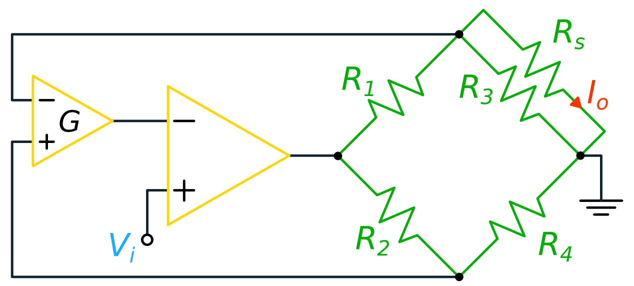
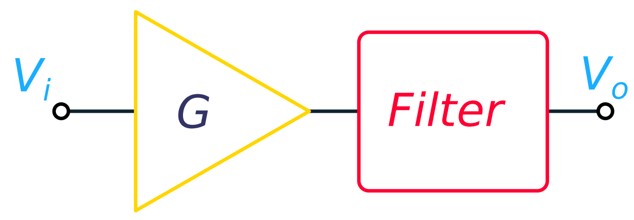
Electronics – September 11 2024 Prof. A. Spinelli

For a correct evaluation, please write your answers in a readable way; thank you





Solving six points correctly gives you 30/30

Problem 1

The scheme in the left figure is a $V - I$ converter. OA parameters are $A_0 = 120$ dB and poles at 10 Hz and 1 MHz. Consider $G = 1$ and balanced bridge condition $R_1R_4 = R_2R_3$.

1. Evaluate the (ideal) gain I_o/V_i.
2. Evaluate the loop gain and discuss the stability for $R_s \to 0$.
3. Compute the output noise current PSD due to the the equivalent voltage and current noise sources of the OA.
4. Propose a compensation scheme for $R_s \to 0$, considering equal bridge resistors for simplicity. If possible, do not change the ideal gain.

Problem 2

A sensor outputs sinusoidal signals $V_i = A\cos 2\pi f_r t$, with $f_r = 80$ Hz and $A \approx 5\ \mu$V. The signal feeds an amplifier with gain G and equivalent input white noise $\sqrt{S_I} = 50$ pA$/\sqrt{\text{Hz}}$, $\sqrt{S_V} = 100$ nV$/\sqrt{\text{Hz}}$ and noise corner frequency of 3 kHz.

1. A band-pass filter is placed at the output. What is the bandwidth needed to measure A with $S/N = 10$?
2. An LIA with square-wave reference at f_r is used to recover the signal. What is the new value of the LPF bandwidth?
3. Two square-wave interferences with frequencies of 1.2 and 1.5 kHz are present at the amplifier input. What is their output signals?
4. The frequency of the signal V_i is not constant but switches between 70 and 90 Hz every 100 ms, while the mixer reference frequency f_r remains at 80 Hz. What is the output signal? Find then a better value for f_r and estimate the new output signal.

Allowed time: 2 hours 45 minutes – Do a good job!

Results will be posted by September 16th **Mark registration: Thursday, September 19th**

Solution

Problem 1

1.1

As the differential input voltage of the OA is zero, the G stage output is V_i. But $G = 1$, so its differential input voltage is also V_i. The midpoint between R_1 and R_3 is at bias I_oR_s, leading to the condition in Fig. 1 (left), from which we can easily express the currents flowing in R_1 and R_2 as

$$I_1 = I_o + I_o\frac{R_s}{R_3} \qquad I_2 = \frac{I_oR_s + V_i}{R_4}.$$

The KVL between ground and the OA output becomes then

$$I_oR_s + I_1R_1 = I_oR_s + V_i + I_2R_2 \Rightarrow I_o = V_i\frac{R_2 + R_4}{R_1R_4} = V_i\frac{R_1 + R_3}{R_1R_3},$$

where the balancing condition $R_1/R_3 = R_2/R_4$ has been used.

1.2

We break the loop at the OA output and apply a test voltage V_T, obtaining the input voltages of the G stage:

$$V^+ = V_T\frac{R_4}{R_2 + R_4} \qquad V^- = V_T\frac{R_p}{R_1 + R_p},$$

where $R_p = R_3 \parallel R_s$. The inverting input of the OA is then $V^+ - V^-$ ($G = 1$), and the loop gain becomes

$$G_{loop} = -A(s)\left(\frac{R_4}{R_2 + R_4} - \frac{R_p}{R_1 + R_p}\right) = -A(s)\frac{R_2}{R_2 + R_4}\frac{R_3 - R_p}{R_1 + R_p}.$$

For large values of R_s, $R_p \to R_3$ and $|G_{loop}| \to 0$; this is obviously undesirable, but does not affect the stability, that remains good with phase margin of 90°.
For $R_s \to 0$, instead, $R_p \to R_s \ll R_1, R_3$ and

$$G_{loop} = -A(s)\frac{R_4}{R_2 + R_4}.$$

The HF pole of $A(s)$ falls at 1 MHz, where $|A(s)| = 10$. To ensure stability for any R_s we must then have

$$\frac{R_4}{R_2 + R_4} < \frac{1}{10} \Rightarrow \frac{R_2}{R_4} = \frac{R_1}{R_3} > 9.$$

1.3

The voltage noise source is subjected to the same transfer as the input signal. Current noise sources are instead short-circuited by the input and the G stage and give no contribution. It is then

$$S_{I_o} = S_V\left|\frac{R_1 + R_3}{R_1R_3}\right|^2.$$

1.4

The worst case is $R_s \to 0$, where we have

$$G_{loop} = -A(s)\frac{R_4}{R_2 + R_4} = -\frac{A(s)}{2},$$

with phase margin of $90 - \arctan(\sqrt{5}) \approx 24°$. If we want to not change the ideal gain, we cannot place any compensation element across the bridge (otherwise, a capacitor in parallel to R_2 could work). The same can

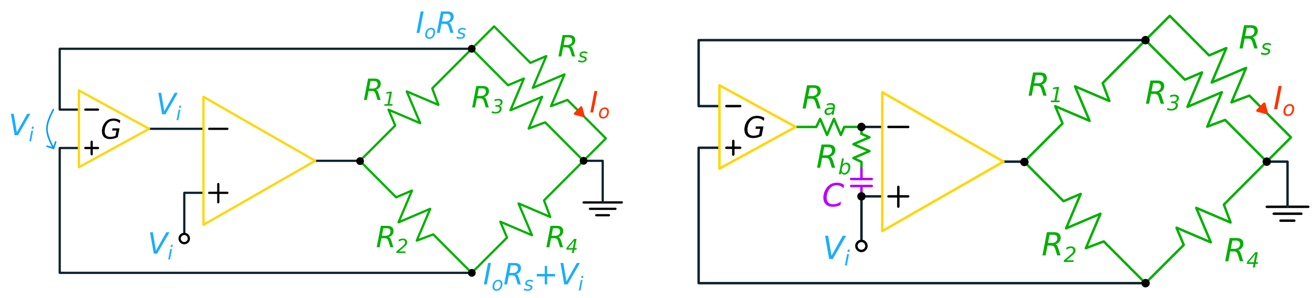


Figure 1: Left = Scheme for gain calculation. Right = possible compensation scheme.

be said for an $R-C$ series network across the G input pins. So, it is better to move such a network across the input pins of the OA. But, since this is driven by the output voltage source of G, an extra resistor is needed, as shown in Fig. 1 (right). We have then a lag network whose zero could be placed before the OA second pole, say at $f_z = 100$ kHz, and the pole to a frequency f_p that gives $|G_{loop}(f_z)| = 0$dB. We have then

$$\begin{cases} G_0 f_{p0} = G_1 f_{p1} \\ G_1 f_{p1}^2 = f_z^2 \end{cases} \Rightarrow f_p = \frac{f_z^2}{G_0 f_{p0}} = 2 \text{ kHz}.$$

The resulting diagram is reportd in Fig. 2 (left). Please note that in this case, even a simple voltage divider (*i.e.*, without the capacitor C) that lowers the loop gain by a factor of 5 would work, giving a larger bandwidth. Usually such a solution is not recommended, but here the reduction is small and can be accepted. Finally, the pole and zero values are given by

$$f_z = \frac{1}{2\pi C R_b} \qquad f_{p1} = \frac{1}{2\pi C(R_a + R_b)}.$$

Problem 2

2.1

If the BP filter is centerd at f_r, the output signal amplitude is obviously $V_o = AG$. Moreover, since $f_r \ll f_{nc}$, the flicker noise is dominant and the noise PSD to consider is actually

$$S_{V_i}(f_r) = S_V \frac{f_{nc}}{f_r} = 3.75 \times 10^{-13} \text{ V}^2/\text{Hz},$$

where we have neglected S_I assuming a low-impedance input. S/N then becomes:

$$\frac{S}{N} = \frac{A}{\sqrt{S_{V_i}(f_r) BW_n}} = 10 \Rightarrow BW_n = \frac{A^2}{100 S_{V_i}(f_r)} = 0.67 \text{ Hz},$$

i.e., a filter $BW = 0.42$ Hz and a quality factor (see Drill #1) $Q = f_r/BW \approx 188$. Such a filter is well beyond the limit of what can be achieved with OAs!

2.2

The output signal is due to the product of the sinusoidal input signal and the harmonic at f_r of the reference square wave (of amplitude B), while the noise is collected by all the harmonics:

$$V_o = GA\frac{2B}{\pi} \qquad \overline{n_o^2} = 2BW_n G^2 \left(\frac{2B}{\pi}\right)^2 \sum_k \frac{S_{V_i}((2k+1)f_r)}{(2k+1)^2},$$

where the factor 2 in the noise expression stems from the fact that S_{V_i} is unilateral. As the noise corner frequency is much larger than f_r, we can consider a pure flicker noise condition, that leads to (see class notes)

$$\left(\frac{S}{N}\right) = \frac{A}{\sqrt{2BW_n\, S_{V_i}(f_r)}} \frac{1}{1.0256} = 10 \Rightarrow BW_n \approx \frac{A^2}{210 S_{V_i}(f_r)} = 0.31 \text{ Hz} \Rightarrow BW = \frac{2}{\pi} BW_n \approx 0.2 \text{ Hz},$$

which means a filter time constant $T_F \approx 0.8$ s.

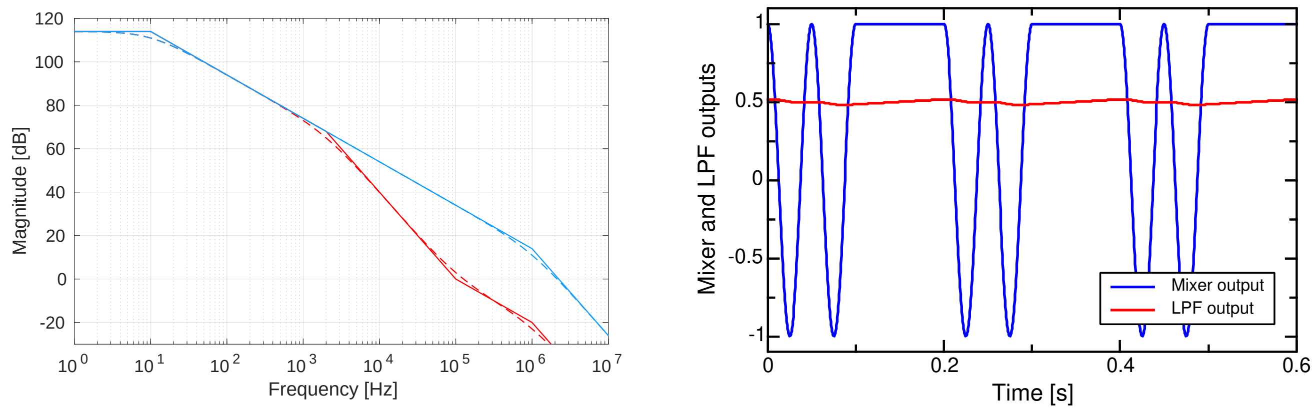


Figure 2: Left = Bode plots of the original and compensated loop gains. Right = mixer and output signal when the frequency is switched as in #2.4.

2.3

The 1.5 kHz interference does not give any output, as none of its harmonics fall within the transmission window of the LIA. In reality, issues such as the saturation of the amplifier and/or of the LIA limit the maximum tolerable amplitude of the interference. These are quantified by the dynamic reserve of the LIA and are not considered here.
The fundamental harmonic of the 1.2 kHz interference, instead, falls right on the 15^{th} harmonic of the 80 Hz reference, resulting in an output signal of

$$V_{int} = G\frac{2B}{15\pi}\frac{4A_i}{\pi},$$

where A_i is the interference amplitude. Please note that in reality we should also consider the higher harmonics (*e.g.*, the third harmonics at 3.6 kHz corresponds to the 45^{th} one of the reference), but the result does not change. Even if not required, we can now set a condition to achieve a a small output error:

$$V_{int} \ll GA\frac{2B}{\pi} \Rightarrow A_i \ll \frac{15}{4}\pi A \approx 59\ \mu\text{V}.$$

2.4

The LF component at the mixer output is a sinusoidal signal with the difference in frequency between the two input signals (we consider the first harmonic of the square wave), i.e.

$$V_M = GA\frac{2B}{\pi}\cos\left((f_r - f_s)t\right).$$

If we consider just the cosine term, we see that in our case, $\Delta f = f_r - f_s$ is ± 10 Hz, which does *not* affect it (cosine is an even function). We then get a sinusoidal output signal that is filtered by the LPF, leading to $V_o = 0$.

A better solution is to move the reference frequency to 70 or 90 Hz, in order to have $\Delta f = 0$ (i.e., $\cos(\Delta f\, t) = 1$) for half of the time. In the other half, $\Delta f = 20$ Hz, and the cosine term is averaged to zero by the LPF. So, the LPF output signal will be one half the previous one. Fig. 2 (left) shows the $\cos(\Delta f\, t)$ term and the (normalized) LPF output.