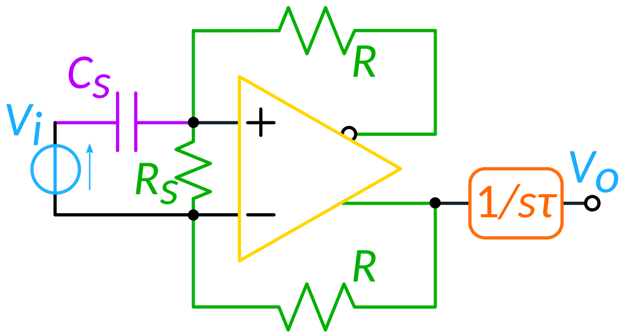
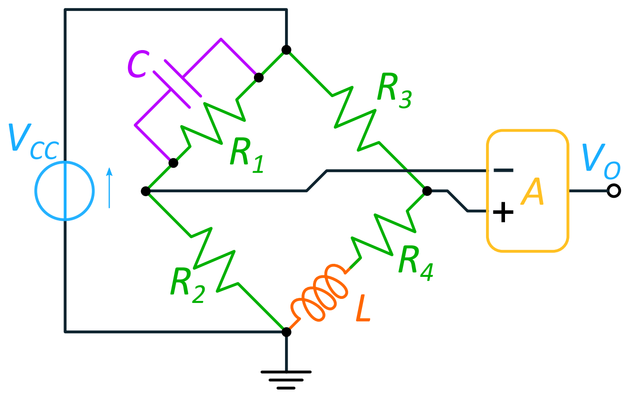
Electronics – January 22 2021 Prof. A. Spinelli





Problem 1

The scheme in the left figure is an amplifier for a piezoelectric sensor, characterized by a series capacitance $C_s = 500$ pF and resistance $R_s = 10$ MΩ. The instrumentation amplifier has low-frequency gain $G = 500$. Other parameter values are $R = 10$ MΩ, $\tau = 1$ ms.

1. Find the expression of the closed-loop gain in the ideal case.
2. Compute the loop gain and evaluate the minimum bandwidth of the amplifier to ensure a phase margin of at least 60°.
3. Compute the output rms noise voltage considering the equivalent voltage noise source of the amplifier, $\sqrt{S_V} = 10$ nV/$\sqrt{\text{Hz}}$. Consider for simplicity the range $f > 1$ Hz.
4. With reference to #1.3, evaluate the output noise when the lower frequency limit is set to zero (make the necessary assumptions on the operation of the stages).

Problem 2

The bridge circuit reported in the right figure is used to measure unknown inductances in terms of resistances and capacitances. Its output is connected to an amplifier having input noise $S(f) = K/f + S_{WN}$ with $K = 10^{-6}$ V^2 and noise corner frequency $f_{nc} = 10$ kHz. Bridge supply is $V_{cc} = 3$ V.

1. Find the relation between the inductance L and the other bridge elements in order to balance the bridge.
2. Consider now a small change in the inductance, from L to $L + \Delta L = L(1 + x)$. What is the output voltage of the bridge (consider a balanced bridge and assume for simplicity all resistors to be equal)?
3. The bridge parameters are $CR = L/R = 1$ ms, $x \approx 10^{-3}$ with bandwidth of 1 Hz. Find the parameters of a LIA system able to recover the signal.
4. Discuss the feasibility of a detection system not LIA-based (e.g. with an HPF to filter the flicker noise).

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by January 27th Mark registration: by Friday, Jan. 29th

Solution

Problem 1

1.1

Ideal case means infinite loop gain, i.e., infinite gain of the INA in this case. As a consequence, no current flows in R_s and the current flowing into C_s and the two R resistors is

$$I_i = sC_s V_i.$$

Kirchhoff voltage law around the two feedback loops reads then:

$$2RI_i = 2V_{INA},$$

where V_{INA} is the output voltage of the INA. This leads to

$$V_{INA} = RI_i = sC_sRV_i \Rightarrow V_o = \frac{V_{INA}}{s\tau} = \frac{C_sR}{\tau}V_i = 5V_i.$$

1.2

Breaking the loop at the OA output and setting $Z = R_s \parallel 1/sC_s$, we can obtain the following expression for G_{loop}:

$$G_{loop} = -G(s)\frac{2Z}{Z+2R} = -G(s)\frac{2R_s}{R_s+2R}\frac{1}{1+sC_s(R_s \parallel 2R)},$$

where the pole added by C_s is located at $f_p = 1/(2\pi C_s(R_s \parallel 2R) \approx 48$ Hz, and the DC gain is 333 (note that this result could have been also obtained via the half-circuit approach). This means that the zero-dB crossing takes place at $f_{0dB} = 333 \times 48 \approx 16$ kHz. To ensure a phase margin of 60°, the contribution of the $G(s)$ pole f_G must be:

$$\phi_m = 60° = 180° - 90° + \measuredangle G(f_{0dB}) \Rightarrow \measuredangle G(f_{0dB}) = -30° \Rightarrow \frac{f_{0dB}}{f_G} = \tan 30° = \frac{1}{\sqrt{3}},$$

which leads to $f_G \approx 28$ kHz.

1.3

We turn off the input voltage source and consider the noise voltage V_n, connected as in Fig. 1 (left). We begin with the ideal transfer: the current flowing in the feedback loops is V_n/Z, because the differential input voltage of the amplifier must be zero. This means that we have:

$$\frac{V_n}{Z}(2R+Z) = 2V_{INA} \Rightarrow V_o = \frac{V_{INA}}{s\tau} = V_n\frac{3}{2}\frac{1+2sC_sR_s/3}{s\tau}.$$

For frequencies higher than the zero at $f_z \approx 47.7$ Hz the noise transfer is constant and equal to 5. This means that we must add a pole at the zero-dB frequency of G_{loop}, $f_{0dB} = 16$ kHz. The resulting transfer is reported in Fig. 1 (right) and leads to

$$\overline{V_o^2} \approx S_V\left(\frac{9}{(4\pi\tau)^2}\frac{1}{f_L} + 25\frac{\pi}{2}f_{0dB}\right) \approx (8\ \mu\text{V})^2,$$

dominated by the white noise contribution.

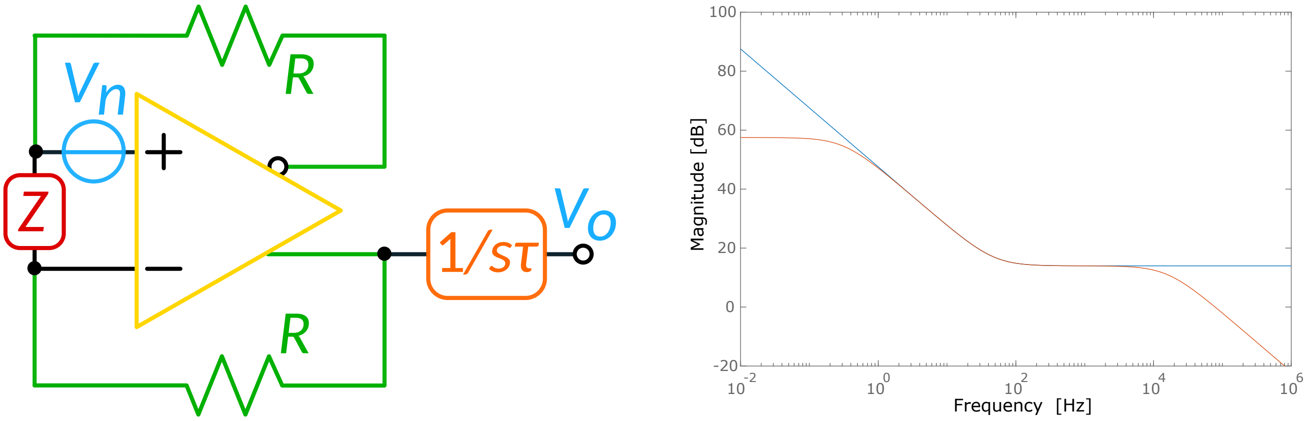


Figure 1: Left = Scheme for noise calculation. Right = Bode plot of the noise transfer in the ideal case (blue) and when accounting for the INA and integration stage limitations (red).

1.4

The problem with extending the limit down to $f = 0$ is of course the $1/s\tau$ term. In reality this stage will not provide infinite gain for zero fequency, but a finite value A_0 (for example, given by the open-loop gain of an OA-based integrator or by a resistor placed in parallel to the integration capacitor). The corresponding pole is $f_p = 1/(2\pi\tau A_0)$, so that the new expression for the output noise becomes:

$$\overline{V_o^2} \approx S_V \left(\frac{9}{4} A_o^2 \frac{\pi}{2} f_p + 25 \frac{\pi}{2} f_{0dB} \right) = S_V \left(\frac{9A_0}{16\tau} + 25 \frac{\pi}{2} f_{0dB} \right).$$

Picking for example $A_0 = 500$ (the noise transfer function in this case is shown in Fig. 1, right) we get $\sqrt{\overline{V_o^2}} \approx 9.5\ \mu$V. Please note that the deviation from the ideal gain at low frequencies is due to the integration stage, and *not* to the INA stage, that provides a unity gain.

Problem 2

2.1

The output of the bridge can be written as:

$$V_B = V_{cc} \left(\frac{Z_4}{R_3 + Z_4} - \frac{R_2}{R_2 + Z_1} \right) = 0 \Rightarrow \frac{Z_1}{R_2} = \frac{R_3}{Z_4}.$$

Of course, calibration must work for every frequency. Considering the DC case (where C is an open circuit and L a short-circuit), we immediately obtain the requirement on resistors:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}. \tag{1}$$

In the general case we have:

$$\frac{R_1}{R_2(1 + sCR_1)} = \frac{R_3}{R_4(1 + sL/R_4)} \Rightarrow CR_1 = \frac{L}{R_4} \Rightarrow L = CR_1R_4.$$

2.2

To begin with, we carry out the calculations. The output of the right-hand side of the bridge is

$$V_R = V_{cc} \frac{R_4 + sL}{R_3 + R_4 + sL} = k \frac{1 + sL/R_4}{1 + sL/(R_3 + R_4)} = k \frac{1 + sL/R_4}{1 + ksL/R_4},$$

where $k = R_3/(R_3 + R_4)$. The left-hand side output reads

$$V_L = V_{cc}\frac{R_2}{R_2 + R_1/(1 + sCR_1)} = k\frac{1 + sCR_1}{1 + sCR_1 \parallel R_2} = k\frac{1 + sCR_1}{1 + ksCR_1} \tag{2}$$

The bridge output voltage is then:

$$V_o = V_R - V_L = kV_{cc}\left(\frac{1 + sL/R_4}{1 + ksL/R_4} - \frac{1 + sCR_1}{1 + ksCR_1}\right). \tag{3}$$

We now consider the change in L, $L \to L(1 + x)$ and call $L/R = CR = \tau$, obtaining

$$V_o = kV_{cc}\left(\frac{1 + s\tau(1 + x)}{1 + ks\tau(1 + x)} - \frac{1 + s\tau}{1 + ks\tau}\right) = \frac{V_{cc}k(1 - k)xs\tau}{(1 + ks\tau(1 + x))(1 + ks\tau)} \approx V_{cc}\frac{x}{4}\frac{s\tau}{(1 + s\tau/2)^2},$$

where we have considered that $k = 0.5$ and that $\Delta L \ll L$, i.e., $x \ll 1$.

2.3

Note that the magnitude of the transfer function $T(s) = s\tau/(1 + s\tau/2)^2$ has its maximum for a frequency $f_M = 1/\pi\tau \approx 318$ Hz, where its magnitude equals 1 and the output signal becomes exactly the same as in a resistive bridge. A possibility could then be to set the reference frequency $f_R = f_M$, while another option is to set $f_R > f_{nc}$. In any case, the output S/N becomes:

$$\frac{S}{N} = \frac{V_{cc}x|T(f_R)|}{4\sqrt{2S(f_R)BW_n}},$$

where $BW_n = 10\pi/2$, considering a single-pole output filter with bandwidth equal to 10 Hz. For $f_R = f_M$ we have $|T(f_R)| = 1$ and

$$\left.\frac{S}{N}\right|_{f_M} = \frac{V_{cc}x}{4\sqrt{2(K/f_M)BW_n}} \approx 2.4.$$

For $f_R = f_{nc}$, $|T(f_R)| = \omega_R\tau/(1 + (\omega_R\tau/2)^2) \approx 6.4 \times 10^{-2}$ and S/N becomes

$$\left.\frac{S}{N}\right|_{f_{nc}} = 6.4 \times 10^{-2}\frac{V_{cc}x}{4\sqrt{2(K/f_{nc})BW_n}} \approx 0.85.$$

This result is not surprising: by increasing f_R beyond f_M we reduce the signal as $1/f$, but the noise is reduced as $1/\sqrt{f}$, hence the decrease in S/N!

2.4

This solution is totally uneffective for a very simple reason: an inductance behaves like a short-circuit at DC! This means that the bridge output signal will be vanishingly small and impossible to measure (in fact the transfer function evaluated in 2.2 has a zero in the origin).

Appendix

It is worth pointing out an approximation in solving 2.2. In fact, when we assume to have a variation in the inductance L, we are considering a time-varying inductance, which means that the circuit is no longer LTI. If Φ is the magnetic flux, the bipole equation becomes now

$$v = \frac{d\Phi}{dt} = \frac{d((L + \Delta L)i)}{dt} = (L + \Delta L)\frac{di}{dt} + i\frac{d\Delta L}{dt}.$$

When the rate of change (i.e., the bandwidth) of ΔL is small, the second term is negligible and these results are correct.