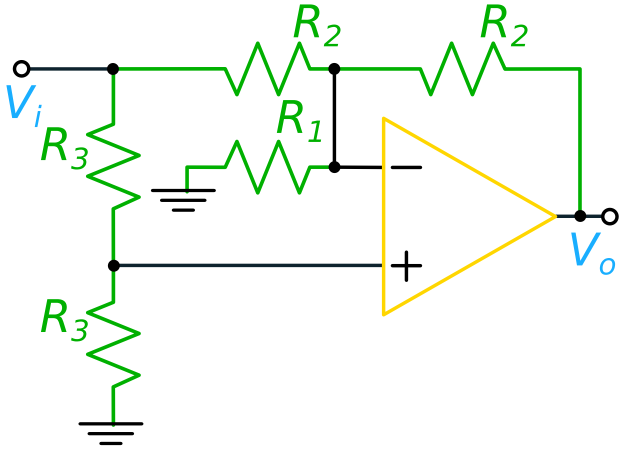
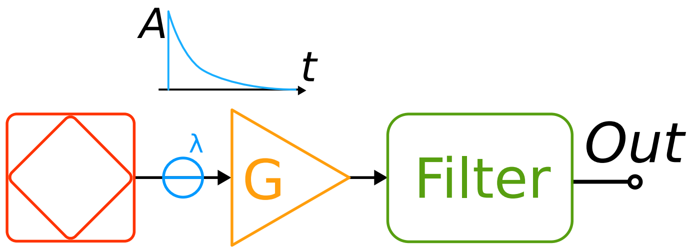
Electronics – June 23 2022 Prof. A. Spinelli





For a correct evaluation, please write your answers in a readable way; thank you!

Problem 1

The scheme in the left figure is a non-inverting amplifier. The OA has low-frequency gain of 100 dB and $GBWP = 1$ MHz. Consider $R_2 = 200$ kΩ and $R_3 = 2$ kΩ.

1. Find the expression of the ideal gain and the range of variation of R_1 in order to change the gain in the range $1 - 100$.
2. Consider the OA input capacitance C_i and find its maximum value that can grant a phase margin of 45° for the whole range of gain.
3. Compute the output rms noise voltage considering the equivalent noise sources of the amplifier $\sqrt{S_V} = 10$ nV/$\sqrt{\text{Hz}}$, $\sqrt{S_I} = 10$ pA/$\sqrt{\text{Hz}}$ and resistors ($4k_BT \approx 1.646 \times 10^{-20}$ J). Consider unity gain for simplicity.
4. We want to double the ideal gain. Propose a modification of the circuit (do not add resistors in series/parallel to R_1 or R_2 nor add an additional gain stage). An approximated solution is also fine (if discussed properly).

Problem 2

A sensor outputs exponential pulses Ae^{-t/T_F}, plagued by a white noise with bilateral PSD λ, as shown in the figure on the right. After a large-bandwidth preamplifier, an LTI optimum filter is applied, that must process the pulse within a time T.

1. Draw the optimum weighting function and compute S/N.
2. Compute the signal at the output of the optimum filter.
3. We want the output signal to go to zero as soon as possible after $t = T$ (without reducing S/N), so that multiple pulses may be processed without pile-up issuess. Propose a new weighting function (approximations are also fine).
4. The white noise is non-stationary and with PSD proportional to the signal. Find the new optimum S/N.

Do a good job!

Results will be posted by June 29th Mark registration: Friday, July 1st

Solution

Problem 1

1.1

A voltage equal to $V_i/2$ is obviously present at the OA inputs. The KCL at the inverting input node leads to:

$$\frac{V_i}{2R_1} = \frac{V_i}{2R_2} + \frac{V_o - V_i/2}{R_2} \Rightarrow V_o = V_i\frac{R_2}{2R_1}.$$

The circuits is a non-inverting amplifier with a gain expression similar to what is obtained from an inverting scheme (i.e., without the "1+" term). The range of values for R_1 is then $1 - 100$ kΩ. R_3 is a biasing resistor and does not affect the gain.

1.2

The input capacitor adds a pole to the loop gain, whose time constant is $C_i R_{eq}$. The resistance seen by the capacitor is

$$R_{eq} = \frac{R_3}{2} + R_1 \parallel \frac{R_2}{2} = \frac{R_3}{2} + \frac{R_1 R_2}{2R_1 + R_2} = \frac{R_3}{2} + \frac{R_2}{2(1+G)},$$

where $G = R_2/2R_1$ is the ideal gain. The second term ranges from 2 (at $G = 100$) to 51 kΩ (at $G = 1$). The loop gain without C_i is instead:

$$G_{loop} = -A(s)\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_2} = -A(s)\frac{1}{2(1+G)} \Rightarrow f_{0dB} = \frac{GBWP}{2(1+G)}.$$

To achieve a phase margin higher than 45° we then set

$$\frac{1}{2\pi C_i R_{eq}} \geq f_{0dB} \Rightarrow C_i \leq \frac{1+G}{\pi R_{eq} GBWP} \leq \frac{2}{\pi 5.1 \times 10^{10}} \approx 12 \text{ pF},$$

where the last inequality stems from the fact that the lowest value is achieved for $G = 1$ and $R_{eq} = 51$ kΩ. Note that a higher values of R_3 could be desired in some cases. Then, a bypass capacitor should be added.

1.3

The circuit for noise calculations is reported in Fig. 1 (left). From it, we can easily derive:

$$S_{V_o} = \left(S_V + S_I^+\frac{R_3^2}{4} + 2k_BTR_3\right)\left(1 + \frac{R_2}{R_1 \parallel R_2}\right)^2 + \left(S_I^- + \frac{4k_BT}{R_2} + \frac{4k_BT}{R_1 \parallel R_2}\right)R_2^2 \approx 4.18 \times 10^{-12}\frac{\text{V}^2}{\text{Hz}},$$

dominated by the $S_I R_2^2$ term. Since in this case we have $f_{0dB} = GBWP/4 = 250$ kHz (see #1.2), we get

$$\sqrt{\overline{V_0^2}} = \sqrt{S_{V_o}\frac{\pi}{2}f_{0dB}} \approx 1.3 \text{ mV}.$$

1.4

If we look at the solution in #1.1, we can see that replacing V_o with $V_o/2$ in the current balance equation would do the job:

$$\frac{V_i}{2R_1} = \frac{V_i}{2R_2} + \frac{V_o/2 - V_i/2}{R_2} \Rightarrow V_o = V_i\frac{R_2}{R_1}.$$

Another way of reaching this conclusion is the following: if we want to multiply the gain by a factor of 2, we can simply *divide* by the same factor the feedback transfer F, as $G_{id} = 1/F$.
In any case, a simple voltage divider is all we need. The scheme is reported in Fig. 1 (right). However, please note that this is an *approximate* solution, working when $R_4 \ll R_2$, so that the voltage divider midpoint is close to $V_o/2$. Working out the exact transfer is left to the reader; the result is

$$\frac{V_o}{V_i} = \frac{R_2}{R_1} + \frac{R_4(R_2 - R_1)}{2R_1R_2}.$$

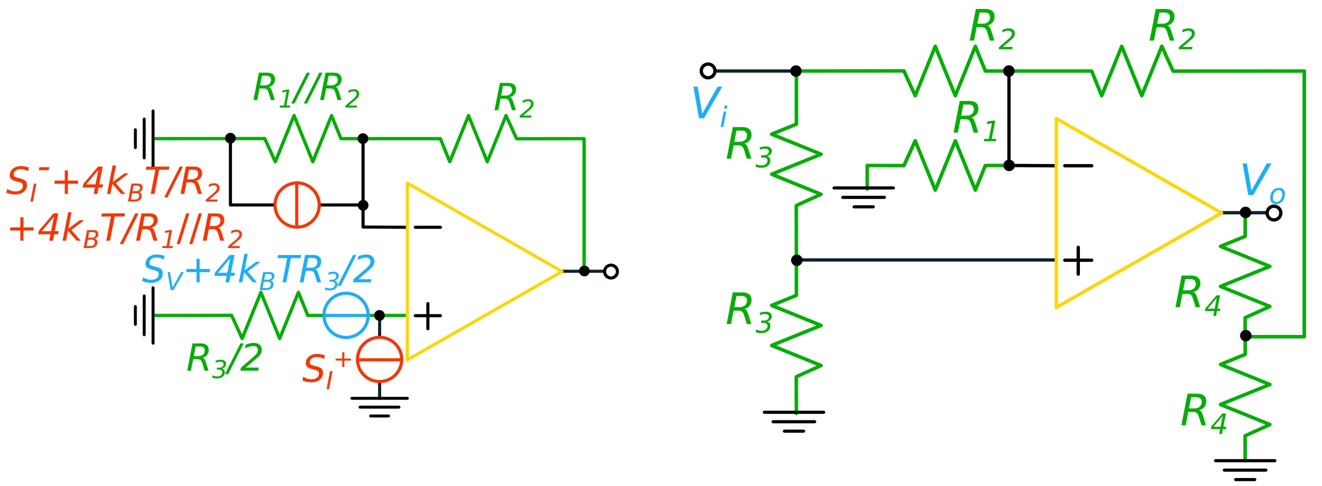


Figure 1: Left = Scheme for noise calculation. Right = Possible scheme for achieving double gain (with $R_4 \ll R_2$).

For example, picking $R_1 = 2$ kΩ ($G = 100$) results in $G = 100.495$ if $R_4 = 20$ kΩ.
If the values of R_1 and R_2 are fixed, an exact solution can be found by tailoring the resistors in the divider to achieve the desired attenuation.

Problem 2

2.1

Since only the signal from $t = 0$ to $t = T$ can be used to provide the output, the optimum weighting function matches the signal only in this time interval. Hence, it is an exponential signal truncated after an interval T, as shown in Fig. 2 (left).
The maximum output is obviously achieved for $t = T$, where we have

$$y(T) = \int x(\tau)w(T,\tau)d\tau = AK\int_0^T x^2(\tau)d\tau = AK\frac{T_F}{2}\left(1 - e^{-2T/T_F}\right),$$

while the output mean square noise is

$$\overline{n_y^2} = \lambda K^2 \int_0^T x^2(t)dt = \lambda K^2 \int_0^T e^{-2t/T_F}dt = \lambda K^2 \frac{T_F}{2}\left(1 - e^{-2T/T_F}\right).$$

We have therefore

$$\left(\frac{S}{N}\right)_T = A\sqrt{\frac{T_F}{2\lambda}}\sqrt{1 - e^{-2T/T_F}} = \left(\frac{S}{N}\right)_{opt}\sqrt{1 - e^{-2T/T_F}}.$$

Note that the prefactor is the optimum value of S/N, achieved when $T \to \infty$. For example, with $T = 0.83T_F$ we already get 90% of the optimum value.

2.2

The outut signal is

$$y(t) = A\int x(\tau)w(t,\tau)d\tau.$$

For $t \le T$ this results in

$$y(t) = AK\int_0^t e^{-\tau/T_F}e^{-(\tau - t + T)/T_F}d\tau = AK\frac{T_F}{2}e^{(t-T)/T_F}\left(1 - e^{-2t/T_F}\right) = AKT_F\, e^{-T/T_F}\sinh\left(\frac{t}{T_F}\right).$$

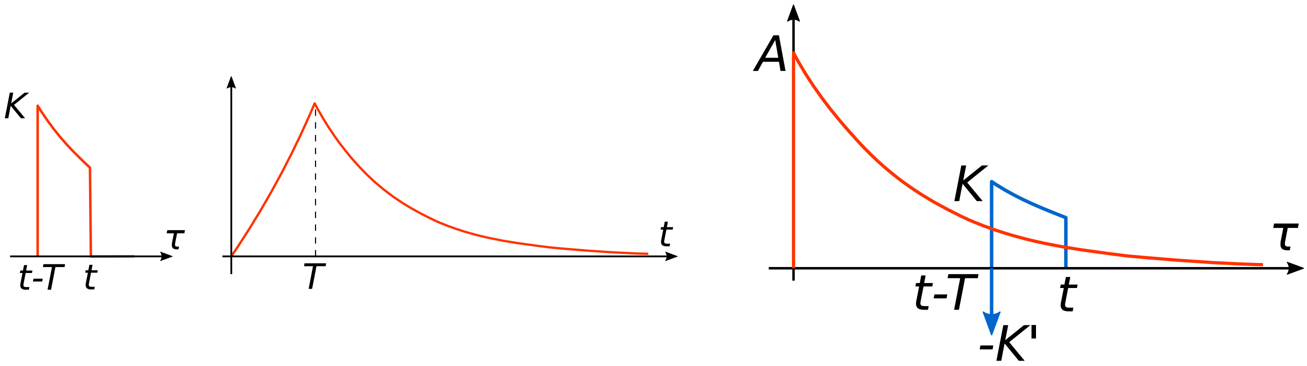


Figure 2: Left = Weighting function and output signal of the optimum filter. Right = Modified weighting function (blue) and input signal (red).

For $t > T$ we obviously have an exponential decay of the signal:

$$y(t) = y(T)e^{-(t-T)/T_F} = AKT_F \sinh\left(\frac{T}{T_F}\right) e^{-t/T_F}.$$

The output signal is also shown in Fig. 2 (left).

2.3

To reduce the output signal, we need to add a negative part to the weighting function. This part must come after the exponential one, so not to affect the optimum S/N. The simpler solution is placing a negative delta function with area $-K'$, as depicted in Fig. 2 (right). For $t > T$ we can exploit the previous results and write

$$y(t) = y(T)e^{-(t-T)/T_F} - K'Ae^{-(t-T)/T_F}.$$

Setting $y(t) = 0$ we obtain

$$K' = \frac{y(T)}{A} = K\frac{T_F}{2}\left(1 - e^{-2T/T_F}\right).$$

2.4

We know that in this case the optimum weighting function is constant: $w(t,\tau) = K$ in the interval $0 - T$. If we label $\lambda(t) = \lambda_0 x(t)$ we have:

$$y(T) = K\int_0^T Ax(\tau)d\tau = KAT_F\left(1 - e^{-T/T_F}\right)$$

while the output mean square value of the noise is

$$\overline{n_y^2}(T) = \int_0^T \lambda(\tau)w^2(T,\tau)d\tau = \lambda_0 K^2 T_F\left(1 - e^{-T/T_F}\right),$$

from which

$$\left(\frac{S}{N}\right)_T = A\sqrt{\frac{T_F}{\lambda_0}}\sqrt{1 - e^{-T/T_F}}.$$