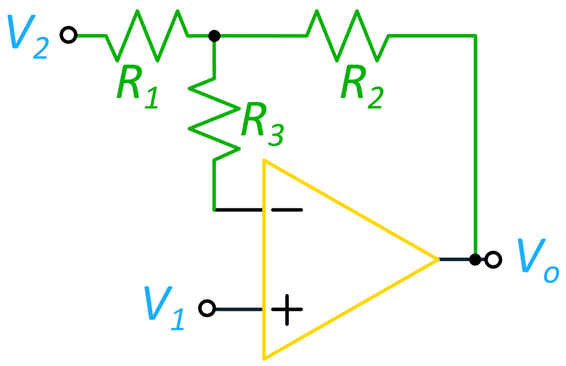
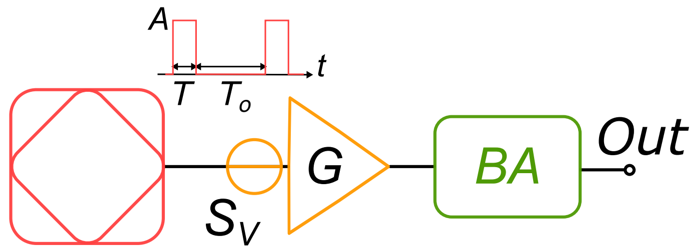
Electronics – June 23 2021 Prof. A. Spinelli





Problem 1

The scheme in the left figure has $R_1 = R_2 = 5\ \text{k}\Omega$, $R_3 = 200\ \text{k}\Omega$. The OA has low-frequency gain of 100 dB and $GBWP = 1$ MHz.

1. Find the expression of the closed-loop gains in the ideal case.
2. The OA has an input capacitance $C_i = 8$ pF. Compute the loop gain and evaluate the stability, compensating the circuit if necessary (do not change the value of R_3).
3. Compute the output rms noise voltage considering the equivalent noise source of the amplifier, $\sqrt{S_V} = 20\ \text{nV}/\sqrt{\text{Hz}}$. What value of S_I would result in a negligible current noise contribution? What kind of OA (BJT/CMOS) would you pick?
4. Consider the OA differential input resistance $R_d = 200\ \text{k}\Omega$ and compute the low-frequency impedance seen at input V_1.

Problem 2

A sensor outputs rectangular pulses with amplitude $A \approx 20\ \mu$V and duration $T \approx 10\ \mu$s, separated by $T_0 = 500\ \mu$s. The signal is sent to a preamplifier with bandwidth of 1 MHz and input noise PSD $\sqrt{S_V} = 50\ \text{nV}/\sqrt{\text{Hz}}$, followed by a boxcar averager.

1. Find a set of BA parameters that grants $S/N = 10$.
2. A low-frequency noise having nearly rectangular autocorrelation over a time $T_n = 1$ s is also present at the BA input. What is the output noise?
3. Find a suitable HPF to be placed before the BA to remove the LF noise. What is the minimum value of T_0 compatible with an error of 1% on the signal?
4. Consider problem 2.1. Would it be convenient to move the BA before the amplifier? Why?

For a correct evaluation, you are asked to write your answers in a readable way; thank you

Do a good job!

Results will be posted by July 2nd

Mark registration: by Tuesday, July 6th

Solution

Problem 1

1.1

Considering that no current flows in R_3, by a simple linear superposition approach we obtain

$$V_o = 2V_1 - V_2.$$

1.2

We break the loop at the OA output. A fast computation is the following: as $R_3 \gg R_1, R_2$, the voltage at midpoint between R_1 and R_2 is $V_T/2$, V_T being the test voltage applied. We then have the $R_3 - C_i$ LPF leading to V^- and to

$$G_{loop}(s) \approx -A(s)\frac{1}{2(1 + sC_iR_3)}.$$

An alternative way is to note that C_i introduces a pole and no zero. We then compute the DC loop gain $-A(s)/2$ and the resistance seen by C_i, $R_3 + R_1 \parallel R_2$, obtaining the correct expression

$$G_{loop}(s) = -\frac{A(s)}{2}\frac{1}{1 + sC_i(R_3 + R_1/2)}.$$

For the formal derivation, we label

$$Z = R_1 \parallel \left(R_3 + \frac{1}{sC_i}\right) = R_1\frac{1 + sC_iR_3}{1 + sC_i(R_1 + R_3)},$$

so that the midpoint voltage between R_1 and R_2 becomes:

$$V_3 = V_s\frac{Z}{Z + R_2} = V_s\frac{R_1(1 + sC_iR_3)}{R_1 + R_2 + sC_i(R_1R_2 + R_1R_3 + R_2R_3)},$$

leading to

$$V_o = V_s\frac{1}{1 + sC_iR_3} = V_s\frac{R_1}{R_1 + R_2 + sC_i(R_1R_2 + R_1R_3 + R_2R_3)}$$

$$G_{loop}(s) = -A(s)\frac{R_1}{R_1 + R_2 + sC_i(R_1R_2 + R_1R_3 + R_2R_3)} = -\frac{A(s)}{2}\frac{1}{1 + sC_i(R_3 + R_1/2)}.$$

The pole frequency is $f_p = 98$ kHz, where $|G_{loop}(f_p)| = 5.1 \approx 14.1$ dB. We have then $f_{0dB} = fp\sqrt{5.1} \approx 221$ kHz and $\phi_m = 90 - \arctan(f_{0dB}/f_p) \approx 24°$.
The easist compensation option is to add a compensation capacitor C_c in parallel to R_3. To find its value, we can follow the first approach of G_{loop} calculation, obtaining

$$G_{loop} \approx -\frac{A(s)}{2}\frac{1}{1 + sC_iR_3} \Rightarrow -\frac{A(s)}{2}\frac{1 + sC_cR_3}{1 + s(C_c + C_i)R_3},$$

where the last expression stems from Replacing R_3 with $R_3/(1 + sC_cR_3)$. Picking $f_z = 221$ kHz we obtain $C_c = 3.6$ pF, $f_p = 68.6$ kHz, $f_{0dB} = 185$ kHz and $\phi_m = 90 - \arctan(f_{0dB}/f_p) + \arctan(f_{0dB}/f_z) \approx 60°$. Bode plots are reported in Fig. 1 (left). Higher values of C_c lead to pole-zero cancellation and even higher ϕ_m.

Note that placing C_c in parallel to R_2 would not be a good solution, as it would result in $\tau_z = R_2C_c$ and $\tau_p \approx (R_1 \parallel R_2)C_c = 0.5\tau_z$, i.e., $f_p = 2f_z$: the additional pole is close to the zero and prevents the achievement of sufficient stability.

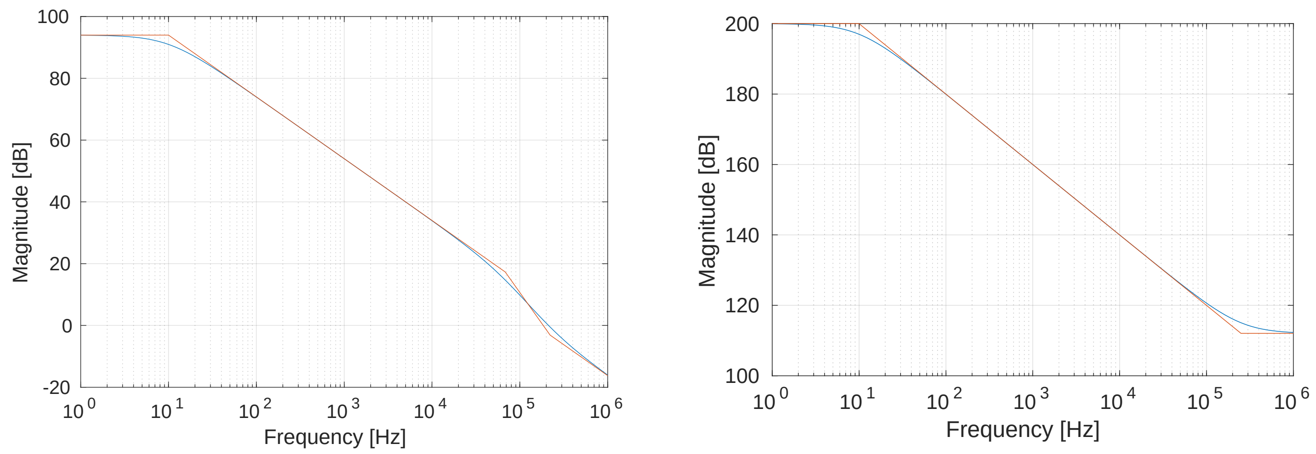


Figure 1: Left = Bode plot of the compensated loop gain (blue) and its asymptotic approximation (red). Right = Bode plot of the input impedance.

1.3

The noise voltage V_n is connected like the input V_1, and the transfer is the same as in 1.1. As for the noise current I_n, the midpoint between R_1 and R_2 is at voltage $I_n R_3$ and the current balance becomes

$$I_n + I_n \frac{R_3}{R_1} = \frac{V_o - I_n R_3}{R_2} \Rightarrow V_o = I_n \left(R_2 + R_3 + \frac{R_2 R_3}{R_1} \right) = I_n \times 405 \text{ k}\Omega.$$

Given that f_{0dB} is the same for both transfers, we can compare the PSDs. The noise current is negligible if

$$\sqrt{S_I}\ 405 \times 10^3 \ll 40 \times 10^{-9} \Rightarrow \sqrt{S_I} \ll 9.8 \times 10^{-14} \approx 0.1 \text{ pA}/\sqrt{\text{Hz}}.$$

A CMOS OA is most suited to this purpose.

1.4

The open-loop impedance is

$$Z_{OL} = R_d + R_3 + R_1 \parallel R_2 \approx R_d + R_3,$$

while G_{loop} is now approximately

$$G_{loop} \approx -A(s) \frac{R_1}{R_1 + R_2} \frac{R_d}{R_d + R_3} = -\frac{A(s)}{4},$$

leading to

$$Z_i = Z_{OL}(1 - G_{loop}(s)) \approx (R_d + R_3) \left(1 + \frac{A(s)}{4} \right).$$

The value is about 10 GΩ at low frequency. It is interesting to note that R_3 increases Z_{OL} but reduces G_{loop} and has no net effect on the low-frequency Z_i.

Problem 2

2.1

The time constant of the preamplifier is 160 ns, meaning that the pulse amplitude is not affected. Moreover, if such time is much shorter than the BA time constant, the BA input noise can be regarded as white. The BA output S/N is then (same formula as the LPF):

$$\left(\frac{S}{N} \right)_{BA} = A \sqrt{\frac{4 T_F}{S_V}} = 10 \Rightarrow T_F = \frac{25 S_V}{A^2} = 156\ \mu\text{s},$$

corresponding to $N_{eq} = 2T_F/T = 31.2$ (considering $T_C = T$).
The same result can be reached going through the single-pulse response, that is (GI approximation):

$$\left(\frac{S}{N}\right)_{sp} = A\sqrt{\frac{2T_C}{S_V}} = 1.79,$$

leading to

$$\left(\frac{S}{N}\right)_{BA} = \left(\frac{S}{N}\right)_{sp}\sqrt{N_{eq}} = 10 \Rightarrow N_{eq} = \left(\frac{10}{1.79}\right)^2 = 31.2.$$

2.2

To compute the output noise we should evaluate the expression

$$\overline{n_y^2} = \int R_{xx}(\gamma)k_{w_{tt}}(\gamma)d\gamma \approx \overline{n_x^2}\int_{-T_n}^{T_n} k_{w_{tt}}(\gamma)d\gamma.$$

We know that the time correlation of the BA weighting function is made up of a series of spikes with exponential envelope characterized by a time constant $T_F(T_C + T_O)/T_C \approx 8$ ms, meaning that $k_{w_{tt}}(\gamma)$ is nearly zero at time T_n (another way to reach this conclusion is to consider the weighting function to be non-zero over a time of the order of $N_{eq}(T_C + T_o) = 16$ ms; the two values are in the same ballpark).
This result means that the integral from $-T_n$ to T_n actually spans over the entire $k_{w_{tt}}$, resulting in (remember that integral in the time domain means zero value in the frequency one):

$$\overline{n_y^2} = \overline{n_x^2}\int k_{w_{tt}}(\gamma)d\gamma = \overline{n_x^2}|W(t,0)|^2 = \overline{n_x^2}.$$

This result makes sense! If the noise is correlated over such a long time, it is collected by the BA as if it were an offset or a very low frequency signal.

2.3

The rectangular autocorrelation has width $2T_n = 2$ s, meaning that the noise equivalent bandwidth f_n is

$$2f_n = \frac{1}{2T_n} \Rightarrow f_n = \frac{1}{4T_n} = 0.25 \text{ Hz}.$$

We can then pick an HPF with a pole at 2.5 Hz (or even higher, if more noise reduction is needed), i.e., a time constant $T_F = 1/5\pi \approx 64$ ms. An alternative derivation comes from recaling that the noise reduction for this case is (see class notes) e^{-T_n/T_F}: if we want a reduction of 10^3 in the rms value, $e^{-T_n/T_F} = 10^{-6} \Rightarrow T_F = T_n/\ln 10^6 \approx 72$ ms.
Because $T_F \gg T + T_0$, and the HPF transfer function has a zero at DC, the output is similar to the input signal, but with zero average value, meaning that the new amplitude is (see class notes)

$$A_H = A\frac{T_0}{T + T_0} > 0.99A \Rightarrow T_0 > 99T = 990\ \mu\text{s}.$$

Note that the result is independent of T_F.

2.4

The option is completely useless! Obviously, if the BA is placed *before* the amplifier (and hence its noise sources), it performs no noise filtering and the output S/N is $A/\sqrt{1.57 \times 10^6\ S_V} = 0.32$.
Perhaps a more interesting question is what happens assuming we could ideally place the BA before the amplifier but *after* its noise source (obviously impossible in reality). In theory, S/N would be the same. In practice, neither this option would work, as the very small voltage stored on the capacitor would make it prone to errors due to leakage currents, bias, offset and so on.