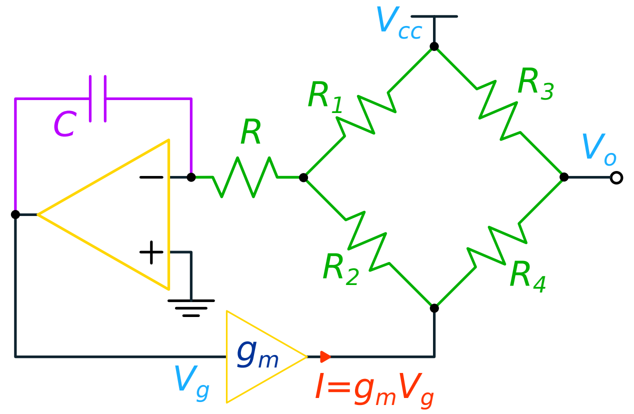
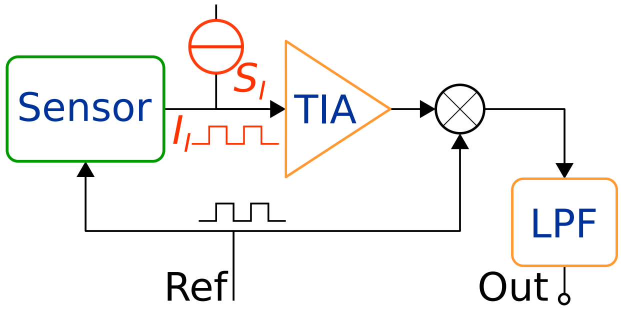
Electronics – September 8 2022 Prof. A. Spinelli





For a correct evaluation, please write your answers in a readable way; thank you!

Problem 1

The scheme in the left figure controls the bridge drive by means of a transconductance amplifier. Bridge resistors are 350 Ω, power supply is $V_{cc} = 10$ V. Other parameters are $R = 100$ kΩ, $C = 20$ nF, $g_m = 1$ S. The zero-drift CMOS OA has low-frequency gain of 120 dB and $GBWP = 2$ MHz.

1. Find the DC value of V_o when the bridge is at rest (i.e., bridge resistors are equal).
2. Compute the loop gain and discuss the stability.
3. Compute the output rms noise voltage considering the equivalent voltage noise source of the amplifier only $\left(\sqrt{S_V} = 100 \text{ nV}/\sqrt{\text{Hz}}\right)$.
4. What are the pros and cons of this solution with respect to the standard one where the bridge is simply connected to ground?

Problem 2

The scheme in the right figure uses synchronous detection to measure a current signal coming from a sensor. A symmetric square wave excitation is used, and the output (symmetric) square-wave current $\pm I_i \approx \pm 1\ \mu$A is amplified by a transimpedance amplifier (TIA). The total input noise current PSD is $S_I = K/f + S_{WN}$, with $K = 10^{-11}$ A^2 and $S_{WN} = 10^{-14}$ A^2/Hz.

1. Find values for the LIA parameters that allow to reach $S/N = 10$.
2. The sensor has its own time constant τ and behaves like an LPF. Sketch the signal at the mixer output and evaluate the new LIA output signal (consider for simplicity the case in which the half-period T of the square wave is $> 5\tau$).
3. In the previous case, would it be convenient to add a phase shift between the two signals at the mixer input? Justify your answer.
4. With reference to #2.2, evaluate the LIA output signal in the general case, when T is not necessarily much larger than τ (hint: start from a symmetric exponential signal at the mixer input).

Do a good job!

Results will be posted by September 13th Mark registration: Friday, September 16th

Solution

Problem 1

1.1

We label V_A the voltage at the midpoint between R_1 and R_2. We have then

$$V_g = -\frac{V_A}{sCR} \Rightarrow I = -V_A \frac{g_m}{sCR}.$$

Neglecting the very small current in R, I will split evenly in the two branches, leading to

$$V_A = V_{cc} + \frac{I}{2} R_1 \Rightarrow V_A = V_{cc} \frac{2sCR}{g_m R_1 + 2sCR}.$$

At DC we have $V_A = 0$, and $V_o = 0$. This result is obvious: at DC the integrator has infinite gain, so the only possible working point is zero voltage at its input, i.e., $V_A = 0$!
In reality, the transconductance stage is easily implemented via a *pnp* BJT and a biasing resistor.

1.2

We ground the power supply and open both loops at the OA output, applying a test signal V_s. Neglecting once again the tiny current flowing through R we obtain

$$V_A = V_s \frac{g_m R_1}{2}$$

which leads to

$$V^- = V_s \frac{sCR}{1+sCR} + V_A \frac{1}{1+sCR} = V_s \frac{g_m R_1}{2} \frac{1 + 2sCR/g_m R_1}{1+sCR} \Rightarrow G_{loop} = -A(s) \frac{g_m R_1}{2} \frac{1 + 2sCR/g_m R_1}{1+sCR}.$$

The pole and zero introduced by C are $f_p = 1/(2\pi RC) \approx 80$ Hz, $f_z = (g_m R_1/2) f_p \approx 14$ kHz. Beyond such frequencies $G_{loop} = -A(s)$ and the phase margin is 90°. The loop gain is plotted in Fig. 1 (left).

1.3

Following the previous convention, we know that $V_A = (g_m R_1/2) V_G$. We can then solve for the OA integration scheme, obtaining

$$V_G = -V_A \frac{1}{sCR} + V_n \frac{1+sCR}{sCR},$$

which leads to

$$V_G = V_n \frac{1+sCR}{g_m R_1/2 + sCR} \Rightarrow V_o = \frac{g_m R_1}{2} V_G = V_n \frac{1+sCR}{1 + 2sCR/g_m R_1},$$

where now $f_p \approx 14$ kHz and $f_z \approx 80$ Hz. Accounting for the additional pole at $GBWP$ we get

$$\overline{V_o^2} \approx S_V \frac{\pi}{2} \left(f_z + \left(\frac{g_m R_1}{2} \right)^2 (GBWP - f_p) \right) \approx S_V \frac{\pi}{2} \left(\frac{g_m R_1}{2} \right)^2 GBWP = (3.1\ \text{mV})^2.$$

1.4

The main advantage of this solution is that it rejects common-mode voltages, allowing to take single-ended measurements. Note in fact that the OA is a zero-drift one, which essentially eliminates offset drift with time and temperature.
The trade-off with respect to a standard design and an INA include more complexity and, more important, the need for a negative power supply.

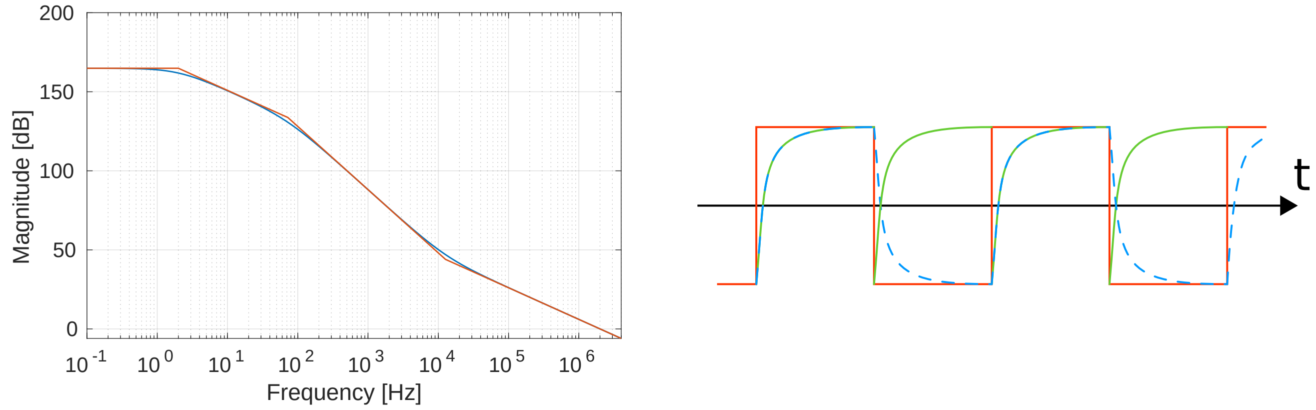


Figure 1: Left = Bode plot of the loop gain (blue) and its asymptotic approximation (red). Right = Pictorial view of the signal at the sensor output (blue dashes) and after the mixer (green).

Problem 2

2.1

We start by picking a reference frequency higher than the 1 kHz noise corner frequency, and remember that in the case of full square wave modulation/demodulation the S/N ratio at the output of the LPF is higher by a factor of $\sqrt{2}$ with respect to the sinusoidal case (remember also that noise PSDs are unilateral):

$$\frac{S}{N} = \frac{I_i}{\sqrt{S_{WN} BW_n}} = 10 \Rightarrow BW_n = \frac{I_i^2}{100 S_{WN}} = 1 \text{ Hz}.$$

2.2

We consider for simplicity a unit amplitude symmetric square wave. The shape of the signals at the sensor and mixer outputs are pictorially shown in Fig. 1 (right) (we neglect the effect of the TIA gain). If we label V_M the amplitude, the signal at the mixer output (green curve) can be expressed as

$$V(t) = -V_M + 2V_M\left(1 - e^{-t/\tau}\right) = V_M\left(1 - 2e^{-t/\tau}\right),$$

obviously valid in a half-period $0 \le t \le T$. The LPF returns the average value, i.e.

$$V_o = \langle V \rangle = \frac{V_M}{T}\int_0^T V(t)dt = V_M - \frac{2V_M}{T}\int_0^T e^{-t/\tau}dt = V_M\left(1 - \frac{2\tau}{T}\left(1 - e^{-T/\tau}\right)\right) \approx V_M\left(1 - \frac{2\tau}{T}\right).$$

Note that even for $T = 10\tau$ we get 20% signal reduction.

2.3

The short answer is yes! With reference to Fig. 1, right, shifting the reference square wave (red) would result in a higher signal, as we would avoid the negative spikes in the green signal. Another way of looking at this is via the Fourier transform: the fundamental component of the FT of the blue signal will obviously be shifted with respect to the reference (think of the phase term added by an LPF under sinusoidal excitation), so compensating for this shift will enhance the signal.

Though not strictly required, we can carry out some calculations: with a positive shift t_s we get (in the $t_s - T$ interval):

$$V_o = V_M\frac{T - t_s}{T} - \frac{2V_M}{T}\int_{t_s}^T e^{-t/\tau}dt = V_M\left(1 - \frac{t_s}{T} - \frac{2\tau}{T}\left(e^{-t_s/\tau} - e^{-T/\tau}\right)\right)$$

plus the contribution in the $T - T + t_s$ one, where the (blue) signal is the opposite:

$$V_o = -V_M\frac{t_s}{T} + \frac{2V_M}{T}\int_0^{t_s} e^{-t/\tau}dt = V_M\left(-\frac{t_s}{T} + \frac{2\tau}{T}\left(1 - e^{-t_s/\tau}\right)\right).$$

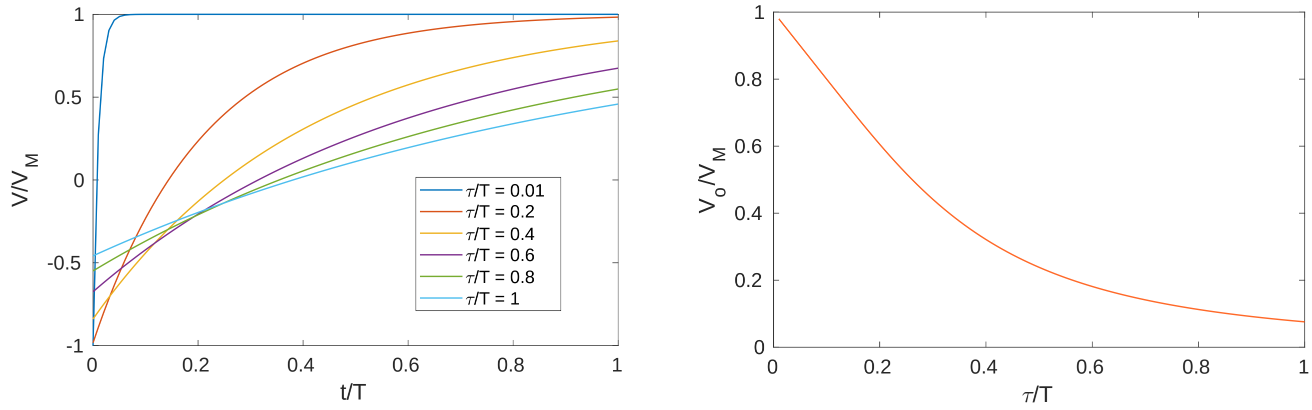


Figure 2: Left = Mixer output signal (on a half-period T) for different values of τ. Right = LPF output signal for different values of τ.

Summing the contributions, we get a negative t_s-dependent term that must be minimized, equal to

$$F = 2V_M \left(\frac{t_s}{T} + \frac{2\tau}{T} e^{-t_s/\tau} \right).$$

Setting $dF/dt_s = 0$ we get $t_s = \tau \ln 2 \approx 0.69\tau$. The relative improvement is higher for larger values of τ. By the way, this result is obvious: looking at Fig. 1 (left), it is clear that the best condition is the one in which the green curve is always positive, i.e., when the square wave is synchronous with its zero crossing. Starting from $t = 0$ and unit amplitude, the green curve is

$$V(t) = -1 + 2\left(1 - e^{-t/\tau}\right),$$

which goes through zero at $t = \tau \ln 2$.

2.4

In the general case the mixer input signal is the response of the sensor LPF to a symmetric square wave, i.e., a symmetric exponential signal, which is then multiplied by a synchronous square wave resulting in the signal reported in Fig. 2 (left). If we label V_H the amplitude, we have

$$V(t) = -V_H + (V_M + V_H)\left(1 - e^{-t/\tau}\right).$$

at $t = T$ we have $V(T) = V_H$ because of symmetry:

$$V_H = -V_H + (V_M + V_H)\left(1 - e^{-T/\tau}\right) \Rightarrow V_H = V_M \frac{1 - e^{-T/\tau}}{1 + e^{-T/\tau}} = V_M \tanh\left(\frac{T}{2\tau}\right).$$

The output signal is of course the temporal average:

$$V_o = \langle V \rangle = V_M - (V_H + V_M)\frac{1}{T}\int_0^T e^{-t/\tau} dt = V_M - (V_H + V_M)\frac{\tau}{T}\left(1 - e^{-T/\tau}\right).$$

The signal is reported in Fig. 2 (right) as a function of τ/T. Note that this technique can be used to extract the value of τ, by performing measurements at different frequencies.