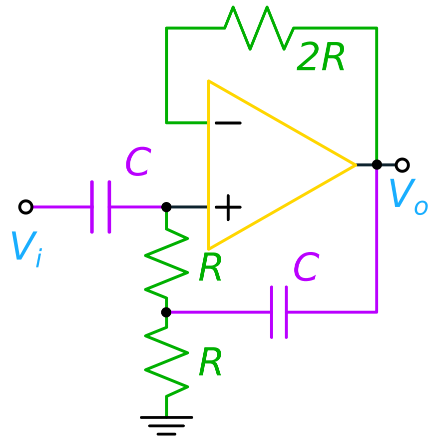
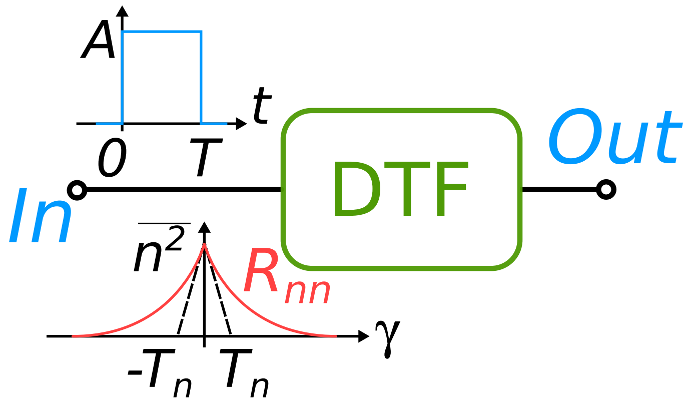
Electronics – February 17 2023 Prof. A. Spinelli

For a correct evaluation, please write your answers in a readable way; thank you





Remember that solving 6 points correctly gives you 30/30

Problem 1

The amplifier in the left figure is an AC-coupled follower, having $R = 100$ kΩ, $C = 3$ μF. The OA has low-frequency gain of 100 dB and $GBWP = 500$ kHz.

1. Find the (ideal) gain of the stage.
2. Compute the loop gain and discuss the stability.
3. Compute the output rms noise voltage considering the equivalent noise sources of the amplifier $\sqrt{S_V} = 10$ nV/$\sqrt{\text{Hz}}$, $\sqrt{S_I} = 1$ pA/$\sqrt{\text{Hz}}$. Consider the high-frequency behavior for simplicity.
4. Consider a parasitic input capacitance $C_i = 4$ pF. Discuss the stability and compensate if necessary (do not change the resistor values).

Problem 2

A sensor outputs a rectangular pulse of amplitude A and width T, affected by a noise with exponential autocorrelation $R_{nn} = \overline{n^2} e^{-|\gamma|/T_n}$. A discrete-time filter with sampling time t_s is used to recover A (right figure). Do not assume $T_n \gg T$ or $T_n \ll T$.

1. Two samples are taken: one with noise, the other with signal + noise. Compute the resulting S/N.
2. We take three uniformly-spaced samples: two for the noise (one before, one after the pulse) and one with signal + noise, subtracting the formers from the latter. Pick the sampling times, sketch the weighting function (with values), and compute S/N.
3. We now take four uniformly-spaced samples: two for the noise, both before the pulse, and two for signal + noise. Pick the sampling times, sketch the weighting function (with values), and compute S/N. Consider unit-area delta functions for ease of calculations.
4. The four samples can be taken at any time. Is there a different smart choice of the sampling times with respect to #2.3? Propose it and evaluate S/N. Extra points for a quantitative comparison between this result and #2.3 (hint: expand in series assuming $t_s \ll T_n$).

Allowed time: 2 hours 45 minutes – Do a good job!

Results will be posted by February 21st Mark registration: Thursday, February 23rd

Solution

Problem 1

1.1

We obviously have $V^+ = V^- = V_o$. To solve the circuit we could use the superposition principle, but the expressions are fairly lenghty. We then proceed by analysis, writing the KCL at the midpoint between the two resistors R (at voltage V_1):

$$\frac{V_o - V_1}{R} + (V_o - V_1)sC = \frac{V_1}{R} \Rightarrow V_1 = V_o \frac{1 + sCR}{2 + sCR}.$$

Equalling the currents in input capacitor C and upper R, we immediately get:

$$sC(V_i - V_o) = \frac{V_o - V_1}{R},$$

where we substitute the previous value of V_1, obtaining:

$$\frac{V_o}{V_i} = \frac{sCR(2 + sCR)}{1 + 2sCR + (sCR)^2} = \frac{sCR(2 + sCR)}{(1 + sCR)^2}.$$

We have one zero at DC and at $f_z = 1/\pi CR \approx 1$ Hz and two coincident poles at $f_p = f_z/2 = 0.5$ Hz. Note that beyond those frequencies the gain is one.

1.2

We ground the input and cut both loops at the OA output, where we apply a test signal V_s and obtain $V^- = V_s$. To get V^+, it is simpler to write a couple of KCLs at the nodes, as in # 1.1. However, let's follow an impedance approach: we call Z the impedance beyond the feedback capacitor, i.e.

$$Z = R \parallel \left(R + \frac{1}{sC}\right) = R\frac{1 + sCR}{1 + 2sCR},$$

we immediately get

$$V^+ = V_s \frac{Z}{Z + 1/sC} \frac{1}{1 + sCR} = V_s \frac{sCR}{1 + 3sCR + (sCR)^2},$$

leading to

$$G_{loop} = -A(s)\frac{(1 + sCR)^2}{1 + 3sCR + (sCR)^2}.$$

This approach is simpler to write, but more prone to errors. We now have two zeros at 0.5 Hz and two negative poles at

$$f_{p_{1,2}} = \frac{1}{2\pi CR}\frac{3 \pm \sqrt{5}}{2} \approx \begin{matrix} 1.4 \text{ Hz} \\ 0.2 \text{ Hz.} \end{matrix}$$

The circuit is obviously stable: poles and zeros fall at very low frequencies, where $|G_{loop}| \gg 1$, and beyond that range we have $G_{loop} = -A(s)$. The phase margin is hence 90° and $f_{0dB} = GBWP$.

1.3

For frequencies much higher than the singularities added by the capacitors, we can regard them as short-circuits. It is then straightforward to obtain

$$S_{V_o} = S_V + S_I(2R)^2 = 10^{-16} + 4 \times 10^{-14} \approx 4 \times 10^{-14} \text{ V}^2/\sqrt{\text{Hz}},$$

$$\overline{V_o^2} = S_{V_o}\frac{\pi}{2}GBWP = (177\ \mu\text{V})^2.$$

If we wanted to account for resistor noise as well, we would see that the two R resistors give no contribution at HF, leaving only the $2R$ element, that gives $S_{V_o} = 4k_BT(2R) \approx 3 \times 10^{-15}$ $\text{V}^2/\sqrt{\text{Hz}}$, negligible indeed.

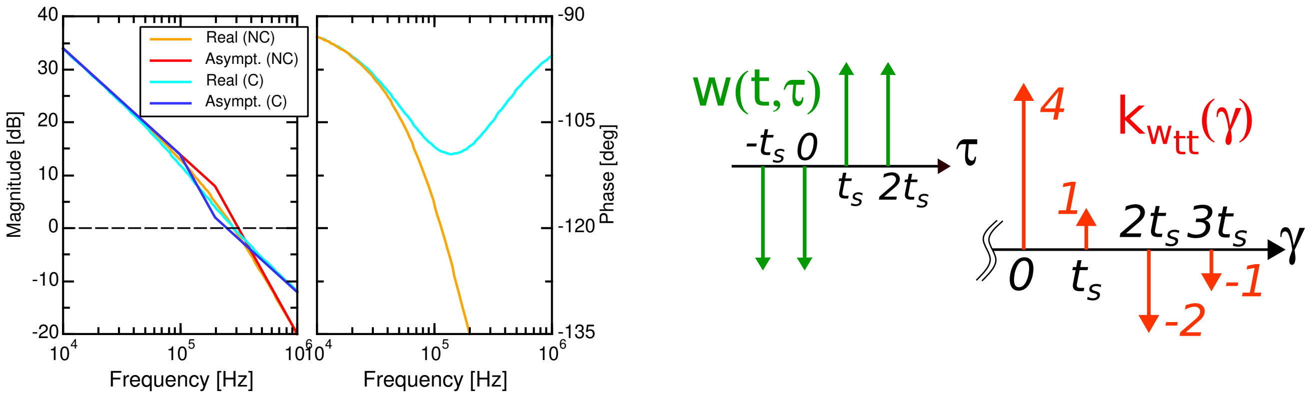


Figure 1: Left = Bode plot of real and asymptotic loop gains for the non-compensated (NC) and compensated (C) condition. Right = Weighting function (green) and its time correlation (red, only shown for non-negative γ) for the case in #2.3.

1.4

The small parasitic capacitance introduces a pole at high frequency, where capacitors C can be safely regarded as short-circuits. The loop scheme then simplifies, with capacitor C_i connected between the inverting input and ground and giving a pole at $f_p = 1/(4\pi C_i R) \approx 200$ kHz, i.e., a phase margin smaller than 45°. We can then use a simple lead compensation, connecting a compensation capacitor C_c in parallel to resistor $2R$. If we pick $C_c = C_i$ we have a zero at 200 kHz while the pole goes to 100 kHz; the circuit is now stable (pole and zero are very close and their phases cancels out). Bode diagrams are plotted in Fig. 1 (left).

Problem 2

2.1

Assuming that the signal starts at $t = 0$, we sample the noise at $t = 0^-$ and the signal at $t = t_s$ (or $-t_s$ and 0^+). The time correlation of the weighting function is then

$$k_{ww}(\gamma) = 2\delta(\gamma) - \delta(|\gamma| - t_s),$$

from which

$$\frac{S}{N} = \frac{A}{\sqrt{2\overline{n^2}\left(1 - e^{-t_s/T_n}\right)}}.$$

2.2

The first and third sample are taken at 0^- and T^+, respectively, so the second one falls at $T/2$. The weighting function (WF) is then

$$w(t,\tau) = -\delta(\tau) + 2\delta(\tau - T/2) - \delta(\tau - T),$$

from which

$$k_{ww}(\gamma) = 6\delta(\gamma) - 4\delta(|\gamma| - T/2) + \delta(|\gamma| - T)$$

and

$$\frac{S}{N} = \frac{2A}{\sqrt{2\overline{n^2}\left(3 - 4e^{-T/2T_n} + e^{-T/T_n}\right)}}.$$

If we consider $T \gg t_s$, the result in # 2.1 is better.

2.3

The best choice is to sample the noise at $-t_s$ and 0^-, followed by the signal (+ noise) samples at t_s and $2t_s$. The WF is then:

$$w(t,\tau) = -\delta(\tau + t_s) - \delta(\tau) + \delta(\tau - t_s) + \delta(\tau - 2t_s),$$

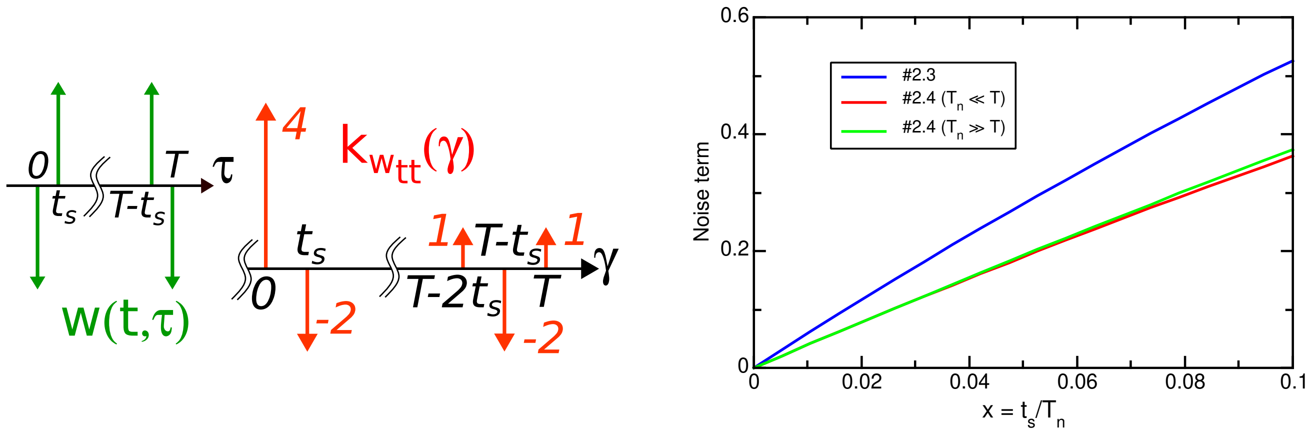


Figure 2: Left = Weighting function (green) and its time correlation (red, only shown for non-negative γ) for the case in #2.4. Right = comparison between #2.3 and #2.4.

whose time correlation is

$$k_{ww}(\gamma) = 4\delta(\gamma) + \delta(|\gamma| - t_s) - 2\delta(|\gamma| - 2t_s) - \delta(|\gamma| - 3t_s).$$

Both quantities are shown in Fig. 1, right. This leads to

$$\frac{S}{N} = \frac{2A}{\sqrt{2\overline{n^2}\left(2 + e^{-t_s/T_n} - 2e^{-2t_s/T_n} - e^{-3t_s/T_n}\right)}}.$$

2.4

An alternative solution is to place one noise and signal (+ noise) delta couple at the beginning of the pulse (0^- and t_s) and one at the end ($T - t_s$ and T^+). The WF is then

$$w(t, \tau) = -\delta(\tau) + \delta(\tau - t_s) + \delta(\tau - (T - t_s)) - \delta(\tau - T),$$

whose time correlation is now

$$k_{ww}(\gamma) = 4\delta(\gamma) - 2\delta(|\gamma| - t_s) + \delta(|\gamma| - (T - 2t_s)) - 2\delta(|\gamma| - (T - t_s)) + \delta(|\gamma| - T),$$

as shown in Fig. 2 (left). The result is then

$$\frac{S}{N} = \frac{2A}{\sqrt{2\overline{n^2}\left(2 - 2e^{-t_s/T_n} + e^{-(T-2t_s)/T_n} - 2e^{-(T-t_s)/T_n} + e^{-T/T_n}\right)}}.$$

For a comparison between the last two results, we can just look at the terms within parenthesis. Calling $x = t_s/T_n$ we have

$$\begin{aligned} \#2.3: &\quad 2 - 2e^{-2x} + e^{-x} - e^{-3x} \\ \#2.4: &\quad 2 - 2e^{-2x} + e^{-T/T_n}\left(1 - e^{x}\right)^2. \end{aligned}$$

If we expand at first order, we get $6x$ for 2.3 and $4x$ for 2.4 (note in fact that the term e^{-T/T_n} multiplies a second-order term in x). Fig. 2 (right) shows such terms as a function of x, in the two extreme cases $T \gg T_n$ and $T \ll T_n$.