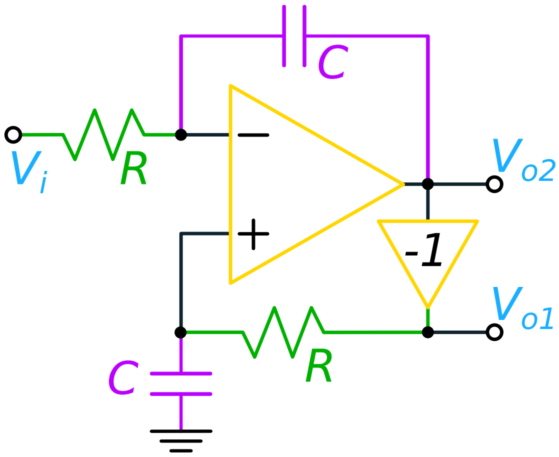
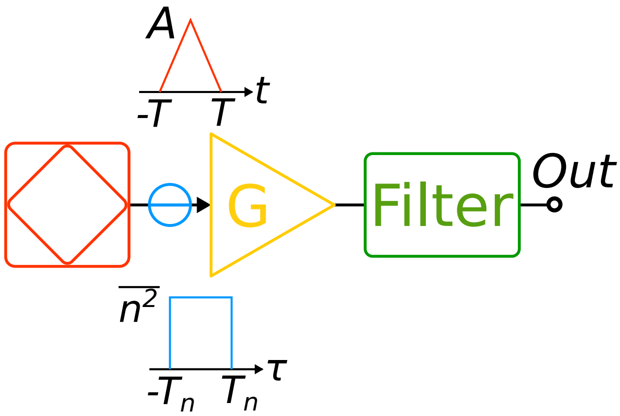
Electronics – September 6 2023 Prof. A. Spinelli

For a correct evaluation, please write your answers in a readable way; thank you





Solving 6 points correctly gives you 30/30

Problem 1

The scheme in the left figure is a single-to-differential converter. The OA has $A_0 = 120$ dB and $GBWP =$ 10 MHz. Parameters are $R = 200$ kΩ, $C = 10$ pF. In the following, consider the differential output $V_0 = V_{o1} - V_{o2}$.

1. Compute the gain.
2. Compute the loop gain and discuss the stability.
3. Compute the output rms noise voltage considering the equivalent noise sources of the OA and the inverting stage $\sqrt{S_V} = 7$ nV/$\sqrt{\text{Hz}}$.
4. Consider the OA input capacitance C_i and discuss its impact on the phase margin (hint: use the Thevenin theorem to avoid lenghty calculations).

Problem 2

A sensor outputs triangular pulses of amplitude $A \approx 1$ mV and duration $T = 100$ ns, on top of a noise having nearly rectangular autocorrelation with $T_n \approx T/2$ and mean square value $\overline{n^2} \approx 10^{-7}$ V^2. The preamplifier has a bandwidth $BW = 10$ MHz. Neglect for simplicity the amplifier effect on the signal and noise when possible.

1. A single sample is taken. Evaluate S/N.
2. A gated integrator is now used. Compute S/N and discuss the difference with the previous result.
3. Consider a flicker noise K/f with $K = 10^{-8}$ V^2. To reduce its effect, three samples are taken (remove the GI). Find suitable values for times and weights and compute S/N (remember that $\cos x = \sum_k (-x^2)^k/(2k)!$). Comment on the accuracy of the result.
4. To reduce the flicker noise, we now place an HPF after the amplifier. Find suitable values for T_F and evaluate the signal undershoot.

Allowed time: 2 hours 45 minutes – Do a good job!

Results will be posted by September 8th Mark registration: Wednesday, September 20th

Solution

Problem 1

1.1

The voltage at the NI input of the OA is

$$V^+ = -\frac{V_{o2}}{1+sCR},$$

equal to the I input bias:

$$V^+ = V^- = \frac{V_i}{1+sCR} + V_{o2}\frac{sCR}{1+sCR} \Rightarrow \frac{V_{o2}}{V_i} = -\frac{1}{1+sCR},$$

from which

$$\frac{V_o}{V_i} = \frac{V_{o1}-V_{o2}}{V_i} = \frac{2}{1+sCR}.$$

The pole frequency is $f_p = 1/(2\pi RC) \approx 80$ kHz.

1.2

We cut both loops by applying a test voltage V_T at the OA output, and easily obtain:

$$V^+ = -V_T\frac{1}{1+sCR} \qquad V^- = V_T\frac{sCR}{1+sCR},$$

from which

$$G_{loop} = A(s)\frac{V^+ - V^-}{V_T} = -A(s),$$

obviously stable with $f_{0dB} = GBWP = 10$ MHz.

1.3

We place the OA noise voltage source V_{n1} at the NI input and write the circuit equations:

$$\begin{aligned} V^+ &= V_{o1}\frac{1}{1+sCR} + V_{n1} = V^- = V_{o2}\frac{sCR}{1+sCR} \\ V_{o1} &= -V_{o2} - V_{n2}, \end{aligned}$$

where V_{n2} is the inverting stage voltage noise. We obtain

$$V_o = V_{o1} - V_{o2} = -2V_{n1} + V_{n2}\frac{1-sCR}{1+sCR},$$

$$S_{V_o} = 4S_{V1} + S_{V2}\left|\frac{1-sCR}{1+sCR}\right|^2 = 4S_{V1} + S_{V2}.$$

The mean square output noise is then

$$\overline{V_o^2} = 5S_V\frac{\pi}{2}f_{0dB} \approx (62\ \mu\text{V})^2.$$

1.4

To evaluate G_{loop}, we need to find V_d, the differential OA input voltage. However, solving the network for V^+ and V^- results in long and complicated expressions, that eventually simplify. This becomes much easier if V_d is used as one network variable, but an even better simplification is achieved by applying the Thevenin theorem at the top and bottom of C_i. This leads to the scheme in Fig. 1 (right), where $Z = R \parallel Z_C = R/(1+sCR)$ and

$$V_L = V_T\frac{sCR}{1+sCR} \qquad V_R = -V_T\frac{1}{1+sCR},$$

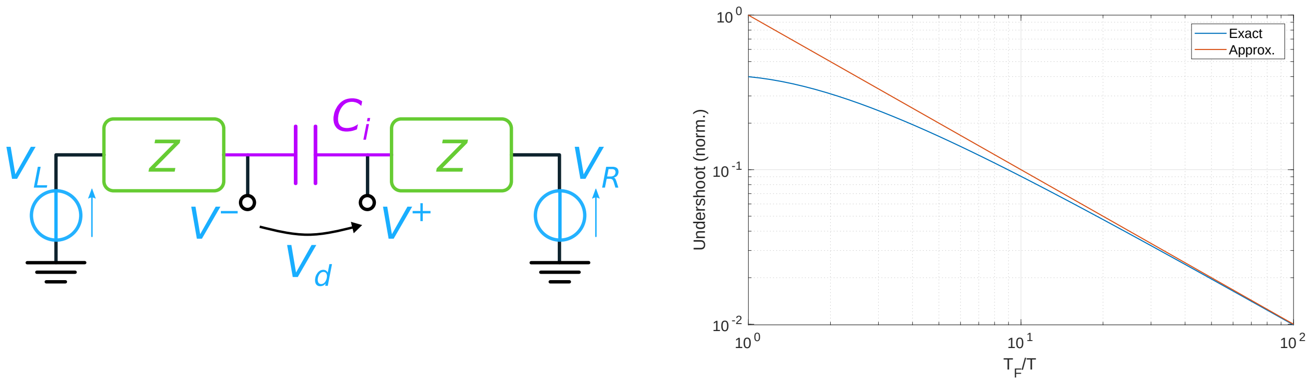


Figure 1: Left = Equivalent scheme for the calculation of G_{loop}. Right = HPF undershoot as a function of T_F/T.

leading to

$$V_d = (V_R - V_L)\frac{1/sC_i}{2Z + 1/sC_i} = -V_T\frac{1+sCR}{1+s(C+2C_i)R} \Rightarrow G_{loop} = -A(s)\frac{1+sCR}{1+s(C+2C_i)R}.$$

It is clear that the circuit is quite immune to the effect of C_i, and that values much larger than C (quite unlikely) are needed to degrade the phase margin.

Problem 2

2.1

If we neglect the effect of the amplifier, whose time constant is $10^{-6}/(20\pi) \approx 16$ ns, we find:

$$\left.\frac{S}{N}\right|_{sample} = \frac{A}{\sqrt{\overline{n^2}}} = 3.16.$$

2.2

We assume the signal peaks at $t = 0$ and consider a symmetric integration window from $-T_G$ to T_G. The output signal is then

$$V_o = 2AK\int_0^{T_G} V_i(t)dt = AKT_G\left(2 - \frac{T_G}{T}\right).$$

For the noise calculation, we need the autocorrelation of the WF, that is a triangular function ranging from $-2T_G$ to $2T_G$ and with amplitude $2T_GK^2$. We then get (for $2T_G \geq T_n$)

$$\overline{n^2_{GI}} = \int R_{xx}(\tau)k_{ww}(t,\tau)d\tau = 2K^2\,\overline{n^2}\,T_GT_n\left(2 - \frac{T_n}{2T_G}\right),$$

and $4K^2T_G^2\,\overline{n^2}$ for $2T_G \leq T_n$. If we take $T_G = T_n = T/2$ we get

$$\left.\frac{S}{N}\right|_{GI} = \frac{A}{\sqrt{\overline{n^2}}}\frac{\frac{3}{2}T_G}{\sqrt{3T_G^2}} = \frac{A}{\sqrt{\overline{n^2}}}\frac{\sqrt{3}}{2} \approx 0.87\frac{A}{\sqrt{\overline{n^2}}}.$$

The result is lower than the previous one, which means that the GI is not effective in reducing the noise, as the integration time is comparable with the noise correlation time.

2.3

Given the shape of the signal, we can take one sample with unit weight at $t = 0$ and two samples with weight $-1/2$ at $\pm T$. The output signal is obviously A, while the weighting function and its time correlation are:

$$w(t,\tau) = \delta(\tau) - \frac{1}{2}\delta(\tau \pm T) \Rightarrow k_{ww}(t,\gamma) = \frac{3}{2}\delta(\gamma) - \delta(\gamma \pm T) + \frac{1}{4}\delta(\gamma \pm 2T),$$

which translates to

$$|W(t,f)|^2 = \frac{3}{2} - \left(e^{j\omega T} + e^{-j\omega T}\right) + \frac{1}{4}\left(e^{j\omega 2T} + e^{-j\omega 2T}\right) = \frac{3}{2} - 2\cos(\omega T) + \frac{1}{2}\cos(2\omega T).$$

To evaluate the output noise, we expand in series the cosine terms $\cos x \approx 1 - x^2/2 + x^4/24$ and obtain

$$|W(t,f)|^2 \approx \frac{3}{2} - 2\left(1 - \frac{(\omega T)^2}{2} + \frac{(\omega T)^4}{24}\right) + \frac{1}{2}\left(1 - \frac{(2\omega T)^2}{2} + \frac{(2\omega T)^4}{24}\right) = \frac{(\omega T)^4}{4},$$

leading to

$$\overline{n_{FN}^2} = \int_0^{f_H} \frac{K}{f}|W(t,f)|^2 df \approx (\pi T)^4 K \int_0^{f_H} 4f^3 df = K(\pi f_H T)^4.$$

taking $f_H = 10$ MHz (the amplifier BW), we get $f_H T = 1$ and $\overline{n_{FN}^2} \approx 97.4\ K \approx 10^{-7}$ V^2, which is to be compared with the original noise contribution equal to $(3/2)\overline{n^2} = 1.5 \times 10^{-7}$ V^2.

The first comment is that the filter eliminates all second-order contributions of the series expansion, so that a higher order expansion is required to get a non-zero result. However, in this case, the FN result is hugely overestimated: at $f_H = 10$ MHz the cosine argument is $\omega_H T = 2\pi = 6.28$, meaning that the Taylor approximation is awful: the exact result is in fact $\overline{n_{FN}^2} \approx 3.3\ K = 3.3 \times 10^{-8}$ V^2, meaning that the filter is effective in reducing LF noise.

2.4

If the time constant T_F is much longer than T, the pulse is barely affected by the HPF, and at the output we have

$$\left(\frac{S}{N}\right) = \frac{A}{\sqrt{K \ln\left(\frac{T_F}{T_H}\right)}} = 4.55,$$

where $T_H = 1/(2\pi f_H) \approx 16$ ns is the time constant of the amplifier pole and we have taken $T_F = 20T = 2\ \mu$s. The output signal undershoot is the response of the filter at $t = T$, and can be exactly calculated from the WF:

$$y(T) = \int_{-\infty}^{T} x(\tau)h(T-\tau)d\tau,$$

but the calculation is boring. An approximated expression can be derived as follows: if the pulse is fast, it will not be affected much by the filter. So, during the pulse, a current equl to $x(t)/R$ flows into the capacitor, that will be charged at a voltage:

$$V_C(t) = \frac{Q}{C} = \frac{1}{RC}\int^{t} x(\tau)d\tau.$$

At $t = T$ we have $V_C(T) = AT/T_F$, which is the amount of undershoot. A comparison between this result and the correct one is reported in Fig. 1 (right).