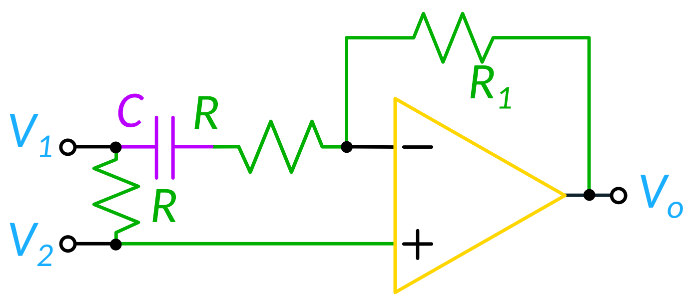
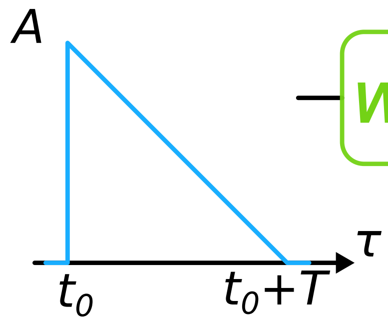
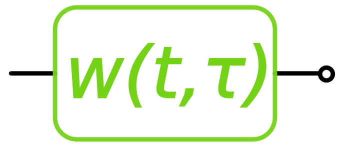
Electronics – February 18 2022

Prof. A. Spinelli







For a correct evaluation, please write your answers in a readable way; thank you!

Problem 1

The scheme in the left figure is a differential amplifier. Parameter values are $R = 10$ kΩ, $R_1 = 100$ kΩ, $C = 15$ nF. The OA has low-frequency gain of 100 dB and $GBWP = 2$ MHz.

1. Find the common- and differential-mode gain in the ideal case.
2. Draw the Bode plot of the loop gain and compute the phase margin.
3. Compute the output rms noise voltage considering the equivalent noise sources of the amplifier $\sqrt{S_V} = 15\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$, $\sqrt{S_I} = 50\ \mathrm{pA}/\sqrt{\mathrm{Hz}}$.
4. Consider a tolerance of 5% in resistor values and evaluate the effect on the amplifier CMRR.

Problem 2

An acquisition system features the triangular weighting function depicted in the figure on the right.

1. The filter is employed with a rectangular signal (starting at t_0) having amplitude V_i and width $T_i < T$, affected by a white noise with bilateral PSD λ. Evaluate the output signal and noise.
2. What is the value of T_i that gives the best S/N? Why?
3. The signal is affected by a noise having nearly rectangular autocorrelation, and correlation time $T_n < T$. Evaluate the mean square value of the output noise.
4. Design a scheme that allows to achieve such a function and set its parameters so that the integral of the weighting function is equal to 10 (hint: cascade two TV filters).

Do a good job!

Results will be posted by February 23rd

Mark registration: Friday, February 25th

Solution

Problem 1

1.1

We first apply a common-mode voltage V_c at both inputs. It is easy to see that no current flows in the feedback network, so that we have

$$V_o = V_c \Rightarrow A_c = 1.$$

For the differential case, we set $V_2 = -V_1 = V_d/2$. Since the input pins of the OA are kept at the same bias, the current flowing through the RC series is

$$I = \frac{V_d}{R + 1/sC} = V_d \frac{sC}{1 + sCR},$$

obviously flowing into resistor R_1. The output becomes then

$$V_o = V_2 + IR_1 = V_d \left(\frac{1}{2} + \frac{sCR_1}{1 + sCR} \right) = \frac{V_d}{2} \frac{1 + sC(R + 2R_1)}{1 + sCR} \Rightarrow A_d = \frac{1 + sC(R + 2R_1)}{2(1 + sCR)}$$

1.2

We ground both inputs and open the loop at the OA output, applying a test signal V_s. We easily get

$$V^- = V_s \frac{R + 1/sC}{R + R_1 + 1/sC} = V_s \frac{1 + sCR}{1 + sC(R + R_1)} \Rightarrow G_{loop} = -A(s) \frac{1 + sCR}{1 + sC(R + R_1)}.$$

The pole and zero introduced by C are $f_p = 1/(2\pi(R + R_1)C) \approx 96$ Hz, $f_z = 1/(2\pi RC) \approx 1$ kHz. Labelling $f_0 = 20$ Hz the OA pole, we also have

$$G_{loop}(0) f_0 = G_{loop}(f_p) f_p \Rightarrow G_{loop}(f_p) = G_{loop}(0) \frac{f_0}{f_p} = 2.08 \times 10^4 = 86.4 \text{ dB}$$

$$G_{loop}(f_p) f_p^2 = G_{loop}(f_z) f_z^2 \Rightarrow G_{loop}(f_z) = G_{loop}(f_p) \frac{f_p^2}{f_z^2} = 192 = 46 \text{ dB}.$$

the Bode plot is shown in Fig. 1 (left). The phase margin is clearly 90° and the zero-dB frequency is $f_{0dB} = G_{loop}(f_z) f_z \approx 192$ kHz. Note that this frequency is equal to $GBWP\ R/(R + R_1)$, as capacitor C behaves like a short-circuit at high frequencies.

1.3

At first, we can note that resistor R placed between the inputs serves no purpose at all, neither from the signal nor from the noise viewpoint, and can be removed from the scheme (it obviously affects the input impedance, but we are not discussing this property here).
Now, the voltage noise of the OA follows the non-inverting gain:

$$V_o = V_n \frac{R_1 + R + 1/sC}{R + 1/sC} = V_n \frac{1 + sC(R + R_1)}{1 + sCR},$$

where $f_z = 96$ Hz, $f_p = 1$ kHz. Its high-frequency gain is limited by the pole at f_{0dB}, leading to

$$\overline{V_0^2} = S_V \frac{\pi}{2} \left(f_z + \left(\frac{R_1 + R}{R} \right)^2 (f_{0dB} - f_p) \right) \approx S_V \frac{\pi}{2} \left(\frac{R_1 + R}{R} \right)^2 f_{0dB} \approx (90\ \mu\text{V})^2.$$

The current noise at the non-inverting input gives no contribution, while the other one flows in R_1, giving

$$\overline{V_0^2} = S_I R_1^2 \frac{\pi}{2} f_{0dB} \approx (2.7\text{ mV})^2,$$

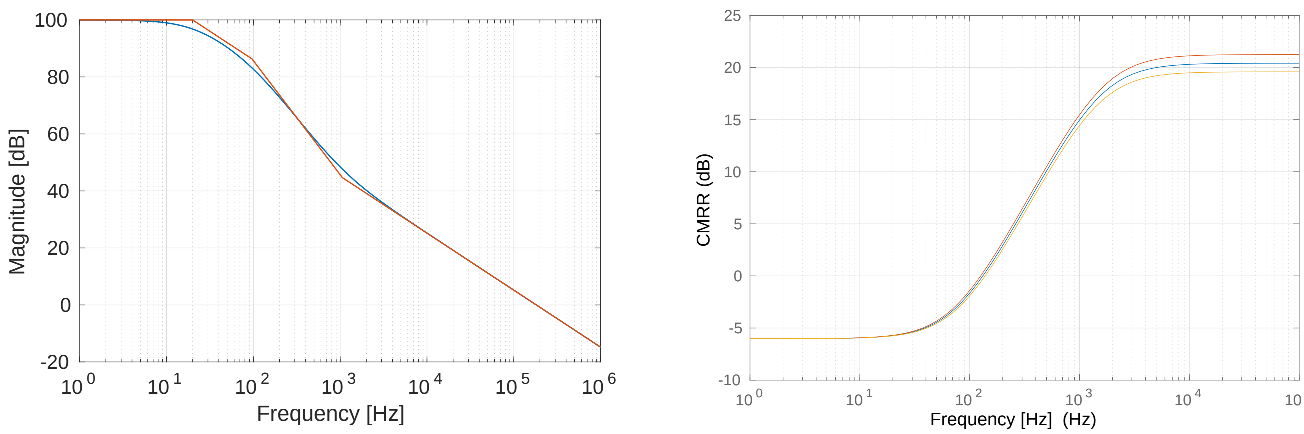


Figure 1: Left = Bode plot of the loop gain (blue) and its asymptotic approximation (red). Right = $CMRR$ as a function of frequency for nominal value of resistors (blue) and the two extreme cases.

which is the dominant term. If resistor values cannot be lowered, an OA with better current noise performance (say, CMOS-based) should be considered.
For the sake of completeness, we discuss resistor noise: the current noise of R_1 (PSD of 0.4 pA/$\sqrt{\text{Hz}}$) has the same transfer as S_I, but is totally negligible (rms output contribution of 22 μV). The voltage noise of resistor R (PSD of 12.8 nV/$\sqrt{\text{Hz}}$) goes with the non-inverting gain:

$$V_o = V_n \frac{R_1}{R + 1/sC} = V_n \frac{sCR_1}{1 + sCR} \Rightarrow \overline{V_0^2} = S_R \left(\frac{R_1}{R}\right)^2 \frac{\pi}{2}(f_{0dB} - f_p) \approx (70\ \mu\text{V})^2.$$

1.4

In common-mode operation, no current flows in the circuit, so resistor values and tolerances have no effect on the (unity) gain. We have then $CMRR = A_d$. Setting $R \Rightarrow R(1 \pm \epsilon)$, $R_1 \Rightarrow R_1(1 \pm \epsilon_1)$ we have (see #1.1):

$$CMRR = A_d = \frac{1 + sC(R(1 \pm \epsilon) + 2R_1(1 \pm \epsilon_1)}{2(1 + sCR(1 \pm \epsilon))}.$$

Picking opposite changes, the high-frequency value changes from 10.5 to 11.55, i.e., by 10% (not unsurprisingly). Similar changes take place in the pole and zero positions. The Bode plot of $CMRR$ for the nominal and the two exteme cases are reported as a reference in Fig. 1 (right). Note that the impact is small, as expected from this kind of stages.

Problem 2

2.1

We set for simplicity $t_0 = 0$, so that the output signal is proportional to the area of the WF between 0 and T_i, i.e.:

$$V_o = V_i \int_0^{T_i} w(T, \tau) d\tau = A \frac{T_i}{2} \frac{2T - T_i}{T} V_i,$$

while the noise is

$$\overline{n_o^2} = \lambda \int w^2(t, \tau) = A^2 \int_0^T \left(\frac{T - \tau}{T}\right)^2 d\tau = \lambda A^2 \frac{T}{3}.$$

2.2

The noise is independent of T_i, so the best solution is to pick as much signal as possible, setting $T_i = T$! Note that this case is different from the apparently similar one of triangular signal and rectangular WF discussed in the class!

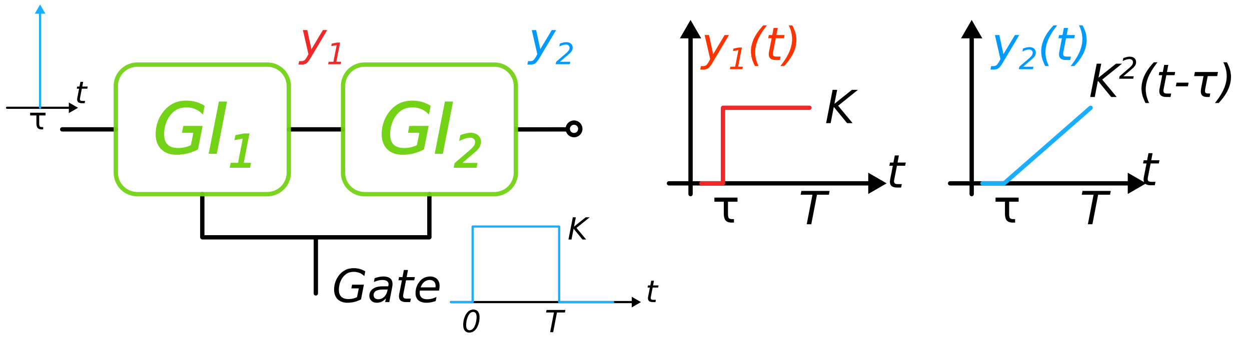


Figure 2: Left = Scheme Bode plot of the loop gain (blue) and its asymptotic approximation (red). Right = Noise transfers (not squared) for OA1 (red) and OA2 (green).

2.3

we should evaluate the well-known integral

$$\overline{n_y^2} = \int R_{xx}(\gamma) k_{w_{tt}}(\gamma) d\gamma,$$

where symbols have the usual meaning. To compute the time correlation of the WF, it is easier to consider the mirrored signal, i.e., a positive ramp from 0 to T. Shifting its replica to the right by γ we obtain

$$k_{w_{tt}}(\gamma) = \frac{A^2}{T^2} \int_{\gamma}^{T} t(t-\gamma) dt = \frac{A^2}{T^2} \left(\frac{T^3}{3} + \frac{\gamma^3}{6} - \frac{\gamma T^2}{2} \right),$$

leading to

$$\overline{n_y^2} = 2\overline{n_x^2} \int_0^{T_n} k_{w_{tt}}(\gamma) d\gamma = 2\overline{n_x^2} A^2 \left(\frac{T_n T}{3} + \frac{T_n^4}{18T^2} - \frac{T_n^2}{4} \right).$$

Note that for $T_n \ll T$ the linear term in T_n dominates and we recover the result of #2.1, with $\lambda = 2\overline{n_x^2} T_n$.

2.4

If we bear in mind that a ramp signal can be obtained as the integral of a constant one, this in turn being the WF of a gated integrator stage, we can consider placing two GIs in series, working with the same gate signal, as displayed in Fig. 2 (left). If we set the integration time to be $0 - T$ for simplicity, and we consider an input delta function applied at time τ within such an interval, the outputs of the two GIs (still in the $0 - T$ interval) are (Fig. 2, right):

$$y_1(t) = K u(t-\tau)$$
$$y_2(t) = K^2 (t-\tau) u(t-\tau),$$

where K is the gain of each GI. When the gate is closed, the output is $y_2(T) = K^2(T-\tau)$, which, seen as a function of τ, reproduces the desired WF. The requirement on the gain becomes then

$$\int w(t,\tau) d\tau = K^2 \frac{T^2}{2} = 10 \Rightarrow K = \frac{\sqrt{20}}{T}$$

and the amplitude of the WF is $A = K^2 T = 20/T$.