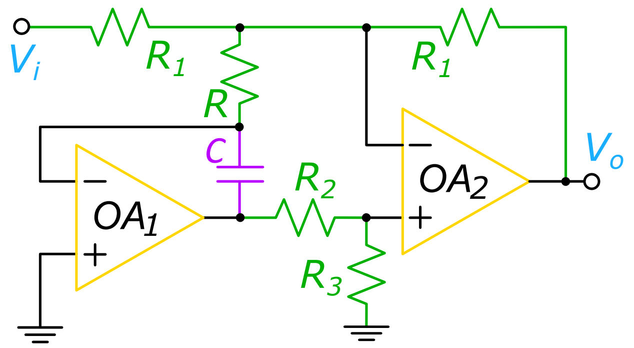
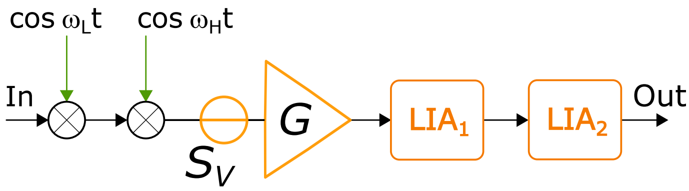
Electronics – September 9 2021 Prof. A. Spinelli





Problem 1

The scheme in the left figure is a composite amplifier. Parameter values are $R = 100$ kΩ, $R_1 = 1$ kΩ, $R_2 = 4.7$ kΩ, $R_3 = 220\ \Omega$, $C = 10$ nF. The OAs have low-frequency gain of 94 dB and $GBWP = 90$ MHz.

1. Find the the closed-loop gain in the ideal case.
2. Compute the loop gain for OA2 with ideal OA1 and evaluate the phase margin.
3. Compute the output rms noise voltage considering the equivalent noise sources of the amplifiers, $\sqrt{S_V} = 20\ \text{nV}/\sqrt{\text{Hz}}$.
4. Evaluate the effect of the OAs offset voltages and bias currents on the output. Discuss then the function of OA1 and its design criteria (bandwidth, HF and LF noise, offset, bias currents).

Problem 2

In several spectroscopy experiments a dual modulation is performed, resulting in a signal $A\cos(\omega_L t)\cos(\omega_H t)$, which can then be recovered by two LIAs in series, as in the figure on the right. Consider $f_L = 65$ Hz, $f_H = 4$ kHz, $A \approx 10\ \mu$V with bandwidth of 0.1 Hz, and an amplifier bilateral input noise PSD $S_V = K/f$ with $K = 2 \times 10^{-9}$ V^2.

1. LIA$_1$ is locked to f_H and LIA$_2$ to f_L. Find suitable values for the two LIA output filter bandwidths.
2. Evaluate the output S/N.
3. The noise source S_V is now located at the input of the *second modulator*. Evaluate the resulting S/N.
4. Repeat problem #2.1 if the demodulation order is swapped (i.e., LIA$_1$ works at f_L and LIA$_2$ at f_H). Is this a better choice? Comment on the result.

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by September 14$^{\text{th}}$ Mark registration: by Friday, September 17$^{\text{th}}$

Solution

Problem 1

1.1

We consider the input pins of OA2 and note that OA1 is connected as an integrator. This means that

$$V^+ = -V^- \frac{1}{sCR}\frac{R_3}{R_2+R_3} = V^- \Rightarrow V^+ = V^- = 0,$$

which leads to

$$G_{id} = -1,$$

as no current flows into R. The circuit behaves like a unity gain inverting amplifier.

1.2

we ground the input and break the loop at the output of OA2, applying a test signal V_T. We also consider that $R \gg R_1$, so that we can neglect the current in R and easily obtain

$$\begin{aligned} V^- &= \frac{V_T}{2} \\ V^+ &= -\frac{V_T}{2}\frac{1}{sCR}\frac{R_3}{R_2+R_3} \end{aligned}$$

which gives

$$G_{loop} = -A(s)\frac{V^- - V^+}{V_T} = -\frac{A(s)}{2}\left(1 + \frac{1}{sCR}\frac{R_3}{R_2+R_3}\right) = -\frac{A(s)}{2}\left(1 + \frac{1}{s\tau_0}\right),$$

where $\tau_0 = CR(R_2+R_3)/R_3 = 22.4$ ms. The resulting Bode plot is shown in Fig. 1 (left). Note that the HF behavior follows $A(s)/2$, with a zero-dB frequency $f_{0dB} = GBWP/2 = 45$ MHz and phase margin of 90°.

1.3

Considering once again $R \gg R_1$, the midpoint between the R_1 resistors is simply $V_0/2$ and the OA2 input pins are

$$\begin{aligned} V^- &= \frac{V_o}{2} - V_{n2} \\ V^+ &= -\frac{V_o}{2}\frac{1}{s\tau_0} + V_{n1}\frac{1+s\tau}{s\tau_0}, \end{aligned}$$

where $\tau = CR = 1$ ms. This leads to

$$V_o = 2V_{n1}\frac{1+s\tau}{1+s\tau_0} + 2V_{n2}\frac{s\tau_0}{1+s\tau_0}.$$

Such transfers are reported in Fig. 1 (right), and lead to

$$\sqrt{\overline{V_o^2}} \approx \sqrt{4S_{V1}\left(\frac{1}{4\tau_0} + \left(\frac{\tau}{\tau_0}\right)^2 \frac{\pi}{2} f_{0dB}\right) + 4S_{V2}\left(\frac{\pi}{2} f_{0dB} - \frac{1}{4\tau_0}\right)} \approx 0.34 \text{ mV}.$$

1.4

Offset is a DC quantity, meaning that we can use the previous results for $s = 0$, i.e., $V_O = 2V_{OS1}$. The OA1 integrator then cancels the offset voltage of OA2. Since $2V_{OS1}$ is present at the output, OA1 should be a *Precision* OA, with low offset voltage.
A simple calculation for bias currents leads to $V_o = 2I_{B1}R + I_{B2}R_1 \Rightarrow I_{B1} \ll I_{B2}(R_1/2R) = 5 \times 10^{-3} I_{B2}$, so a small bias current is needed in order not to give a contribution to the total value.

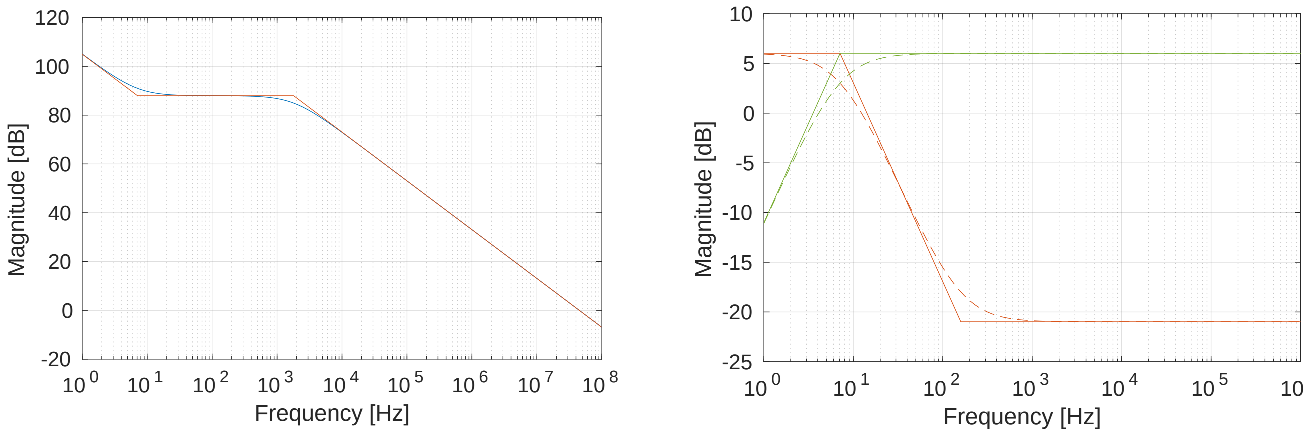


Figure 1: Left = Bode plot of the loop gain (blue) and its asymptotic approximation (red). Right = Noise transfers (not squared) for OA1 (red) and OA2 (green).

As for noise, we see from the previous result that it is dominated by S_{V2}, thanks to the $R_2 - R_3$ divider that quenches the OA1 output noise. So, there is no specific requirement on HF noise, though low flicker noise is desirable (OA1 cancels OA2 low-frequency noise).
And as for its bandwidth, note that the integrator $1/sCR$ has unity gain at about 160 Hz, so a $GBWP$ equal to OA2 is definitely not needed: OA1 can easily have a much lower bandwidth.

All these results suggest that OA1 must possess good DC performance (low offset, low bias, low flicker noise), while good AC performance is not needed. In fact, note that the signal is processed by OA2, while the integrator job is to control the DC operating point. OA2, on the contrary, must have good AC performance.

Problem 2

2.1

When LIA_1 demodulates the signal at f_H, we must be sure that its output signal (still modulated at f_L) is not filtered: the output filter of LIA_1 must then have a BW larger than f_L, say $BW_H = 650$ Hz. The BW of the second LIA LPF is instead tailored on the signal, say $BW_L = 1$ Hz.

2.2

The output signal is

$$y = A\langle\cos^2(\omega_L t)\cos^2(\omega_H t)\rangle = \frac{A}{4}\langle(1+\cos 2\omega_L t)(1+\cos 2\omega_H t)\rangle = \frac{A}{4},$$

while the noise spectrum after the first demodulation is

$$S_d(f) = \frac{1}{4}S_V(f-f_H) + \frac{1}{4}S_V(f+f_H),$$

that becomes, after the second one:

$$S_d(f) = \frac{1}{16}\left(S_V(f-f_H-f_L) + S_V(f-f_H+f_L) + S_V(f+f_H-f_L) + S_V(f+f_H+f_L)\right).$$

Please note that the above spectrum has to be multiplied by the absolute square of the two filters transfer functions. The output mean square noise value is then

$$\overline{n_y^2} \approx 2BW_L\, S_d(0) = \frac{BW_L}{4}\left(S_V(f_H+f_L) + S_V(f_H-f_L)\right) \approx \frac{BW_L}{2}S_V(f_H),$$

eventually leading to

$$\frac{S}{N} = \frac{A}{\sqrt{8BW_L\, S_V(f_H)}} = 5.$$

2.3

The inner modulation and demodulation operations result in a multiplication of signal and noise by a $\cos^2(\omega_H t) = (1 + \cos(2\omega_H t))/2$ term, meaning that the noise at the input of LIA_2 is

$$S_x = \frac{1}{4} S_V(f) + \frac{1}{16} S_V(f \pm 2f_H).$$

This noise is then demodulated by $\cos \omega_L t$, leading to a final PSD

$$S_d = \frac{1}{16} S_V(f \pm f_L) + \frac{1}{64} S_V(f \pm 2f_H \pm f_L)$$

and to

$$\overline{n_y^2} \approx 2BW_L\, S_d(0) \approx BW_L \left(\frac{1}{4} S_V(f_L) + \frac{1}{8} S_V(2f_H) \right) \approx \frac{BW_L}{4} S_V(f_L),$$

with a worse S/N with respect to the previous case:

$$\frac{S}{N} = \frac{A}{\sqrt{4BW_L\, S_V(f_L)}} = 0.9.$$

Please note that this is exactly the expected S/N of the LIA having reference frequency f_L.

2.4

If the first demodulation is carried out at f_L, we must ensure that its output signal still contains the component at f_H, to be demodulated by LIA_2. This means that LIA_1 should have an output filter bandwidth larger than f_H, say, 40 kHz. LIA_2 can then have $BW = 1$ Hz.
In principle, this is fine (and used in some radio demodulation schemes). However, this solution can suffer from several practical problems in signal recovery applications: first, a BW of 40 kHz means a time constant of about 4 μs, which is not easily achieved in standard LIAs. Moreover, an LIA with an output filter bandwidth much larger than its reference frequency does not remove the component at $2f_L$ nor the high-frequency noise. In fact, this filter is totally useless, and the entire job of cleaning the signal is left to LIA_2, which can suffer from linearity error. The solution of #2.1 is much better.