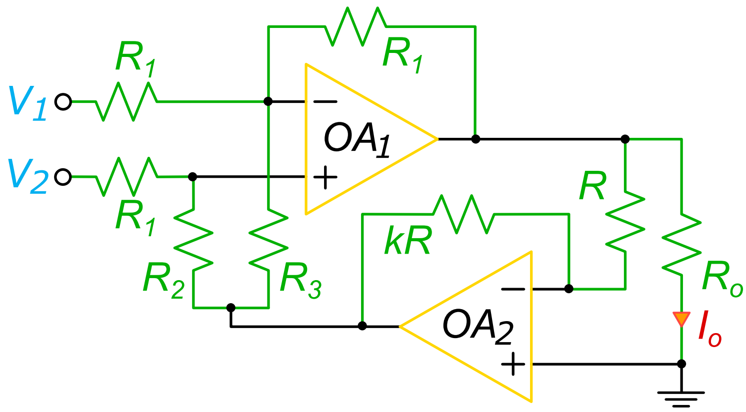
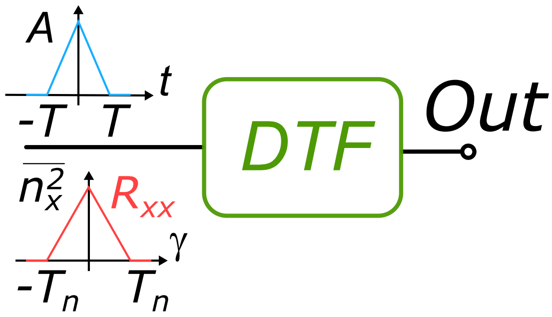
Electronics – July 22 2021

Prof. A. Spinelli





Problem 1

The scheme in the left figure is a transconductance amplifier. Parameter values are $R_1 = 380$ kΩ, $R_2 = 20$ kΩ, $R_3 = 21.1$ kΩ, $R_0 = 100$ Ω, $k = 9$. OA1 has low-frequency gain of 100 dB and $GBWP = 500$ kHz.

1. Find the <u>value</u> of the closed-loop gain in the ideal case. Please note that $R_3 \parallel R_1 = R_2$.
2. Compute the loop gain for OA1 with ideal OA2. What is the minimum $GBWP$ value of OA2 that grants a phase margin larger than 45°?
3. Compute the output rms noise current considering the equivalent noise source of the amplifiers, $\sqrt{S_V} = 20\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$.
4. Consider a difference in ground potential between input and output ground nodes. What is its effect on the output current?

Problem 2

A discrete-time filter with sampling time T_s works on a triangular signal affected by a noise with nearly triangular autocorrelation, as in the figure on the right.

1. A single sampling is performed. Evaluate S/N.
2. Three samples are now taken and added. Consider the noise only: what is the minimum rms value at the output?
3. Consider also the signal, and assume $T = T_n$. What is the best S/N achievable with the three-sample filter? (hint: consider a few cases)
4. Consider the case $T_n \gg T$. Find suitable values for T_s and for the three sample weights to improve S/N (and estimate it).

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by July 28th

Mark registration: by Friday, July 30th

Solution

Problem 1

1.1

We label V_o the output of OA1, so that $-kV_o$ is the output of OA2. Voltages at the input pins of OA1 are

$$V^+ = V_2\frac{R_2}{R_1+R_2} - kV_o\frac{R_1}{R_1+R_2} \tag{1}$$

$$V^- = V_1\frac{R_1 \parallel R_3}{R_1+R_1 \parallel R_3} - kV_o\frac{R_1/2}{R_1/2+R_3} + V_o\frac{R_1 \parallel R_3}{R_1+R_1 \parallel R_3}. \tag{2}$$

We consider a common/differential mode representation of the input voltages and note that the common-mode transfer ($V_1 = V_2 = V_c$) is zero. We then set $V_1 = -V_2 = V_d/2$ and recall that $R_1 \parallel R_3 = R_2$, obtaining

$$-V_d = V_o + kV_o\frac{R_1}{R_2}\left(1-\frac{R_1+R_2}{R_1+2R_3}\right) = V_o + kV_o\frac{R_1}{R_2}\left(\frac{R_2}{R_1}\right) = V_o(1+k),$$

which leads to

$$I_o = \frac{V_o}{R_o} = -\frac{V_d}{(k+1)R_o} = -\frac{V_d}{1\ \text{k}\Omega}.$$

1.2

we break the loop at the output of OA1 and obtain (replace R_3 with $R_1R_2/(R_1-R_2)$):

$$V^+ = -kV_T\frac{R_1}{R_1+R_2} \tag{3}$$

$$V^- = V_T\frac{R_1 \parallel R_3}{R_1+R_1 \parallel R_3} - kV_T\frac{R_1/2}{R_1/2+R_3} = V_T\frac{R_2}{R_1+R_2} - kV_T\frac{R_1-R_2}{R_1+R_2}, \tag{4}$$

which gives

$$G_{loop} = -A(s)\frac{R_2}{R_1+R_2}(k+1) = -0.5A(s).$$

The zero-dB frequency is then $f_{0dB} = 250$ kHz. To address the effect of the OA2 bandwidth, we can replace $k = 9$ with $k = 9/(1+s\tau)$, obtaining

$$\frac{k+1}{20} = \frac{\frac{9}{1+s\tau}+1}{20} = 0.5\frac{1+s\tau/10}{1+s\tau}.$$

We see then that G_{loop} has the same pole as OA2. To not lower the phase margin below 45°, the pole must fall after f_{0dB}, meaning that $GBWP$ must be

$$GBWP_2 \geq 9f_{odB} = 2.25\ \text{MHz}.$$

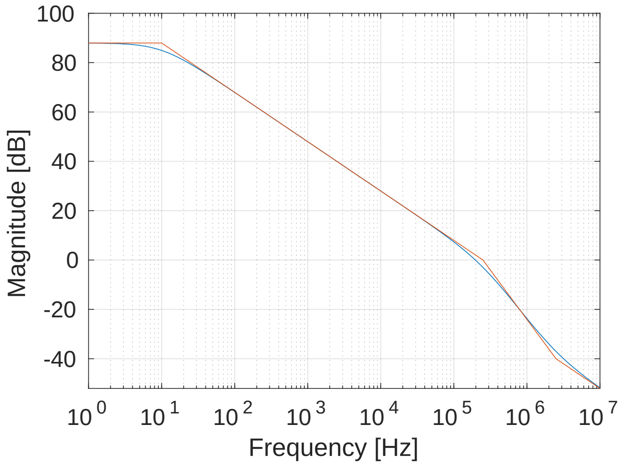
Please note that in reality the relation for an inverting amplifier involves $G_{id}+1$, so the correct result is 2.5 MHz. Note also that we are neglecting the small phase contribution from the zero. The resulting loop gain is shown in Fig. 1 (left).

1.3

The scheme for noise calculation is in Fig. 1 (right). We note that the OA2 output can be written as $V_{n2}(k+1) - kV_o$ and follow the approach of 1.1, obtainig

$$V^+ = (V_{n2}(k+1) - kV_o)\frac{R_1}{R_1+R_2} + V_{n1} \tag{5}$$

$$V^- = (V_{n2}(k+1) - kV_o)\frac{R_1-R_2}{R_1+R_2} + V_o\frac{R_2}{R_1+R_2}. \tag{6}$$



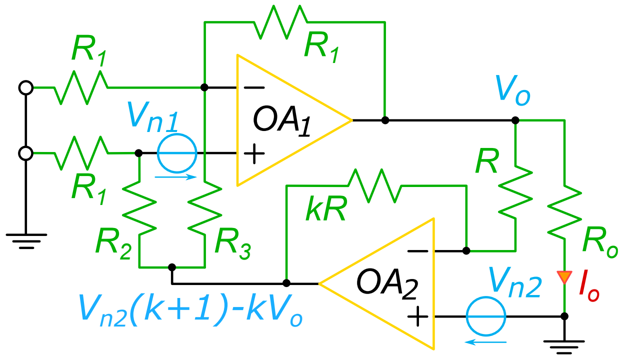


Figure 1: Left = Bode plot of the compensated loop gain (blue) and its asymptotic approximation (red). Right = Scheme for noise calculations.

Equalling the input voltages of OA1 we obtain

$$V_o = 2V_{n1} + V_{n2} \Rightarrow S_{I_o} = 5\frac{S_V}{R_o^2} \approx \left(0.45\ \mathrm{nA}/\sqrt{\mathrm{Hz}}\right)^2,$$

leading to $\sqrt{\overline{I_o^2}} = \sqrt{S_{I_o}}\sqrt{(\pi/2)f_{0dB}} = 0.28\ \mu\mathrm{A}$.

1.4

Fluctuations in ground potential behave as common-mode signals and are rejected by the circuit, so that the output is unaffected.

Problem 2

2.1

For a single sampling of the signal (obviously at its peak), the resulting S/N is simply

$$\left(\frac{S}{N}\right) = \frac{A}{\sqrt{\overline{n_x^2}}}.$$

2.2

The time correlation of the weighting function consists of delta functions as in Fig. 2 (left; we consider for simplicity unit-area delta functions), while the input noise autocorrelation is $R_{xx}(\gamma) = \overline{n_x^2}(1 - |\gamma|/T_n)$. Clearly, the minimum noise corresponds to minimum overlap between the functions, i.e., when only the central delta is sampling $R_{xx}(0)$. In any case, let us evaluate all the possible cases:

a) $T_s \leq 0.5T_n$, i.e., all delta functions are sampling correlated noise:

$$\overline{n_y^2} = \int R_{xx}(\gamma)k_{w_{tt}}(\gamma)d\gamma = \overline{n_x^2}\left(3 + 4\left(1 - \frac{T_s}{T_n}\right) + 2\left(1 - 2\frac{T_s}{T_n}\right)\right) = \overline{n_x^2}\left(9 - 8\frac{T_s}{T_n}\right).$$

b) $0.5T_n \leq T_s \leq T_n$, i.e., only the central and the delta functions at $\pm T_s$ are sampling correlated noise:

$$\overline{n_y^2} = \int R_{xx}(\gamma)k_{w_{tt}}(\gamma)d\gamma = \overline{n_x^2}\left(3 + 4\left(1 - \frac{T_s}{T_n}\right)\right) = \overline{n_x^2}\left(7 - 4\frac{T_s}{T_n}\right).$$

c) $T_s \geq T_n$:

$$\overline{n_y^2} = \int R_{xx}(\gamma)k_{w_{tt}}(\gamma)d\gamma = 3\overline{n_x^2}.$$

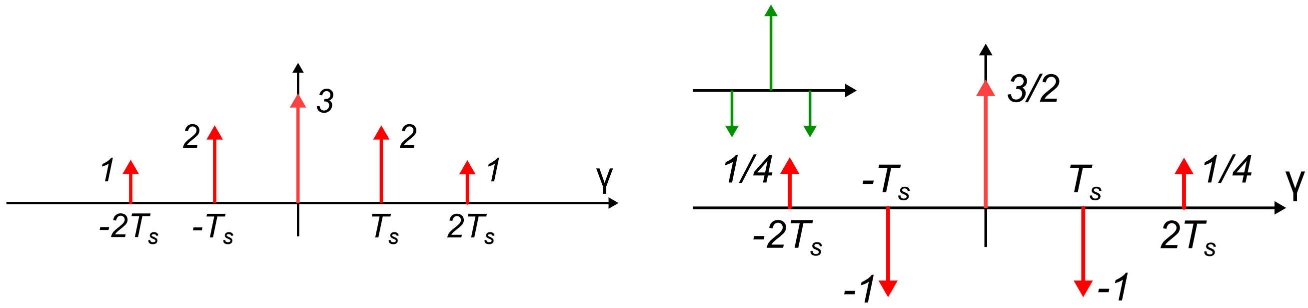


Figure 2: Left = Time correlation of the weighting function. Right = Same for the case of the WF shown in green in the inset.

As said, the minimum noise corresponds (unsurprisingly) to the last case, when we are sampling non-correlated noise.

2.3

The signal can be expressed as $A(1 - |t|/T)$, and sampling is performed at $\pm T_s$ and 0, leading to:

$$y = A + 2A\left(1 - \frac{T_s}{T_n}\right) = A\left(3 - 2\frac{T_s}{T_n}\right).$$

To evaluate the best S/N we can just sample its value in a few points, taking advantage of the previous results, obtaining:

$$T_s \approx 0 \Rightarrow \frac{S}{N} = \frac{A}{\sqrt{\overline{n_x^2}}}$$

$$T_s = 0.5T_n \Rightarrow \frac{S}{N} = \frac{A}{\sqrt{\overline{n_x^2}}}\frac{2}{\sqrt{5}}$$

$$T_s = T_n \Rightarrow \frac{S}{N} = \frac{A}{\sqrt{\overline{n_x^2}}}\frac{1}{\sqrt{3}}$$

Clearly the best value is achieved for $T_s = 0$, i.e., when the three samples are equivalent to a single sampling operation.
This result becomes clear if we consider the optimum filter in the frequency domain: its weighting function is given by

$$|W(t, f)| = \frac{|X(f)|}{S_n(f)} = \text{constant},$$

as both quantities are proportional to $\text{sinc}^2(\pi f T_n)$. A single sampling operation is indeed the optimum filter for this case.

2.4

If $T_s \ll T_n$, the previous filter always returns a value for S/N similar to the $T_s = 0$ case. To reduce the correlated noise, we must then *subtract* the samples. An idea could be to have two negative samples (weight -0.5) at $\pm T_s$ and a central sample (weight 1) at 0. Given the weighting function time correlation (Fig. 2, right), the output noise becomes now

$$\overline{n_y^2} = \overline{n_x^2}\left(\frac{3}{2} - 2\left(1 - \frac{T_s}{T_n}\right) + \frac{1}{2}\left(1 - 2\frac{T_s}{T_n}\right)\right) = \overline{n_x^2}\frac{T_s}{T_n}.$$

Of course we pick $T_s = T$, obtaining a signal amplitude A and:

$$\left(\frac{S}{N}\right) = \frac{A}{\sqrt{\overline{n_x^2}}}\sqrt{\frac{T_n}{T}}$$