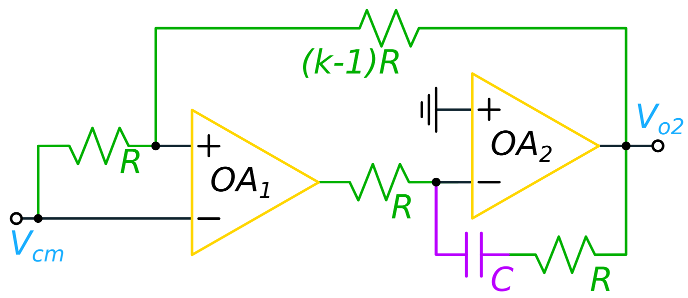
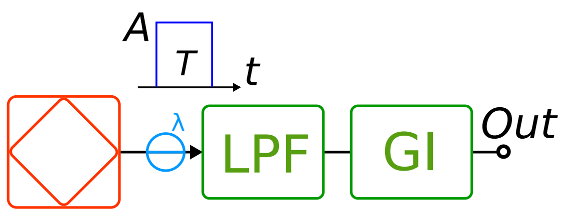
Electronics – July 19 2023 Prof. A. Spinelli

For a correct evaluation, please write your answers in a readable way; thank you





Solving 6 points correctly gives you 30/30

Problem 1

The scheme in the left figure is used to measure the CMRR of OA1 in DC. Parameters are $R = 1$ kΩ, $k = 500$, $C = 1$ μF and $V_{cm} = 5$ V. OAs have low-frequency gain of 114 dB and $GBWP = 10$ MHz. The output is defined as $V_o = V_{o2} - V_{cm}$.

1. Find the gain of the stage under ideal conditions.
2. Compute the loop gain at DC. Extend then the calculation to the general case.
3. Compute the output rms noise voltage considering the equivalent noise sources of the OAs $\sqrt{S_V} = 15$ nV/$\sqrt{\text{Hz}}$, $\sqrt{S_I} = 25$ pA/$\sqrt{\text{Hz}}$ and resistors ($4k_BT \approx 1.646 \times 10^{-20}$ J).
4. Consider the finite CMRR of the OAs and evaluate V_o at DC.

Problem 2

A sensor outputs rectangular signals with amplitude $A \approx 1$ mV and duration $T = 500$ ns. A WN with bilater PSD $\lambda = 10^{-13}$ V^2/Hz is also present at the sensor output. The signal is sent to the cascade of an LPF and a GI.

1. Pick a (reasonable) value for T_F and evaluate S/N at the LPF output.
2. Consider the filter made up of LPF + GI. Compute the output signal and the weighting function.
3. Evaluate the output noise and the resulting S/N. What is the best value of T_F?
4. The signal has now a right triangular shape, decaying from A to zero in a time T. Suggest the best choice for T_F and evaluate the resulting S/N (hint: find a smart solution to avoid lengthy calculations).

Allowed time: 2 hours 45 minutes – Do a good job!

Results will be posted by July 24th

Mark registration: Thursday, July 27th

Solution

Problem 1

1.1

Under ideal conditions, V_{cm} is applied to both input of OA1, meaning that no current flows in the feedback resistors and $V_{o2} = V_{cm}$. The output voltage $V_{o2} - V_{cm}$ is then zero.

1.2

We open the external loop, replacing the OA2 block with its closed-loop transfer $G_2(s)$, obtaining:

$$G_{loop} = \frac{1}{k} A(s) G_2(s).$$

The ideal transfer of the OA2 block is $-(R + 1/sC)/R = -(1 + sCR)/sCR$, i.e., an integrator at low frequencies and unity gain for frequencies larger than $1/(2\pi CR) \approx 160$ Hz. To evaluate the poles added by OA2, we can consider the high-and low-frequency limits. At HF (C is a short-circuit):

$$|G_{id}| = 1,\ |G_{OL}| = |A(s)|/2 \Rightarrow f_p = GBWP/2,$$

which means that the zero-dB crossing frequency of G_{loop} is determined by $A(s)/k$: $f_{0dB} = GBWP/k = 20$ kHz. At LF (C is an open circuit) we have instead:

$$|G_{id}| = 1/(\omega CR),\ |G_{OL}| = A_0 \Rightarrow f_p = 1/(2\pi A_0 CR),$$

that is the same behavior as the standard integrator, discussed in class. The DC value of the loop gain is then A_0^2/k. The loop gain is reported in Fig. 1 (left).

We now carry on the full calculations for G_2, just for the sake of completeness:

$$G_{loop} = -A(s)\frac{R}{2R + 1/sC} = -A(s)\frac{sCR}{1 + 2sCR}$$

$$G_2 = \frac{G_{id}}{1 - 1/G_{loop}} = -\frac{\dfrac{1+sCR}{sCR}}{1 + \dfrac{1+2sCR}{sCRA(s)}} = -\frac{A(s)(1+sCR)}{1 + (A(s)+2)sCR} =$$

$$-\frac{A_0(1+sCR)}{1 + s(\tau + 2CR + A_0 CR) + 2s^2 CR\tau} \approx -\frac{A_0(1+sCR)}{1 + sA_0CR + 2s^2CR\tau},$$

where τ is the OA pole (at 20 Hz) time constant. Approximated solutions for the poles are then

$$f_{p1} = \frac{1}{2\pi A_0 CR} = 0.3 \text{ mHz} \quad f_{p2} = \frac{1}{2\pi}\frac{A_0 CR}{2CR\tau} = \frac{A_0}{4\pi\tau} = \frac{GBWP}{2} = 5 \text{ MHz},$$

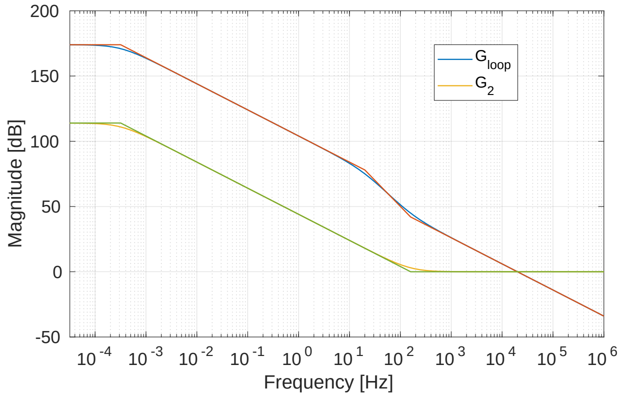
and G_{loop} becomes

$$G_{loop} \approx -\frac{A_0}{k}\frac{A_0(1+sCR)}{(1+s\tau)(1+sA_0CR+2s^2CR\tau)},$$

If we had cut both loops, we would have obtained

$$G_{loop} = -\frac{A(s)}{1+2sCR}\left(sCR + \frac{A(s)}{k}(1+sCR)\right) \approx -\frac{A_0}{k}\frac{A_0 + sCRA_0 + s^2kCR\tau}{(1+s\tau)^2(1+2sCR)}.$$

As expected, the two expressions differ, but the stability condition (not requested here) is the same: the reader can verify it by computing the zeros of $1 - G_{loop}$. Also remember that the latter expression cannot be used with G_{id} to obtain the closed-loop gain, but that G_{OL} is required.



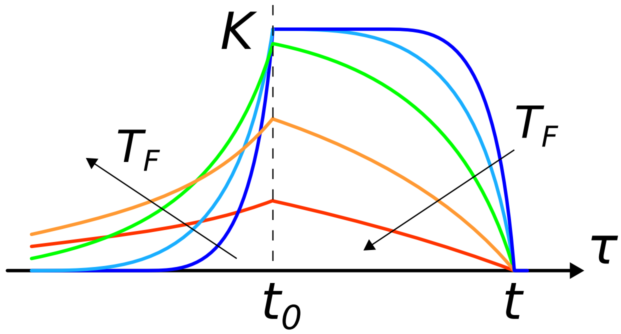


Figure 1: Left = Bode plot of real and asymptotic gains. Right = weighting function of the cascade filter for different values of T_F.

1.3

Noise sources due to the OA2 block do not give any contribution to the output noise, as they are placed inside the external feedback loop (in other words, their ideal gain is zero). We are then left to the voltage and NI input current source of OA1 plus external resistors, resulting in:

$$V_o = V_{o2} = kV_n + (k-1)RI_n \Rightarrow S_{V_o} = k^2 S_V + (k-1)^2 R^2 S_{I_n} = 14.7\ \mu\text{V}/\sqrt{\text{Hz}},$$

where $S_{I_n} = S_I + 4k_BT/R + 4k_BT/((k-1)R) \approx 25.3\ \text{pA}/\sqrt{\text{Hz}}$. The total output noise is then

$$\overline{V_o^2} = S_{V_o}\frac{\pi}{2}f_{0dB} = (2.6\ \text{mV})^2$$

1.4

The voltage at the NI input of OA1 is

$$V^+ = V_{cm}\frac{k-1}{k} + V_{o2}\frac{1}{k} \approx V_{cm} + \frac{V_o}{k}.$$

Given that $V^- = V_{cm}$, we obtain a common-mode voltage equal to V_{cm} and a differential mode voltage $V_{dm} = V_o/k$. OA1 output voltage is then

$$V_{o1} = A_{cm}V_{cm} + A_{dm}\frac{V_o}{k}.$$

OA2 has $V_{dm} = V_{o1}$ and $V_{cm} = V_{o1}/2$, hence

$$V_o = A_{dm}V_{o1} + A_{cm}\frac{V_{o1}}{2} = V_{o1}\left(A_{dm} + \frac{A_{cm}}{2}\right) \Rightarrow V_{o1} = V_o\frac{1}{A_{dm} + \frac{A_{cm}}{2}}.$$

Substituting the expression, we get

$$A_{cm}V_{cm} = -V_o\left(\frac{A_{dm}}{k} - \frac{1}{A_{dm} + \frac{A_{cm}}{2}}\right) \approx -V_o\frac{A_{dm}}{k} \Rightarrow \frac{A_{cm}}{A_{dm}} = -\frac{V_o}{kV_{cm}} \Rightarrow CMRR = 20\log_{10}\frac{V_o}{kV_{cm}}.$$

For example, if $V_o = 250$ mV, the resulting CMRR would be 80 dB. Please note that neglecting the second term in the parenthesis amounts to consider $V_{o1} = 0$, i.e., neglecting the non-idealities of OA2.

Problem 2

2.1

The expression for S/N is

$$\frac{S}{N} = A\left(1 - e^{-T/T_F}\right)\sqrt{\frac{2T}{\lambda}}.$$

If we pick the filter time constant as $T_F = T/5 = 100$ ns, we get $S/N \approx 1.4$.

2.2

We remember that $w(t,\tau)$ is the response in t to a delta function applied in τ and consider a time τ_1: the LPF response to a delta function is a decaying exponential function,

$$y_{LPF}(\tau) = \frac{1}{T_F} e^{-(\tau-\tau_1)/T_F} h(\tau - \tau_1),$$

which is in turn integrated in the $t_o - t$ window. We set $t_0 = 0$ for simplicity in the calculations, obtaining

$$y_{GI}(t,\tau_1) = K\int_0^t y_{LPF}\ d\tau = \begin{cases} K\displaystyle\int_0^{(t-\tau_1)/T_F} e^{-x}dx = K\left(1 - e^{-(t-\tau_1)/T_F}\right) \forall \tau_1 > 0 \\ Ke^{\tau_1/T_F}\displaystyle\int_0^{t/T_F} e^{-x}dx = Ke^{\tau_1/T_F}\left(1 - e^{-t/T_F}\right) \forall \tau_1 < 0 \end{cases}$$

When seen as a function of the delta arrival time τ_1, this is the WF, sketched in Fig. 1 (right). If a formal derivation is desired, we can work in the frequency domain. In the GI case we have

$$y(t) = \int X(f)W^*_{GI}(t,f)df = \int V_i(f)W_{LP}(f)W^*_{GI}(t,f)df,$$

from which

$$W(t,f) = W_{GI}(t,f)W^*_{LP}(f) \Rightarrow w(t,\tau) = w_{GI}(t,\tau) * w_{LP}(t,-\tau)$$

The output signal can be computed from the WF, i.e.

$$V_o(t) = \int V_i(t)w(t,\tau)d\tau = AK\int_0^t \left(1 - e^{-(t-\tau)/T_F}\right)d\tau = AK\left(t - T_F\left(1 - e^{-t/T_F}\right)\right).$$

Of course, this result can also be obtained by looking at the block scheme: the output is the integral of the LPF step response.

2.3

For the case of input WN, the output mean square noise is

$$\overline{n_o^2} = \lambda\int w^2(t,\tau)d\tau = \lambda K^2\int_0^t \left(1 - e^{-(t-\tau)/T_F}\right)^2 d\tau + \lambda K^2\left(1 - e^{-t/T_F}\right)^2\int_{-\infty}^0 e^{2\tau/T_F}d\tau =$$

$$\lambda K^2 T_F\left(\int_0^{t/T_F}\left(1 - e^{-x}\right)^2 dx + \int_{-\infty}^0 e^{2x}dx\right) = \lambda K^2 T_F\left(\frac{t}{T_F} + e^{-t/T_F} - 1\right).$$

As for the signal, the result could have been obtained by considering a GI with an input noise with exponential autocorrelation (output of the LPF fed with WN).
Picking the integration time t equal to the pulse duration $T = 500$ ns we have $S/N = 2$ for $T_F = 100$ ns. The best value of T_F is obviously zero, because the GI is the optimum filter for the rectangular pulse. In this case we obtain $(S/N)_{opt} = 2.24$.

2.4

For the case of a right triangular signal, the best choice is to pick $T_F \gg T$, so that the shape of the WF in the $t_0 - t$ interval mimics the signal, creating again an optimum filter. The resulting S/N will then be:

$$\frac{S}{N} = \frac{A}{\sqrt{\lambda}}\sqrt{\int_0^T\left(1 - \frac{t}{T}\right)^2 dt} = A\sqrt{\frac{T}{3\lambda}} = 1.29.$$