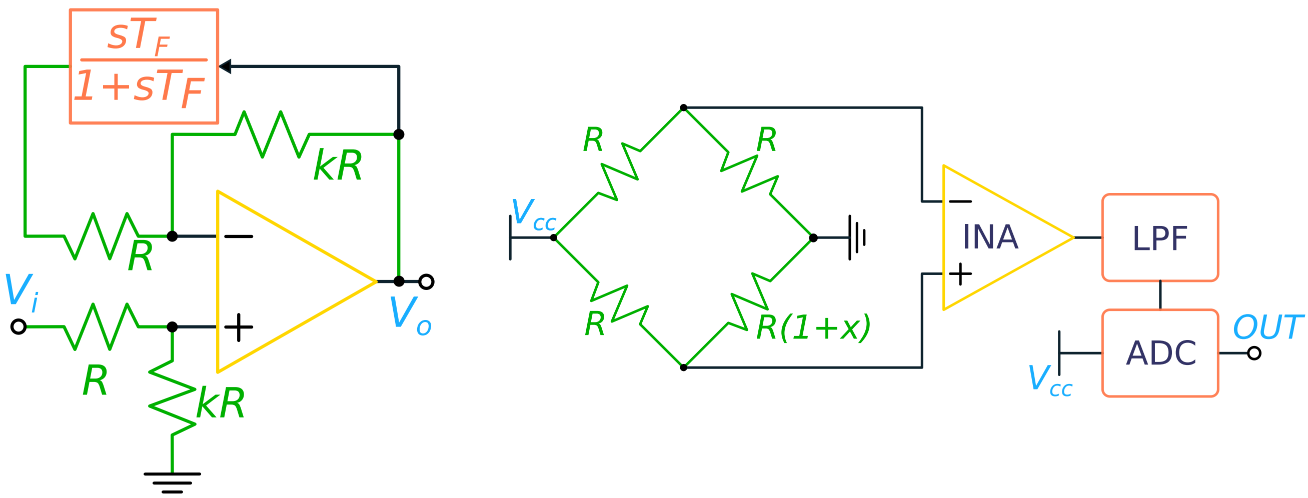
Electronics – June 23 2023 Prof. A. Spinelli

For a correct evaluation, please write your answers in a readable way; thank you



Solving 6 points correctly gives you 30/30

Problem 1

The amplifier in the left figure is an integrator, having $R = 10$ kΩ, $k = 10^3 \gg 1$ and $T_F = 16$ μs. The OA has low-frequency gain of 106 dB and $GBWP = 1$ MHz.

1. Find the (ideal) gain of the stage.
2. Compute the loop gain and discuss the stability.
3. Compute the output rms noise voltage considering the equivalent noise sources of the OA $\sqrt{S_V} = 20\ \text{nV}/\sqrt{\text{Hz}}$, $\sqrt{S_I} = 1\ \text{pA}/\sqrt{\text{Hz}}$.
4. Design a scheme for the block in Fig. 1.

Problem 2

The bridge in the right figure measures a signal with $x_{max} = 0.01$ and bandwidth of 1 Hz, sending it to a 12-bit ADC. Parameters are $R = 350\ \Omega$, $V_{cc} = 5$ V. The INA has input noise $\sqrt{S_V} = 40\ \text{nV}/\sqrt{\text{Hz}}$, $\sqrt{S_I} = 5\ \text{pA}/\sqrt{\text{Hz}}$, and $GBWP = 1$ MHz ($4k_BT \approx 1.646 \times 10^{-20}$ J).

1. Set the LPF bandwidth and evaluate S/N.
2. From now on, V_{cc} is pulsed with $T_{on} = 1$ ms at a frequency of 100 Hz. Is the previous solution still working? If not, design a different one and evaluate S/N. What is the minimum value of the pulse frequency?
3. A flicker noise $S_{LF} = K/f$ with $K = 10^{-14}$ V^2 is also present at the INA input. We subtract two samples (taken after the INA) to reduce the noise. Find the (maximum) sampling times that makes the flicker noise negligible with respect to the white one.
4. The reference used to modulate V_{cc} is employed in a synchronous detection, to recover the signal. Sketch the system, including all filters and signals, and compute the output signal.

Allowed time: 2 hours 45 minutes – Do a good job!

Results will be posted by June 30th Mark registration: July 7th

Solution

Problem 1

1.1
The voltage at the NI input of the OA is:

$$V^+ = V_{in}\frac{k}{k+1} = V^-,$$

while looking at the upper resistors and applying the superposition principle, we get:

$$V^- = V_o\frac{1}{1+k} + V_o\frac{sT_F}{1+sT_F}\frac{k}{1+k} = V_o\frac{1}{1+k}\frac{1+(k+1)sT_F}{1+sT_F},$$

from which

$$\frac{V_o}{V_i} = k\frac{1+sT_F}{1+s(k+1)T_F} \approx k\frac{1+sT_F}{1+skT_F}.$$

The LF gain is 60 dB, with a pole at $f_p = 1/(2\pi kT_F) = 10$ Hz and a zero at $f_z = 1/(2\pi T_F) = 10$ kHz. In this frequency range, the stage behaves as an integrator.

1.2
We ground the input (hence V^+) and cut both loops at the OA output, where we apply a test signal V_s. The expression for V^- is the one obtained previously from linear superposition, i.e.:

$$V^- \approx V_o\frac{1}{k}\frac{1+ksT_F}{1+sT_F} \Rightarrow G_{loop} = -A(s)\frac{1}{k}\frac{1+skT_F}{1+sT_F}.$$

This loop gain is shown in Fig. 1 (left). The high-frequency behavior is the same as $A(s)$, hence $f_{0dB} = GBWP = 1$ MHz and $\phi_m = 90°$.

1.3
If we place the voltage noise source at the NI input, we immediatly notice that its transfer is the same as V_i (apart from a factor $k/k+1 \approx 1$). The same holds for the NI input current noise source, multiplied by R^2. We then only need to compute the output noise for the I input current noise. We write the KCL at the I input node (which is a virtual ground) as follows:

$$V_o\frac{sT_F}{1+sT_F}\frac{1}{R} + V_o\frac{1}{kR} = I_n \Rightarrow V_o = I_nRk\frac{1+sT_F}{1+skT_F},$$

which is again the same transfer as V_i (apart from R)! This is because the stage is balanced, with equal impedances seen from each input. The final result is then

$$\overline{V_o^2} \approx \left(S_V + 2S_IR^2\right)\frac{\pi}{2}\left(k^2f_p + GBWP\right) = (102\ \mu\text{V})^2.$$

1.4
The block represents a high-pass filter, i.e., a CR filter having – say – $R = 16$ kΩ and $C = 1$ nF. The only precaution that must be taken is to place a buffer stage to decouple the filter from resistor R. The scheme is reported in Fig. 1 (right). Note that a buffer stage is not required at the input, as the filter is driven by the OA.

Problem 2

2.1
The maximum and minimum signal at the sensor output (INA input) are

$$V_M = V_{cc}\frac{x_{max}}{4} = 12.5\text{ mV} \qquad V_{min} = \frac{V_M}{2^{12}} = 3\ \mu\text{V}.$$

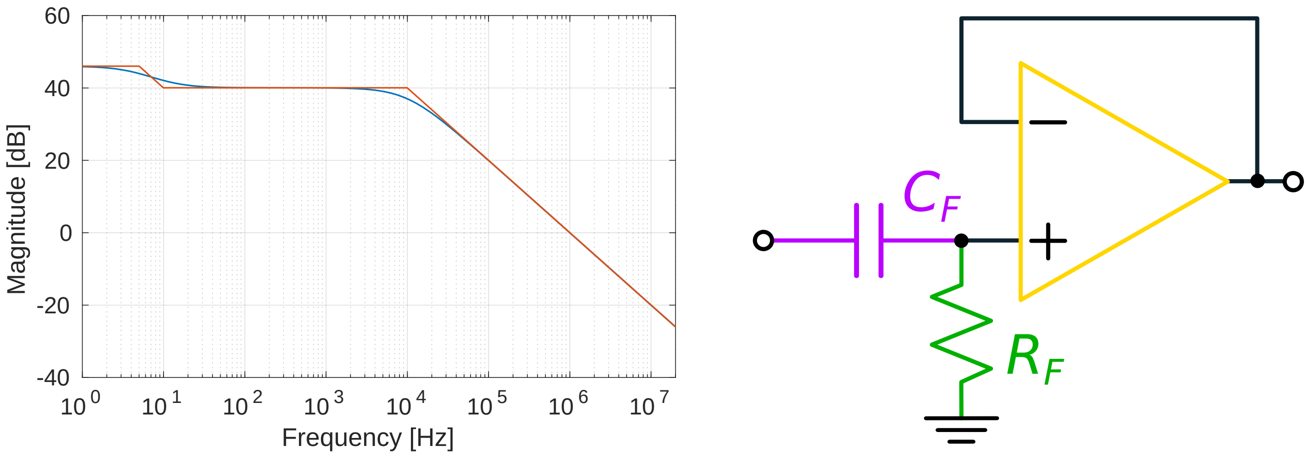


Figure 1: Left = Bode plot of real (blue) and asymptotic (red) loop gain. Right = Proposed scheme for the realization of the HPF.

meaning that the INA gain must be 10 V/25 mV = 400, where we assume that the input range of the ADC goes from 0 to V_{cc}. The total INA input noise PSD is

$$S_{in} = S_V + 2S_I \frac{R^2}{4} + 2 \times 4k_B T \frac{R}{2} = 16 \times 10^{-16} + 1.53 \times 10^{-18} + 5.76 \times 10^{-18} \approx 1.6 \times 10^{-15} \ \mathrm{V}^2/\mathrm{Hz},$$

dominated by the INA itself. Picking a 10 Hz LPF not to degrade the signal, we get:

$$\frac{S}{N} = \frac{3 \times 10^{-6}}{\sqrt{16 \times 10^{-16} \times 5\pi}} \approx 19.$$

2.2

The LPF has a time constant of $1/(20\pi) \approx 16$ ms, meaning that the output will not reach the steady-state condition during T_{on}. If we replace the LPF with a GI working over T_{on}, the new S/N becomes:

$$\frac{S}{N} = V_{min} \sqrt{\frac{2T_{on}}{S_{in}}} \approx 3.4,$$

where the GI input noise, filtered by the INA pole at $f_p = 10^6/400 = 2.5$ kHz (i.e., with a correlation time of about 64 μs), is regarded as white. The minimum pulse frequency is limited by the signal bandwidth, as we are now performing a periodic sampling. The theoretical limit is then 2 Hz (remember the sampling theorem), but a better choice is somewhat higher, say 5 Hz.
If a BA is used to further improve S/N, we can retain the previous value of T_F to get the same S/N as in #2.1. A check should be performed to ensure that the total acquisition time remain within 1 s, dictated by the signal BW $f_s = 1$ Hz limit. The minimum pulse frequency is limited by the previous requirement: the total acquisition time is

$$5T_F \frac{T_C + T_O}{T_C} < 1 \text{ s} = \frac{1}{f_s} \Rightarrow f_{min} = \frac{1}{T_C + T_O} = \frac{5T_F f_p}{T_C} = 80 \text{ Hz}.$$

2.3

The weighting function of the sampling operation is

$$w(t, \tau) = -\delta(\tau) + \delta(\tau - t_s),$$

from which

$$W(t, f) = -1 + e^{-j2\pi f t_s} = -1 + \cos(2\pi f t_s) - j \sin(2\pi f t_s) \Rightarrow |W|^2 = 2(1 - \cos(2\pi f t_s))$$

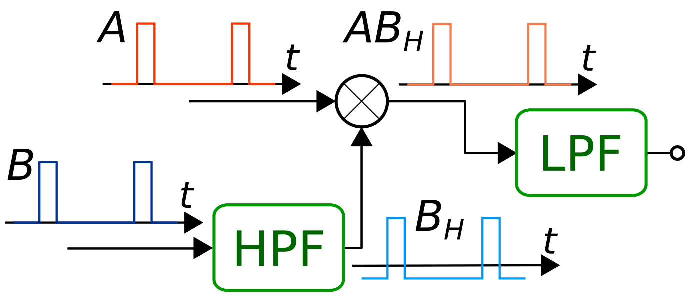


Figure 2: LIA scheme with the relevant signals.

and

$$\overline{n_{FN}^2} = \int_0^{f_p} \frac{K}{f} |W|^2 df = 2K \int_0^{x_p} \frac{1-\cos x}{x} dx,$$

where $x = 2\pi f t_s$. To reduce the FN, t_s must be small, and we expand the cosine in series, i.e., $\cos x \approx 1 - x^2/2$ (see class notes), from which

$$\overline{n_{FN}^2} \approx 2K \int_0^{x_p} \frac{x}{2} dx = K \frac{(2\pi f_p t_s)^2}{2}.$$

The WN contribution at the INA output is doubled by the sampling operation, resulting in

$$\overline{n_{WN}^2} = 2 S_{in} \frac{\pi}{2} f_p,$$

and the condition becomes

$$K \frac{(2\pi f_p t_s)^2}{2} \ll S_{in} \pi f_p \Rightarrow t_s \ll \sqrt{\frac{S_{in}}{2\pi f_p K}} = 3.2 \text{ ms}.$$

Note that the WN approximation holds as long as $t_s \gg 64$ μs.

2.4

We must consider that the signal used to modulate V_{cc} is not symmetric, and its DC value is not zero. Therefore, we cannot simply use this signal to drive the reference channel, but we need to place an HP or BP filter. This is mandatory; otherwise the LIA will not push the noise away from the baseband!
If we place an HPF (with a pole much lower than 100 Hz) to simply remove the DC component, the reference is a square-wave signal as in Fig. 2, where

$$B_H T_{on} = (B_H - B)(T - T_{on}) \Rightarrow B_H = 0.9B,$$

because $T_{on} = 1$ ms, $T = 10$ ms. The LPF output is the average value of the mixer output, i.e.

$$V_o = A B_H \frac{T_{on}}{T} = \frac{A B_H}{10} = 0.09 AB.$$

Though not requested, we can also compute the noise, but we need the Fourier expansion of a pulse series with amplitude B and duty cycle $d = T_{on}/T$, that is

$$w_R(t) = Bd + \sum_n \frac{2B}{n\pi} \sin(n\pi d) \ \cos(2n\pi f_r t),$$

where $f_r = 1/T = 10$ Hz. The DC component is removed by the HPF and the total output noise becomes

$$\overline{V_o^2} = 4BW_n \sum_n B_n^2 S_x(nf_r) = 4BW_n \sum_n \left(\frac{B}{n\pi}\right)^2 \sin^2(n\pi d) \frac{K}{nf_r} \approx 1.24 \times 10^{-15} \text{ V}^2,$$

which means:

$$\left(\frac{S}{N}\right) = \frac{0.09 \times 3 \times 10^{-6}}{\sqrt{1.24 \times 10^{-15}}} \approx 7.7.$$

The BP case is left to the reader.