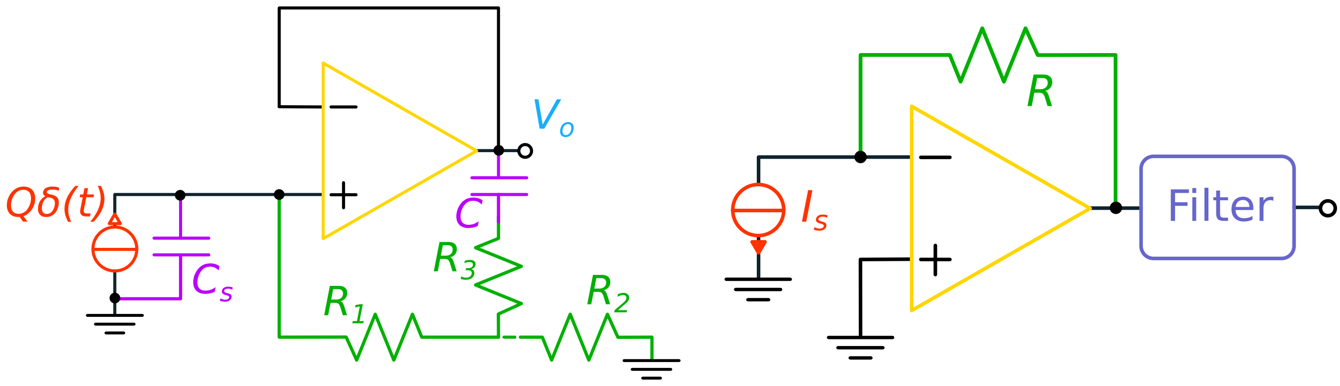
Electronics – July 21 2022 Prof. A. Spinelli



For a correct evaluation, please write your answers in a readable way; thank you!

Problem 1

The scheme in the left figure is an amplifier for a capacitive sensor having $C_s = 33$ pF. Other parameters are $R_1 = 10$ MΩ, $R_2 = 100$ kΩ, $R_3 = 1$ kΩ, $C = 1$ μF. The OA has low-frequency gain of 100 dB and $GBWP = 100$ kHz. Consider initially the circuit without C and R_2 (both assumed to be of infinite value.

1. Find the (ideal) output voltage vs. time when the sensor outputs delta-like pulses.
2. From now on, add capacitor C. Compute the loop gain and evaluate the phase margin.
3. Compute the output rms noise voltage considering the equivalent voltage noise source of the amplifier $\left(\sqrt{S_V} = 15\ \mathrm{nV}/\sqrt{\mathrm{Hz}}\right)$ and resistors ($4k_BT \approx 1.646 \times 10^{-20}$ J).
4. Consider the OA bias currents and discuss the purpose of R_2.

Problem 2

A sensor outputs a rectangular current pulse of width T and small amplitude $I_s \approx 100$ fA. The signal is affected by shot noise and is amplified by a transimpedance stage, followed by an LPF.

1. Find a suitable value for T that grants $S/N = 10$, and the value of R that allows to neglect its thermal noise.
2. Remove the LPF and replace R with a capacitor C (and a switch). Evaluate the new value of T.
3. The current signal is now $I_s = I_0 e^{-t/T}$ with $I_0 \approx 100$ fA, still affected by shot noise. The current noise source of the CMOS OA is $\sqrt{S_{OA}} = 0.3\ \mathrm{fA}/\sqrt{\mathrm{Hz}}$. What is the optimum weighting function (remove the LPF)?
4. Compute S/N in the optimum case. Remember that $\int \frac{e^{-2x}}{e^{-x}+a} dx = a \ln(e^{-x} + a) - e^{-x}$ (for $a \geq 0$).

Do a good job!

Results will be posted by July 26th **Mark registration: July 27th**

Solution

Problem 1

1.1

Without resistor R_2 there is no current flowing in the feedback path, meaning that the input current flows into the capacitor C_s, providing a step output voltage:

$$V_o = \frac{Q}{C_s} u(t).$$

1.2

We disconnect the current source and open both loops at the OA output, applying a test signal V_s. We easily get

$$V^+ = V_s \frac{1/sC_s}{R_1 + R_3 + 1/sC + 1/sC_s} \approx V_s \frac{C}{C + C_s + sCC_sR_1},$$

which leads to

$$G_{loop} = -A(s) \left(1 - \frac{C}{C + C_s + sCC_sR_1}\right) = -A(s) \frac{C_s}{C + C_s} \frac{1 + sCR_1}{1 + sC_tR_1},$$

where $C_t = CC_s/(C + C_s) \approx C_s$ is the series capacitance. The pole and zero are now $f_p = 1/(2\pi R_1 C_t) \approx 482$ Hz, $f_z = 1/(2\pi R_1 C) \approx 16$ mHz. At higher frequencies $G_{loop} = -A(s)$ and the phase margin is 90°. The loop gain is plotted in Fig. 1 (left).

1.3

The scheme for noise calculation is depicted in Fig. 1 (right), where V_n and V_R are the noise sources associated to the OA and resistor $R_1 + R_3$. Current I can be written as

$$I = \frac{V_n - V_R}{R_1 + R_3 + 1/sC} \approx (V_n - V_R) \frac{sC}{1 + sCR_1},$$

and flows into the capacitor C_s, leading to:

$$V_o - V_n = I \frac{1}{sC_s} = (V_n - V_R) \frac{C/C_s}{1 + sCR_1} \Rightarrow V_o = V_R \frac{C/C_s}{1 + sCR_1} + V_n \frac{C}{C_t} \frac{1 + sC_tR_1}{1 + sCR_1}.$$

Note that the second transfer has a pole and a zero, beyond which the gain is 1. The output mean square noise is then

$$\begin{aligned}\overline{V_0^2} &= S_R \left(\frac{C}{C_s}\right)^2 \frac{1}{4CR_1} + S_V \left[\left(\frac{C}{C_t}\right)^2 \frac{1}{4CR_1} + \frac{\pi}{2}(f_{0dB} - f_z)\right] \\ &\approx 2.3 \times 10^7 S_R + (2.3 \times 10^7 + 1.57 \times 10^5) S_V = 5.2 \times 10^{-9} + 3.8 \times 10^{-6} \approx (2 \text{ mV})^2.\end{aligned}$$

Note that the noise is dominated by the low-frequency terms, controlled by the ratio C/C_s. A smaller value of C could then be employed, but this scheme has other problems: for example, if the OA current noise contribution were computed, the result would be $V_o = I_n/sC_s$, i.e., a divergence at low frequency.

1.4

The main purpose of resistor R_2 is to provide a DC path for the OA bias currents. In its absence, I_B^+ would charge the capacitor C_s leading to saturation of the output. It is also nice to note that this resistor reduces the noise DC gains, that become 1 for V_n and zero for V_R, removing also the singularity for the current noise. As a bonus comment, R_2 also decouples the two capacitors, that now give rise to two poles (and two zeroes) in the loop, and the system remains stable. However, the closed-loop transfer has complex poles, which is not desirable. A better design is achieved by increasing the value of C, to counteract the reduction in resistance seen with the addition of R_2.

If you wish to solve the circuit, the full transfer function becomes

$$\frac{V_o}{I_s} = \frac{R_1 + R_2 + sC(R_1R_2 + R_1R_3 + R_2R_3)}{1 + s(CR_3 + C_s(R_1 + R_2)) + s^2CC_s(R_1R_2 + R_1R_3 + R_2R_3)} \approx \frac{R_1(1 + sCR_2)}{1 + s(CR_3 + C_sR_1) + s^2CC_sR_1R_2}.$$

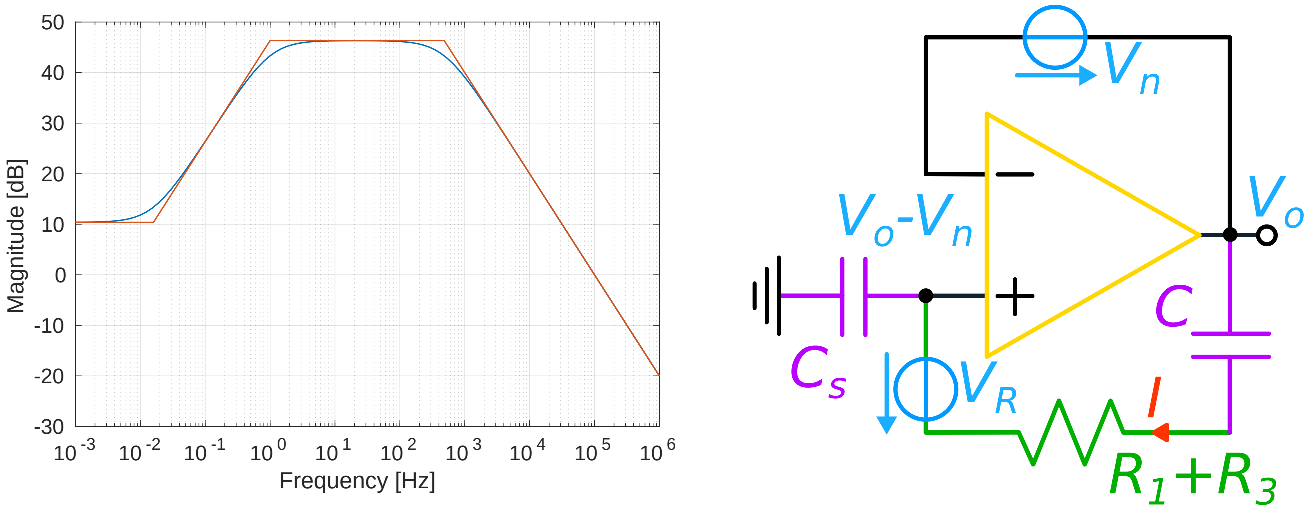


Figure 1: Left = Bode plot of the loop gain (blue) and its asymptotic approximation (red). Right = Scheme for noise calculation.

Problem 2

2.1

We begin with the LPF. The unilateral shot noise PSD associated to a current I_s is $S_I = 2qI_s$. The S/N ratio at the output of the LPF is then

$$\frac{S}{N} = \frac{I_s R}{\sqrt{2qI_s R^2 BW_n}} = \sqrt{\frac{I_s}{2qBW_n}} = 10 \Rightarrow BW_n = \frac{I_s}{200q} \approx 3.1 \text{ kHz}.$$

If we are using a single-pole LPF, its bandwidth will be about 2 kHz, i.e., a time constant of about 80 μs. The signal will reach the steady state after about $5 \times 80 = 400$ μs, that is the (minimum) value for T. Resistor R must be such that

$$\frac{4k_bT}{R} \ll 2qI_s \Rightarrow R \gg \frac{2k_BT}{qI_s} = 5.1 \times 10^{11}\ \Omega.$$

This requirement is very tight, and a better solution is always used (see next point).

2.2

The OA now works as a gated integrator. Signal and noise are then (GI with a gain of $1/C$):

$$V_0 = \frac{I_s}{C}T \qquad \overline{V_o^2} = \frac{S_I}{2}\frac{1}{C^2}T,$$

leading to:

$$\frac{S}{N} = \frac{I_sT}{\sqrt{qI_sT}} = \sqrt{\frac{I_sT}{q}} = 10 \Rightarrow T = \frac{100q}{I_s} = 160\ \mu\text{s}.$$

Note that this is the optimum filter for this case.

2.3

For the case of non-stationary white noise with PSD $\lambda(t)$, we have

$$V_o(t) = \int I_s(\tau)w(t,\tau)d\tau \qquad \overline{V_o^2(t)} = \int \lambda(\tau)w^2(t,\tau)d\tau,$$

where the input current noise bilateral PSD is $\lambda(t) = qI_s(t) + \dfrac{S_{OA}}{2}$, leading to the optimum choice

$$w(t,\tau) \propto \frac{I_s(\tau)}{\lambda(\tau)} = \frac{I_0e^{-t/T}}{qI_0e^{-t/T} + S_{OA}/2} \propto \frac{e^{-t/T}}{e^{-t/T} + S_{OA}/2qI_0} = \frac{e^{-t/T}}{e^{-t/T} + a},$$

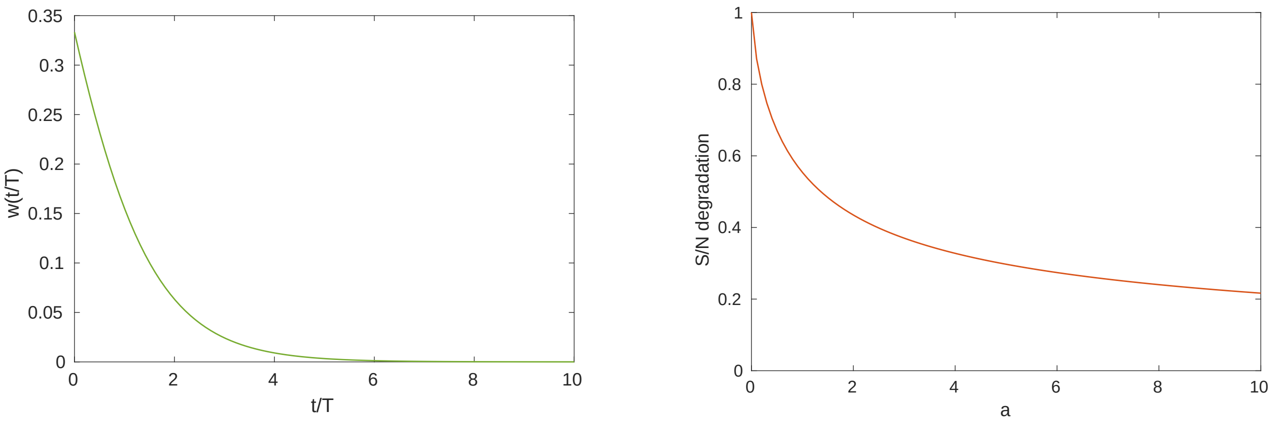


Figure 2: Left = Optimum weighting function. Right = S/N degradation due to the OA current noise.

where $a = S_{OA}/2qI_0 = 2.8$. The weighting function is reported in Fig. 2 (left).

2.4

The output signal and noise are

$$V_o = K\int_0^\infty I_0 e^{-t/T}\frac{e^{-t/T}}{e^{-t/T}+a}dt = KTI_0\int_0^\infty \frac{e^{-2x}}{e^{-x}+a}dx = KTI_0\left(a\ln a - a\ln(1+a)+1\right)$$

$$\overline{V_o^2(t)} = K^2T\int_0^\infty \left(qI_0e^{-x}+\frac{S_{OA}}{2}\right)\left(\frac{e^{-x}}{e^{-x}+a}\right)^2 dx = qI_0K^2T\int_0^\infty \frac{e^{-2x}}{e^{-x}+a}dx =$$
$$= qI_0K^2T\left(a\ln a - a\ln(1+a)+1\right),$$

from which

$$\frac{S}{N} = \sqrt{\frac{I_0T}{q}}\sqrt{a\ln a - a\ln(1+a)+1}.$$

For $a = 2.8$ the second term is about 0.38. The degradation factor due to the OA noise is shown in Fig. 2 (right) as a function of a.