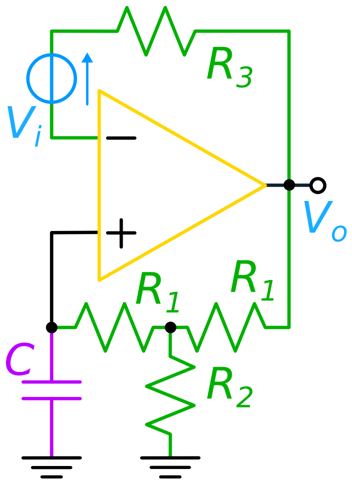
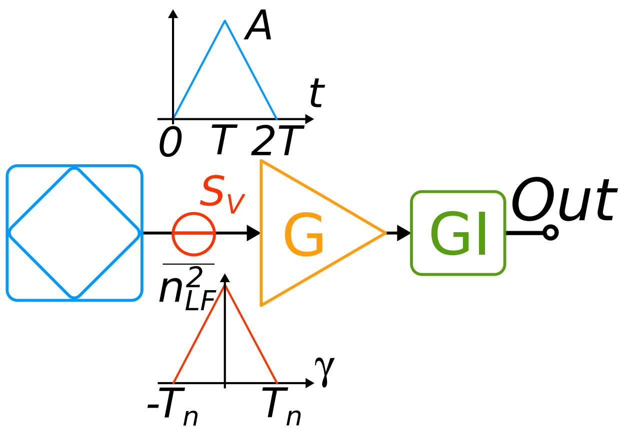
Electronics – January 20 2023 Prof. A. Spinelli

For a correct evaluation, please write your answers in a readable way; thank you





Remember that solving 6 points correctly gives you 30/30

Problem 1

The amplifier in the left figure has $R_1 = 1$ kΩ, $R_2 = 9$ kΩ, $R_3 = 1.9$ kΩ, $C = 10$ nF. The OA has low-frequency gain of 120 dB and $GBWP = 10$ MHz.

1. Find the (ideal) gain of the stage.
2. Compute the loop gain and discuss the stability.
3. Compute the output rms noise voltage considering the equivalent voltage noise source of the amplifier only $\left(\sqrt{S_V} = 10\ \mathrm{nV}/\sqrt{\mathrm{Hz}}\right)$.
4. Compute the (ideal) step voltage response of the circuit.

Problem 2

A sensor outputs symmetric triangular pulses of amplitude A and width $2T$, sent to a preamplifier with white input noise voltage PSD S_V (see figure on the right). A gated integrator is used to recover the pulse amplitude.

1. Find the optimum integration time and compute S/N.
2. Evaluate the output noise when a low-frequency noise with nearly triangular autocorrelation (with correlation time $T_n > T_G$) is present at the input (no white noise). Compute then the new optimum T_G and comment on the result. Consider $T_n \gg T_G$ in the T_G calculation only.
3. The LF noise is sampled right before the GI acquisition and subtracted from the output. Sktetch the new weighting function and evaluate the output noise.
4. We now subtract two noise samples, taken at the beginning and end of the GI window, each weighted one half the one in #2.3. Does this further reduce the LF noise? Why?

Allowed time 2 hours 45 minutes – Do a good job!

Results will be posted by January 24th Mark registration: Friday, January 27th

Solution

Problem 1

1.1

If we consider that no current can flow in R_3 (it is there for bias current compensation purposes), we see that the input pins of the OA are at bias $V_0 - V_i$, which is also the voltage drop across C. The current flowing in C is then $sC(V_o - V_i)$ and the voltage at the midpoint between the resistors R_1 becomes

$$V_1 = sC(V_o - V_i)\left(\frac{1}{sC} + R_1\right) = (V_o - V_i)(1 + sCR_1),$$

which is obvious considering the $R_1 - C$ LPF. The KCL at this node then reads

$$sC(V_o - V_i) + \frac{V_1}{R_2} = \frac{V_o - V_1}{R_1} \Rightarrow V_0 = V_i \frac{R_1 + R_2}{R_1} \frac{1 + sC(R_1 + R_1 \parallel R_2)}{1 + sC(R_1 + 2R_2)}.$$

The low-frequency gain is equal to 10, with a pole at $f_p = 1/(2\pi C(R_1 + 2R_2)) \approx 838$ Hz and a zero at $f_z = 1/(2\pi C(R_1 + R_1 \parallel R_2)) \approx 8.38$ kHz. The high-frequency gain is equal to one, as can be seen from the scheme when C is replaced by a short-circuit.

1.2

We replace the source V_i with a short-circuit and disconnect the OA output, opening both loops and applying a test voltage V_s. We immediately get $V^- = V_s$. To compute V^+, we need instead to solve the network: midpoint voltage V_1 is now

$$V_1 = V_s \frac{(R_1 + 1/sC) \parallel R_2}{(R_1 + 1/sC) \parallel R_2 + R_1} = V_s \frac{R_2(1 + sCR_1)}{R_1 + R_2 + sCR_1(R_1 + 2R_2)},$$

which leads to

$$V^+ = V_s \frac{1}{1 + sCR_1} = V_s \frac{R_2}{R_1 + R_2 + sCR_1(R_1 + 2R_2)}$$

and to

$$G_{loop} = -A(s)\left(1 - \frac{R_2}{R_1 + R_2 + sCR_1(R_1 + 2R_2)}\right) = -A(s) \frac{R_1}{R_1 + R_2} \frac{1 + sC(R_1 + 2R_2)}{1 + sC(R_1 + R_1 \parallel R_2)}.$$

Please note that this result could have been easily obtained by noticing that the open-loop gain is $G_{OL} = A(s)$ (just ground the the feedback paths) and recalling that $G_{loop} = -G_{OL}/G_{id}$.
The loop gain is plotted in Fig. 1 (left). Note that beyond the pole frequency at 8.38 kHz, the ideal gain is 1 and $G_{loop} = -G_{OL} = -A(s)$, so that the phase margin is 90° and $f_{0dB} = GBWP$.

1.3

The noise voltage V_n of the OA has the same transfer as the input voltage:

$$V_0 = V_n \frac{R_1 + R_2}{R_1} \frac{1 + sC(R_1 + R_1 \parallel R_2)}{1 + sC(R_1 + 2R_2)},$$

to which we should add a pole at $GBWP$ given by the OA. The transfer is then

$$\overline{V_o^2} \approx S_V \frac{\pi}{2}\left(\left(\frac{R_1 + R_2}{R_1}\right)^2 f_p + (GBWP - f_z)\right) \approx S_V \frac{\pi}{2} GBWP = (39.6\ \mu\text{V})^2.$$

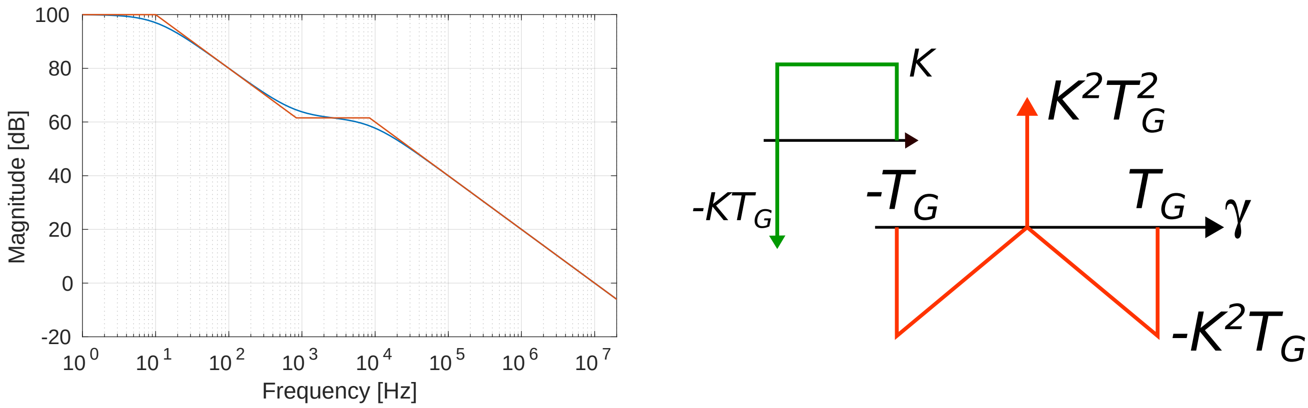


Figure 1: Left = Bode plot of the loop gain (blue) and its asymptotic approximation (red). Right = Weighting function (green) and its time correlation (red) when a sampling delta in introduced to remove the LF noise.

1.4

For a unit amplitude step, the output voltage in the Laplace domain becomes

$$V_o(s) = G\frac{1+s\tau_z}{1+s\tau_p}\frac{1}{s} = G\left(\frac{\tau_z-\tau_p}{1+s\tau_p}+\frac{1}{s}\right),$$

where $G = 10$, $\tau_z = 1/(2\pi f_z) = 19$ μs, and $\tau_p = 1/(2\pi f_p) = 190$ μs. Recalling the elementary function transform, we get

$$v_o(t) = G\left(\frac{\tau_z-\tau_p}{\tau_p}e^{-t/\tau_p}+1\right)h(t),$$

i.e., an exponential function starting from $v_o(0) = G\tau_z/\tau_p = 1$ V and going to $v_o(\infty) = 10$ V. Note that these values can be computed from the initial/final value theorems, or from the value of the transfer function (only for the step case).

Problem 2

2.1

If one wants to do the calculations, it is convenient to shift the signal so to have a symmetric signal with the maximum in $t = 0$. However, the case of triangular signal with white noise is well-known and discussed in the drills: the optimum integration width is 2/3 of the pulse width. This is also true in the symmetric case, so that we have a window of $\pm(2/3)T$, i.e.,

$$\frac{S}{N} = A\frac{\left(2-\frac{2}{3}\right)\frac{2}{3}T}{\sqrt{S_V(2/3)T}} = A\sqrt{\frac{T}{S_V}}\frac{4\sqrt{2}}{3\sqrt{3}}.$$

Note that S/N is larger by a factor of $\sqrt{2}$ with respect to what obtained from a one-sided triangular signal.

2.2

The GI WF time correlation is the triangular signal

$$k_{ww}(\gamma) = K^2T_G\left(1-\frac{\gamma}{T_G}\right) \qquad \forall\ |\gamma| \le T_G,$$

which leads to

$$\overline{n_o^2} = 2K^2\overline{n_{LF}^2}T_G\int_0^{T_G}\left(1-\frac{\gamma}{T_G}\right)\left(1-\frac{\gamma}{T_n}\right)d\gamma = K^2\overline{n_{LF}^2}T_G^2\left(1-\frac{T_G}{3T_n}\right),$$

that can be approximated by $K^2\overline{n_{LF}^2}T_G^2$ when $T_n \gg T_G$. The new S/N becomes then

$$\left(\frac{S}{N}\right)^2 = A^2\frac{\left(2-\frac{T_G}{2T}\right)^2\frac{T_G^2}{4}}{\overline{n_{LF}^2}T_G^2} = \frac{A^2}{4\overline{n_{LF}^2}}\left(2-\frac{T_G}{2T}\right)^2,$$

which is maximized for $T_G = 0$! This is not surprising: the noise is highly correlated and its contribution increases linearly with T_G (think of an unknown offset), while the triangular signal decreases from the maximum value. Hence, the best choice is to minimize the integration time.

2.3

The new weighting function is a negative delta function followed by the usual rectangular profile (see inset in Fig. 1, right). To reduce the LF noise contribution, the area of the WF must be zero, and the delta function integral is $-KT_G$. The time correlation of the WF is shown in Fig. 1 (right) and can be written as:

$$k_{ww}(\gamma) = K^2T_G^2\delta(\gamma) - K^2|\gamma| \qquad \forall\ |\gamma| \le T_G,$$

leading to

$$\overline{n_o^2} = K^2T_G^2\overline{n_{LF}^2} - 2\overline{n_{LF}^2}\int_0^{T_G} K^2\gamma\left(1-\frac{\gamma}{T_n}\right)d\gamma = K^2\overline{n_{LF}^2}\left(T_G^2 - 2\left(\frac{T_G^2}{2} - \frac{T_G^3}{3T_n}\right)\right) = K^2\overline{n_{LF}^2}\frac{2T_G^3}{3T_n}.$$

Note that the mean square noise is smaller with respect to the previous case by a factor of roughly T_G/T_n.

2.4

It does indeed! Taking two noise samples with weight 1/2 means that we are subtracting the average noise (over two samples) rather than one instantaneous value. This makes no difference from the viewpoint of the LF components (that are cancelled in both cases), but reduces the contribution at higher frequencies. In other words, this choice tracks the noise fluctuation over a time T_G rather than relying on a single sample.

To perform the calculations (not required), we consider the new WF, which is the usual rectangular function with the addition of two delta function at its sides. Let us consider for simplicity $K = 1/T_G$ (unity gain GI), meaning the two deltas have area of $-1/2$. The time correlation is similar to the one in Fig. 1 (right), but the central delta has area of 1/2 and there are two delta at $\pm T_G$ with area 1/4. So, the integral result in the last equation is still valid, but the first term $K^2T_G^2\overline{n_{LF}^2} = \overline{n_{LF}^2}$ is replaced by

$$\frac{1}{2}\overline{n_{LF}^2} + \frac{2}{4}\overline{n_{LF}^2}\left(1-\frac{T_G}{T_n}\right) = \overline{n_{LF}^2}\left(1-\frac{T_G}{2T_n}\right),$$

meaning that the final result is

$$\overline{n_o^2} = \overline{n_{LF}^2}\left(1-\frac{T_G}{2T_n}\right) - \overline{n_{LF}^2}\left(1-\frac{2T_G}{3T_n}\right) = \overline{n_{LF}^2}\frac{T_G}{6T_n}.$$