem 1

The scheme in the left figure is a differential amplifier. Parameter values are $R_1 = 1$ kΩ, $R_2 = 10$ kΩ, $R_3 = 10$ kΩ, k ranging from 1 to 9. The OAs have $GBWP = 10$ MHz and $A_0 = 100$ dB.

1. Find the expression of the closed-loop (ideal) gain.
2. Compute the OA input capacitance that grants a phase margin of 45° (consider the OA1 loop).
3. Compute the output rms noise voltage considering the equivalent noise sources of the amplifiers, $\sqrt{S_V} = 10\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$.
4. Repeat problem 1.2, but for the input capacitance of OA2 (with OA1 as ideal).

Problem 2

The scheme in the right figure is controlled by two switches, ϕ_1 and ϕ_2 (consider $T_D \geq T_G$). The input is a constant signal during the first integration phase.

1. Compute the weighting function and its time correlation.
2. Consider a noise with nearly triangular autocorrelation and correlation time $T_n \gg T_D$. Provide an estimate of the output rms noise.
3. There are now two noise sources: a low-frequency one (same as previous case) and a white one, with bilateral PSD λ. Evaluate the resulting S/N and find the best values of T_D and T_G.
4. The two GI are now replaced by two Boxcar averagers, working with $T_D = 0$. Sketch the new weighting function and its time correlation.

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by June 23rd Mark registration: Friday, June 26th

Solution

Problem 1

1.1

We begin with the common-mode amplification, equalling the expression for V^+ and V^- of OA1 and considering that the output voltage of OA2 is $-kV_o$. Exploiting the linear superposition principle and setting $V_d = 0$, we get

$$V^- = V_c \frac{R_2}{R_1 + R_2} + V_o \frac{R_1}{R_1 + R_2} = V^+ = V_c \frac{R_2}{R_1 + R_2} - kV_o \frac{R_1}{R_1 + R_2} \Rightarrow V_o = 0.$$

Writing the same equations under voltage source V_d we obtain

$$V_d \frac{R_2}{R_1 + R_2} + V_o \frac{R_1}{R_1 + R_2} = -kV_o \frac{R_1}{R_1 + R_2} \Rightarrow V_o = -\frac{R_2}{(k+1)R_1} V_d.$$

Note that changing the value of k allows to change the gain of the stage (from 1 to 5) without affecting the resistor matching needed to reject the common-mode signal.

1.2

We first compute the loop gain of OA1, opening both loops at its output. We easily get

$$G_{loop} = -A(s) \frac{R_1}{R_1 + R_2} (1 + k),$$

where the zero-dB crossover frequency f_{0dB} changes from 0.18 $GBWP = 1.8$ MHz ($k = 1$) to 0.91 $GBWP = 9.1$ MHz ($k = 9$). When we add the input capacitance of OA1, C_i, we note that it introduces a pole in G_{loop} while not adding any zero. The equivalent resistance seen by C_i is $2(R_1 \parallel R_2)$, so that the condition on the pole satisfies

$$f_p = \frac{1}{4\pi C_i R_1 \parallel R_2} \geq f_{odB} \Rightarrow C_i \leq 9.7 \text{ pF}.$$

1.3

The output of OA2 can be expressed as (see Fig. 1, left)

$$V_2 = -kV_o + V_{n2}(k + 1),$$

from which we can proceed in analogy with # 1.1, obtaining

$$V_o \frac{R_1}{R_1 + R_2} = V_2 \frac{R_1}{R_1 + R_2} + V_{n1} \Rightarrow V_o = V_{n1} \frac{R_1 + R_2}{(1 + k)R_1} + V_{n2}.$$

In the case $k = 9$, the output rms noise becomes

$$\overline{V_o^2} = 2.21\ S_V \frac{\pi}{2} f_{0dB} \approx (0.56\ \mu\text{V})^2.$$

For the $k = 1$ case ($f_{0dB} = 1.8$ MHz) we get instead $\overline{V_o^2} = 31.25\ S_V \frac{\pi}{2} f_{0dB} \approx (0.94\ \mu\text{V})^2$.

1.4

Breaking both loops at the output of OA2, it is straightforward to obtain

$$G_{loop} = -A(s),$$

meaning that $f_{0dB} = GBWP$. The resistance seen by C_i is now $R_i = R_3 \parallel kR_3$, leading to (worst case for $k = 9$)

$$\frac{1}{2\pi R_i C_i} \geq GBWP \Rightarrow C_i \leq \frac{k + 1}{2k\pi R} = 1.77 \text{ pF}$$

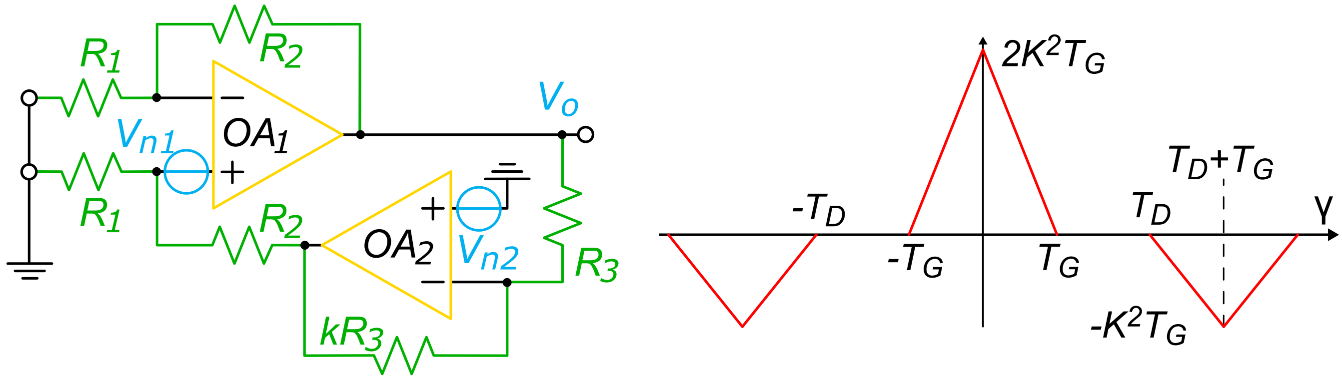


Figure 1: Left = Scheme for noise calculation. Right = Time correlation of the weighting function.

Problem 2

2.1

Recalling that the weighting function of a GI is a rectangular function, the overall WF can be expressed as

$$w(t,\tau) = K(\mathrm{rect}(0,T_G) - \mathrm{rect}(T_D+T_G, T_D+2T_G)),$$

where $\mathrm{rect}(T_1,T_2)$ is the unit-amplitude rectangular function between T_1 and T_2. The time correlation of the WF is shown in Fig. 1 (right). In the frequency domain we have (shifting the time axis by $T_G + T_D/2$):

$$W(t,f) = KT_G\,\mathrm{sinc}(\pi f T_G)\left(e^{j\pi f(T_G+T_D)} - e^{-j\pi f(T_G+T_D)}\right) = 2jKT_G\,\mathrm{sinc}(\pi f T_G)\sin(\pi f(T_G+T_D)).$$

2.2

The mean square value of the output noise is given by

$$\overline{n_{out}^2} = \int R_{nn}(\gamma)k_{ww}(\gamma)d\gamma,$$

which can be solved analytically, if one bothers to. However, a rough approximation can be obtained considering the noise autocorrelation R_{nn} to be nearly constant over a time comparable to T_G, having values $R_{nn}(0) = \overline{n_x^2}$ at $\gamma = 0$ and $R_{nn}(T_D+T_G) = \overline{n_x^2}(1-(T_D+T_G)/T_n)$ at $\gamma = T_D+T_G$ (in other words, we are assuming the triangular functions to behave as sampling deltas with equal area). This means that the total noise can be written as

$$\overline{n_{out}^2} \approx R_{nn}(0)2(KT_G)^2 - 2R_{nn}(T_D+T_G)(KT_G)^2 = 2\overline{n_x^2}(KT_G)^2\frac{T_D+T_G}{T_n}.$$

2.3

The output noise due to the white input noise is readily calculated from the expression of the WF time correlation:

$$\overline{n_y^2} = 2K^2T_G\lambda,$$

which leads to the following expression for S/N:

$$\left(\frac{S}{N}\right)^2 = \frac{(AT_G)^2}{2T_G\lambda + 2\overline{n_{LF}^2}T_G^2(T_G+T_D)/T_n}.$$

From this expression, it is clear that the best choice for T_D is $T_D = 0$. Setting this, we obtain

$$\left(\frac{S}{N}\right)^2 = \frac{A^2T_G^2}{2T_G\lambda + 2\overline{n_{LF}^2}T_G^3/T_n} \propto \frac{T_G}{\lambda + \overline{n_{LF}^2}T_G^2/T_n}.$$

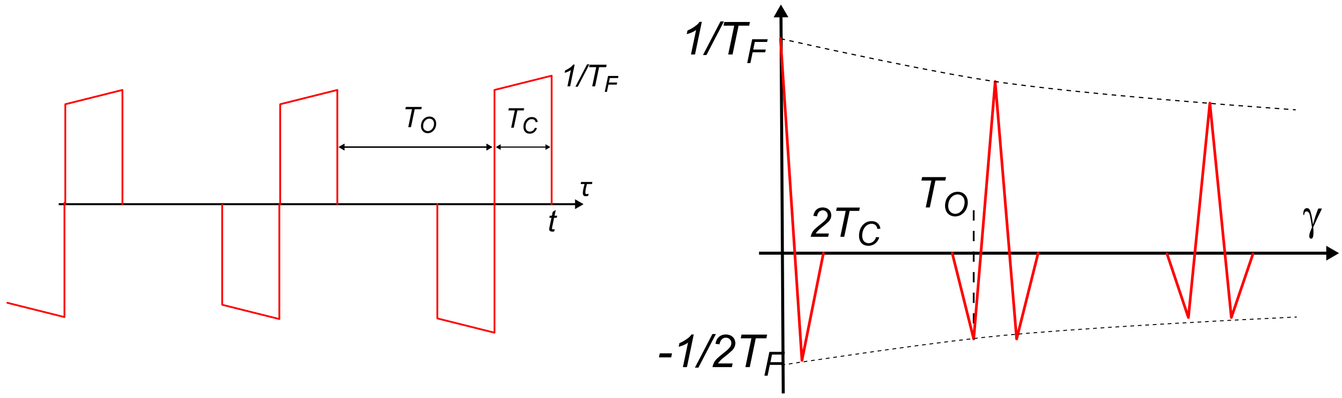


Figure 2: Left = Scheme for noise calculation. Right = Time correlation of the weighting function.

An optimization of this term leads to

$$T_G = \sqrt{\frac{\lambda T_n}{\overline{n_{LF}^2}}}.$$

It is interesting to express this result in term of the PSD of the LF noise, whose LF value is $\lambda_{LF} = \overline{n_{LF}^2} T_n$:

$$T_G = T_n \sqrt{\frac{\lambda}{\lambda_{LF}}}.$$

Note that the optimum T_G increases as the WN increases, and decreases as the LF noise gains importance. Also note that the above result is obtained for $T_D = 0$. If the condition $T_D \geq T_G$ is retained, we set $T_D = T_G$ and obtain

$$T_G = \sqrt{\frac{\lambda T_n}{2\overline{n_{LF}^2}}}.$$

2.4

The weighting function is obviously made up of two BA weighting functions, shifted by T_C, as shown in Fig.2 (left). Its time correlation (positive values only) is shown in Fig. 2 (right).