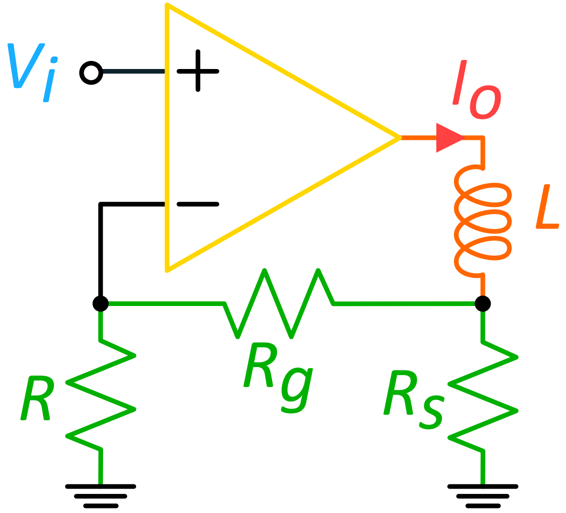
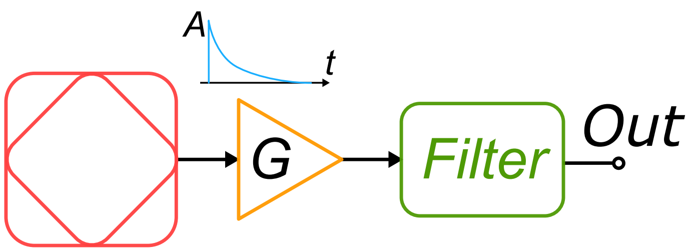
Electronics – February 18 2021 Prof. A. Spinelli





Problem 1

The scheme in the left figure is a power OA driving an inductive load. Component values are $R = 10$ kΩ, $R_g = 30$ kΩ, $R_s = 1$ Ω, $L = 16$ μH. The OA has low-frequency gain of 100 dB and $GBWP = 1$ MHz.

1. Find the expression of the closed-loop gain in the ideal case.
2. Compute the loop gain and evaluate the stability, compensating the circuit if necessary.
3. Compute the output rms noise current considering the equivalent noise sources of the amplifier, $\sqrt{S_V} = 50\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$ and $\sqrt{S_I} = 1\ \mathrm{pA}/\sqrt{\mathrm{Hz}}$.
4. V_i increases linearly from 0 to 250 mV in a time T, then remains constant. Sketch the (ideal) output voltage of the OA and compute the minimum value of T if the output dynamic is ±10 V.

Problem 2

A sensor outputs an exponential signal $Ae^{-t/T}$ with $A \approx 10$ μV and $T \approx 10$ μs. The signal is sent to a large-bandwidth preamplifier having input white noise with bilateral PSD $\lambda = 2 \times 10^{-17}$ V^2/Hz.

1. An optimum filter is applied right after the preamplifier. Compute the resulting S/N.
2. An LPF with time constant equal to T is placed after the preamplifier, followed by an optimum filter. Compute the LPF output signal and noise PSD. Compute then the weighting function and resulting S/N of the optimum filter.
3. Replace the optimum filter with a gated integrator. Select a suitable gate time and compute the resulting S/N.
4. Consider the case of #2.1, but with a bilateral noise PSD given by $S(f) = K/f + \lambda$ (i.e., flicker + white noise). Find the weghting function of the optimum filter and compute the new S/N (hint: work in the frequency domain and approximate the noise behavior in the S/N calculation).

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by February 23rd Mark registration: by Friday, Feb. 26th

Solution

Problem 1

1.1

The differential input voltage of the OA is zero, which implies $V^- = V_i$. This means that the voltage at the node right after the inductor is $V_1 = V_i(R + R_g)/R$. The total current flowing in the inductor becomes then

$$I_o = V_1 \left(\frac{1}{R_s} + \frac{1}{R + R_g} \right) \approx \frac{V_1}{R_s} \Rightarrow I_o = \frac{V_i}{R_s} \left(1 + \frac{R_g}{R} \right) = \frac{V_i}{0.25\ \Omega},$$

where we considered that $R_s \ll R_g$.
The purpose of R_s is now clear: it senses the inductor current, providing a feedback signal that is proportional to it, so that the quantity that is controlled by the feedback loop is the OA output current, not voltage!

1.2

Breaking the loop at the OA output, the loop gain can be easily obtained:

$$G_{loop} = -A(s) \frac{R_s \parallel (R_g + R)}{R_s \parallel (R_g + R) + sL} \frac{R}{R + R_g} \approx -A(s) \frac{R_s}{R_s + sL} \frac{R}{R + R_g} = -\frac{A(s)}{4} \frac{1}{1 + sL/R_s}.$$

The Bode diagram is shown in Fig. 1 (left; red curve). Note that the pole added by L is located at $f_p = R_s/(2\pi L) \approx 10$ kHz, well before the $GBWP$. At f_p we have $|G_{loop}| = 25$ and the 0-dB crossing takes place at

$$f_{0dB} = f_p \sqrt{25} = 50 \text{ kHz},$$

where the phase margin is $\phi_m = 180 - 90 - \arctan(f_{0dB}/f_p) \approx 11°$.
One possibility to compensate the stage is adding a lag network (i.e., an RC series) between the OA input pins. This network adds a pole and a zero (at a higher frequency). To find the values of the compensating elements R_c and C_c, we can proceed in this way: at high frequencies, higher than those of the pole and zero added by the compensation network, C_c can be approximated as a short-circuit, and the network reduces to resistor R_c placed in parallel to R. G_{loop} is then the same as before, but with R replaced by $R \parallel R_c$. To get $\phi_m = 45°$ it has to be $f_{0dB} = f_p = 10$ kHz, down by a factor of 25 with respect to the original scheme, and down by 100 with respect to $A(s)$. This simply means:

$$\frac{R \parallel R_c}{R_g + R \parallel R_c} = \frac{1}{100} \Rightarrow \frac{R_g}{R \parallel R_c} = 99 \Rightarrow R_c \approx 312\ \Omega.$$

C_c can now be picked with the simple requirement $f_z \le f_{0dB}$. If we pick $f_z = 1$ kHz, we have

$$C_c = \frac{1}{2\pi R_c f_z} = 510 \text{ nF}.$$

Note that this network does not affect the ideal gain, but limits the bandwidth to the new $f_{0dB} = 10$ kHz.

1.3

The noise voltage V_n is connected as the input, and the transfer is the same as in 1.1. The output rms noise current is then

$$\sqrt{\overline{I_o^2}} = 4V_n \sqrt{\frac{\pi}{2} f_{0dB}} \approx 25\ \mu\text{A},$$

where we have used the previous value $f_{0dB} = 10$ kHz. The noise current source connected at the non-inverting input obviously gives no contribution, leaving us with noise current I_n connected at the inverting OA input. Given that in R flows no current, voltage V_1 (see 1.1) is simply $I_n R_g$, and the total current flowing through L becomes

$$I_o = I_n + \frac{V_1}{R_s} = I_n \left(1 + \frac{R_g}{R_s} \right) = 3 \times 10^4 I_n \Rightarrow \sqrt{\overline{I_o^2}} = 3 \times 10^{-8} \sqrt{\frac{\pi}{2} f_{0dB}} \approx 3.8\ \mu\text{A},$$

negligible with respect to the previous one.

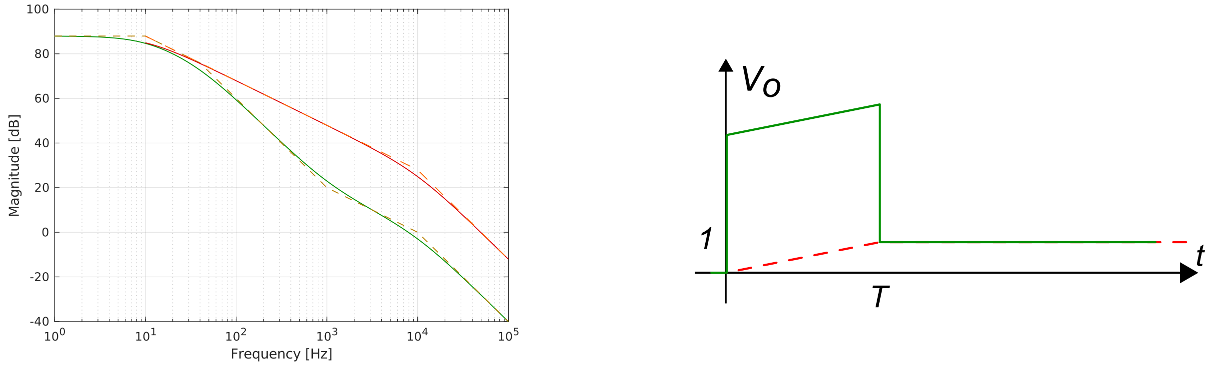


Figure 1: Left = Bode plot of the loop gains before (red) and after (green) compensation. Right = OA output voltage (green). The red dashed line is the voltage drop across R_s.

1.4

The output voltage of the OA equals the voltage drops across R_s and L. The first is just equal to $4V_i$ and increases linearly up to 1 V. The second is given by (in the $0-T$ interval)

$$V_L = L\frac{dI_L}{dt} = L\frac{I}{T},$$

because the inductor current I_L (flowing into R_s) increases linearly from 0 to $I = 1$ A in a time T (see Fig. 1, right). The output limit is not exceeded if

$$V_L < V_{cc} - 1 \Rightarrow T > \frac{LI}{V_{cc}-1} \approx 1.8\ \mu\text{s}.$$

Problem 2

2.1

Regarding the preamp output noise as white, the expression for the optimum S/N is

$$\left(\frac{S}{N}\right)_{opt} = \frac{A}{\sqrt{\lambda}}\sqrt{\int_0^\infty e^{-2t/T}dt} = A\sqrt{\frac{T}{2\lambda}} = 5,$$

,

2.2

The output signal is

$$y(t) = x(t) * h(t) = \int x(\tau)h(t-\tau)d\tau = \frac{A}{T}\int_0^t e^{-\tau/T}e^{-(t-\tau)/T}d\tau = A\frac{t}{T}e^{-t/T}.$$

The noise PSD is

$$S_y(f) = \lambda|H(j\omega)|^2 = \frac{\lambda}{1+(2\pi fT)^2}, \tag{1}$$

and its autocorrelation is $(\lambda/2T)e^{-\gamma/T}$. The optimum filter can be seen as the cascade of a whitening filter (an HPF) plus the matched filter stage. This latter one would be exactly the same as in #2.1, meaning that S/N takes exactly the same value there computed.

2.3

We set $x = t/T$ and integrate between x_1 and x_2. It is easy to see that the signal (Fig. 2, left) reaches its peak at $x = 1$, so that the limits should be $x_1 < 1$ and $x_2 > 1$. The output signal is then

$$y = KAT\int_{x_1}^{x_2} xe^{-x}dx = KAT\left(e^{-x_1}(1+x_1) - e^{-x_2}(1+x_2)\right)$$

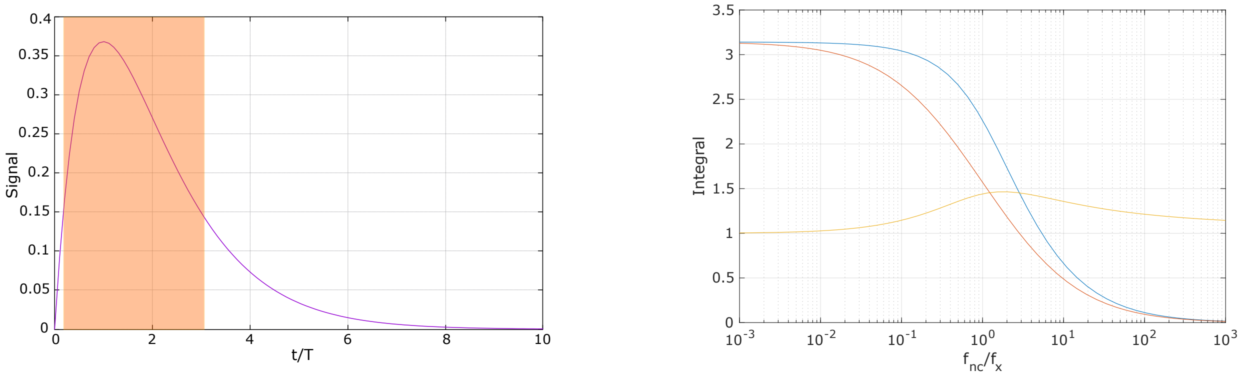


Figure 2: Left = LPF output signal and optimum integration window. Right = Approximated (blue) and correct (red) solution for the noise integral. The yellow line is the ratio between the two.

For the noise calculation we must consider the exponential autocorrelation of the input noise (see previous point), so that we have

$$n_y^2 = \frac{\lambda K^2 T_G}{T} \int_0^{T_G} e^{-\gamma/T} \left(1 - \frac{\gamma}{T_G}\right) d\gamma = \frac{\lambda K^2 T_G}{T} \left(T \int_0^{x_2 - x_1} e^{-x} dx - \frac{T^2}{T_G} \int_0^{x_2 - x_1} x e^{-x} dx \right)$$

$$= \lambda K^2 T \left((x_2 - x_1) \left(1 - e^{-(x_2 - x_1)}\right) + e^{-(x_2 - x_1)}(1 + x_2 - x_1) - 1 \right)$$

For example, between 0 and 2 the x-function becomes 0.55. The optimum value is $x_1 \approx 0.4, x_2 \approx 2.1$, with function value ≈ 0.59 (note that it is 0.7 for the optimum filter case).

2.4

The optimum weighting function in the frequency domain is given by

$$|W(f)| \propto \frac{|X(f)|}{S(f)} = \frac{AT}{\sqrt{1 + (2\pi T f)^2}} \frac{1}{K/f + S_W},$$

and the resulting S/N is given by (calling $K/S_W = f_{nc}$ and $1/(2\pi T) = f_x$)

$$\left(\frac{S}{N}\right)^2 = 2 \int_0^\infty \frac{|X(f)|^2}{S(f)} df = \frac{2A^2T^2}{K} \int_0^\infty \frac{1}{1 + (f/f_x)^2} \frac{f}{1 + f/f_{nc}} df.$$

This integral can be analitically solved, but it is easier to break it down in two regimes: one where flicker noise is dominant ($f \ll f_{nc}$) and the other where white noise gives the main contribution ($f \gg f_{nc}$). This leads to (only for the integral)

$$\int_0^\infty \approx \int_0^{f_{nc}} \frac{f}{1 + (f/f_x)^2} df + \int_{f_{nc}}^\infty \frac{f_{nc}}{1 + (f/f_x)^2} df = \frac{\ln(1 + (f_{nc}/f_x)^2)}{2(2\pi T)^2} + \frac{f_{nc}}{2\pi T} \left(\frac{\pi}{2} - \arctan\left(\frac{f_{nc}}{f_x}\right) \right)$$

$$\left(\frac{S}{N}\right)^2 = \frac{A^2}{(2\pi)^2 \lambda f_x} \frac{\ln(1 + x^2) + x(\pi - 2\arctan x)}{x},$$

where $x = f_{nc}/f_x$. The correct solution is instead:

$$\left(\frac{S}{N}\right)^2 = \frac{A^2}{(2\pi)^2 \lambda f_x} \frac{x \ln x^2 + \pi}{1 + x^2},$$

and the two results (only the x-dependent part) are compared in Fig. 2 (right).