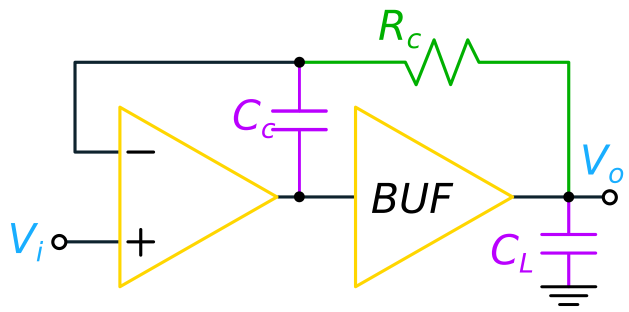
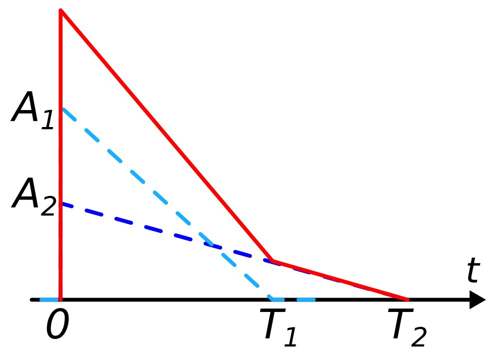
Electronics – February 7 2024 Prof. A. Spinelli

For a correct evaluation, please write your answers in a readable way; thank you





Solving six points correctly gives you 30/30

Problem 1

The scheme in the left figure is a power buffer for capacitive loads. The OA has $A_0 = 126$ dB and $GBWP = 20$ MHz. The buffer stage (BUF) has unity gain and output resistance $R_o = 6\ \Omega$. Other parameters are $R_c = 2\ \text{k}\Omega$, $C_c = 1$ nF.

1. Consider $R_o = 0$ and compute the ideal and loop gains.
2. Evaluate the maximum value of C_L that grants a phase margin larger than 45° when R_o is not zero.
3. Compute the output rms noise voltage considering the equivalent noise sources of the OA and BUF $\sqrt{S_V} = 20\ \text{nV}/\sqrt{\text{Hz}}$, $\sqrt{S_I} = 10\ \text{pA}/\sqrt{\text{Hz}}$. Consider again $R_0 = 0$ for simplicity.
4. Would it be convenient to use a simpler scheme without R_c and C_c, having the OA connected as follower and BUF driving the load? Why?

Problem 2

Because of different electron and hole mobilities, a semiconductor sensor outputs a signal made up of two triangular pulses with different amplitudes (A_1, A_2) and durations (T_1 T_2, with $T_2 > T_1$). T_1 and T_2 are due to the sensor design and are hence known. We want to measure the amplitude A_1 in presence of white noise with bilateral PSD λ. Note that A_2 is not known.

1. Consider two GIs plus basic electronics (sum, difference, etc.). Propose a measurement scheme with its parameter values and sketch the resulting weighting function.
2. Evaluate S/N.
3. Consider now a shot-like white noise with PSD λ proportional to the signal, *i.e.*, with two triangular components with PSD amplitudes λ_1 and λ_2. Compute the new output noise.
4. Consider a repetitive signal. Would it be convenient to employ boxcar averagers in the non-stationary white noise case of #2.3? If yes, would the improvement in S/N follow the $\sqrt{N_{eq}}$ law even in this case, or not? Justify your answers.

Allowed time: 2 hours 45 minutes – Do a good job!

Results will be posted by February 9th **Mark registration: Tuesday, February 13th**

Solution

Problem 1

1.1

As the BUF gain is one and $R_o = 0$, V_o is the voltage at the OA output. Hence, there can be no current flowing into R_c and C_c, meaning that $V_i = V_0$. The ideal gain is therefore one.
To compute G_{loop}, we cut the loop at the OA output and apply a test voltage V_s. Once again, no current can flow into the feedback network, and we obtain $V^- = V_s$, *i.e.*

$$G_{loop} = -A(s).$$

1.2

The scheme for G_{loop} calculation is shown in Fig. 1 (left). We label V_1 the voltage drop on C_L and write (remember that the BUF output is also at V_s):

$$\begin{aligned} \frac{V_s - V_1}{R_o} &= sC_L V_1 + \frac{V_1 - V^-}{R_c} \\ \frac{V_1 - V^-}{R_c} &= sC_c(V^- - V_s) \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_1\left(\frac{1}{R_c} + \frac{1}{R_o} + sC_L\right) &= \frac{V_s}{R_o} + \frac{V^-}{R_c} \\ \frac{V_1}{R_c} &= V^-\left(\frac{1}{R_c} + sC_c\right) - sC_cV_s \end{aligned}$$

from which, after some algebra

$$\frac{V^-}{V_s} = \frac{1 + sC_c(R_c + R_o) + s^2C_cC_LR_cR_o}{1 + sC_c(R_c + R_o) + sC_LR_o + s^2C_cC_LR_cR_o} \approx \frac{1 + sC_cR_c + s^2C_cC_LR_cR_o}{1 + s(C_cR_c + C_LR_o) + s^2C_cC_LR_cR_o},$$

that means

$$G_{loop} = -A(s)\frac{1 + sC_cR_c + s^2C_cC_LR_cR_o}{1 + s(C_cR_c + C_LR_o) + s^2C_cC_LR_cR_o} = -A(s)\frac{1 + s\tau_c + s^2\tau_c\tau_L}{1 + s(\tau_c + \tau_L) + s^2\tau_c\tau_L},$$

where we set for simplicity $\tau_c = C_cR_c$ e $\tau_L = C_LR_o$. If $\tau_L \ll \tau_c$, *i.e.*, $C_L \ll C_cR_c/R_o = 0.33\ \mu$F, $G_{loop} \approx -A(s)$ and stability is ensured. We then consider the other extreme, where $\tau_L \gg \tau_c$ and

$$G_{loop} \approx -A(s)\frac{1 + s\tau_c + s^2\tau_c\tau_L}{1 + s\tau_L + s^2\tau_c\tau_L}.$$

Approximate pole positions are $f_1 = 1/2\pi\tau_L$ and $f_2 = 1/2\pi\tau_c \approx 80$ kHz, while zeros are complex at a frequency $f_z = 1/(2\pi\sqrt{\tau_L\tau_c})$. This means that for frequencies higher than f_2, we still have $G_{loop} \approx -A(s)$, and the system remains stable for any capacitor value. Loop gain is shown in Fig. 1 (right). Further discussion is in the Appendix.

1.3

The OA voltage noise is transferred with unity gain, like the signal, while its NI current noise gives no contribution. So does the current noise of the BUF, leaving the OA I input current noise and BUF voltage noise. With simple calculations we get

$$S_{V_o} = S_{V_{OA}} + S_{I_{OA}}\left|\frac{R}{1 + sC_cR_c}\right|^2 + S_{V_{BUF}}\left|\frac{sC_cR_c}{1 + sC_cR_c}\right|^2.$$

Considering that the zero-dB frequency of G_{loop} is equal to the $GBWP$, we have

$$\begin{aligned} \overline{V_o^2} &= S_V\frac{\pi}{2}GBWP + S_IR_c^2\frac{1}{4C_cR_c} + S_V\left(\frac{\pi}{2}GBWP - \frac{1}{4R_cC_c}\right) \\ &\approx 1.26 \times 10^{-8} + 5 \times 10^{-11} + 1.26 \times 10^{-8} \approx 2.52 \times 10^{-8}\ \mathrm{V}^2 = (158\ \mu\mathrm{V})^2. \end{aligned}$$

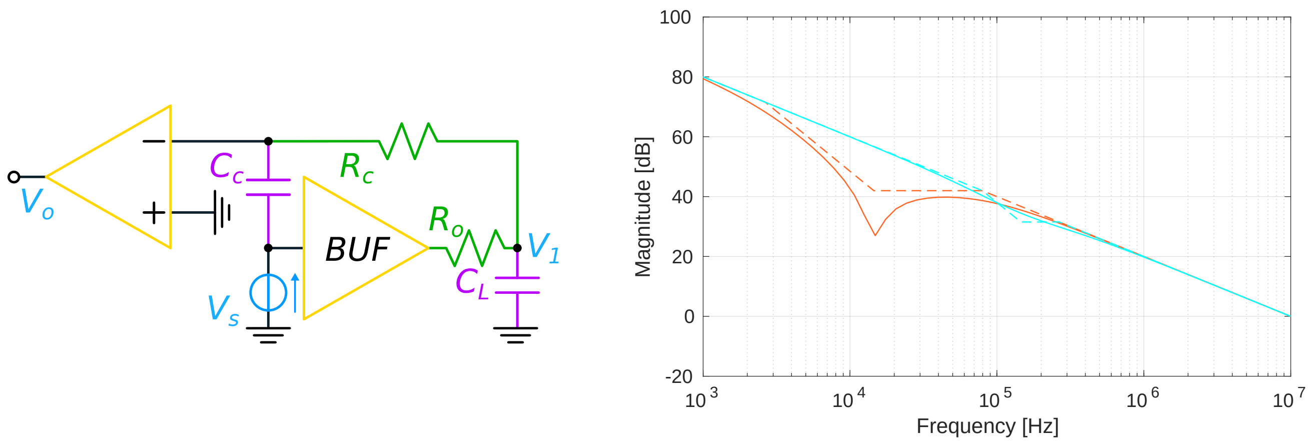


Figure 1: Left = Loop gain calculation scheme. Right = Resulting loop gain in the $10^3 - 10^7$ Hz range for $C_L = 100$ nF (blue) and 10 μF (red). Solid lines = actual gain, dashes = asymptotic approximation.

1.4

The proposed solution has the disadvantage of having a pole set by the R_oC_L time constant, hence depending on the load conditions. Moreover, keeping BUF outside the loop means that its offset, non-linearity and other unwanted drawbacks affect the output.

Problem 2

2.1

In principle, any two measurements can be used to extract A_1. However, a simple choice is to evaluate A_2 from an integration in the $T_1 - T_2$ interval (or 2/3 of it) and subtract this value from an integration in the $0 - T_1$ interval. The GI outputs are

$$V_1 = A_1 G_1 \frac{T_1}{2} + A_2 G_1 T_1 \left(1 - \frac{T_1}{2T_2}\right) = A_1 \frac{T_1}{2} G_1 + A_2 \frac{T_1(2T_2 - T_1)}{2T_2} G_1$$
$$V_2 = A_2 G_2 \left(1 - \frac{T_1}{T_2}\right) \frac{T_2 - T_1}{2} = A_2 \frac{(T_2 - T_1)^2}{2T_2} G_2,$$

where G_1 and G_2 are the GI gains. We then set

$$G_2 = G_1 \frac{T_1(2T_2 - T_1)}{(T_2 - T_1)^2}$$

and subtract V_2 from V_1. Setting $G_1 = 2K/T_1$ returns $V_o = KA_1$. The WF is sketched in Fig. 2 (left).

2.2

The output noise is simply

$$\overline{V_0^2} = \lambda \int_0^{T_2} w^2(t,\tau) d\tau = \lambda \left(G_1^2 T_1 + G_2^2 (T_2 - T_1)\right) = \lambda G_1^2 T_1 \left(1 + \frac{T_1(2T_2 - T_1)^2}{(T_2 - T_1)^3}\right) =$$
$$\lambda G_1^2 T_1 T_2 \frac{T_2^2 + T_1 T_2 - T_1^2}{(T_2 - T_1)^3},$$

from which

$$\left(\frac{S}{N}\right) = \frac{A_1}{2\sqrt{\lambda}} \sqrt{\frac{T_1(T_2 - T_1)^3}{T_2(T_2^2 + T_1 T_2 - T_1^2)}}.$$

For example, considering $T_1 = 1$ μs, $T_2 = 2$ μs and $K = 1$ the noise becomes $\overline{V_0^2} = \lambda \times 40 \times 10^6$.

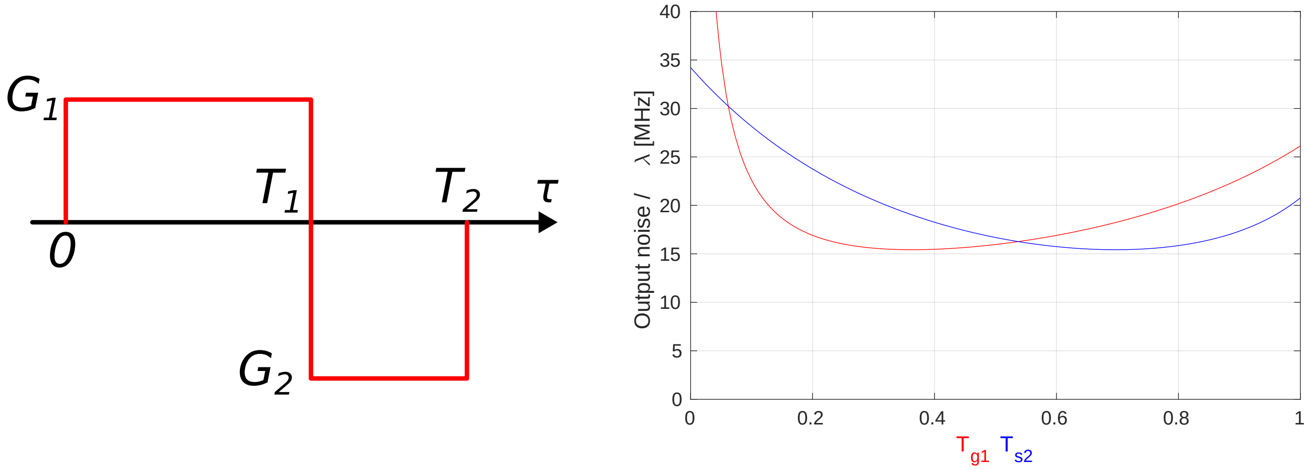


Figure 2: Left = possible weighting function for measuring A_1. Right = output noise as a function of T_{g1} (red) and T_{s2} (blue) for $T_1 = 1$ μs, $T_2 = 2$ μs and $K = 1$.

2.3

The output mean square noise for a non-stationary white noise is (see class notes):

$$\overline{n_{out}^2(t)} = \int \lambda(\tau) w^2(t,\tau) d\tau.$$

The filter output is collected at $t = T_2$, where we get

$$\begin{aligned}\overline{n_{out}^2(T_2)} =& G_1^2 \int_0^{T_1} \left[\lambda_1 \left(1 - \frac{\tau}{T_1}\right) + \lambda_2 \left(1 - \frac{\tau}{T_2}\right)\right] d\tau + G_2^2 \lambda_2 \int_{T_1}^{T_2} \left(1 - \frac{\tau}{T_2}\right) d\tau \\ =& G_1^2 \left[\lambda_1 \frac{T_1}{2} + \lambda_2 T_1 \left(1 - \frac{T_1}{2T_2}\right) + \frac{G_2^2}{G_1^2} \lambda_2 \frac{(T_2 - T_1)^2}{2T_2}\right] = G_1^2 \frac{T_1}{2} \left[\lambda_1 + \lambda_2 \frac{T_2(2T_2 - T_1)}{(T_2 - T_1)^2}\right].\end{aligned}$$

2.4

The answer is yes to both questions. BAs improve S/N (by the square root of the number of equivalent samples) whenever the noise is not correlated over different samples, which is indeed the case here!

To conclude, we briefly discuss the optimization of the GI parameters. It makes sense to start the first integration from 0, as the signal is maximum there. Analogously, it is reasonable to stop the second integration at 2/3 of the $T_1 - T_2$ interval. We are then left with the gate time of the first GI, T_{g1}, and the starting time of the second GI, T_{s2}. The output noise as a function of such parameters is shown in Fig. 2 (right) for $T_1 = 1$ μs, $T_2 = 2$ μs and $K = 1$; note that the optimum values are $T_{g1} \approx 0.36$ μs and $T_{s2} \approx 0.7$ μs.

Appendix

The results in #1.2 do not guarantee that the circuit is actually useful for any value of C_L. In fact, if we compute the (ideal) closed-loop gain, we get (labelling V_1 the OA output):

$$\begin{aligned}\frac{V_1 - V_o}{R_o} &= sC_L V_o + \frac{V_o - V_i}{R_c} \\ \frac{V_1 - V_o}{R_c} &= sC_c(V_i - V_1),\end{aligned} \Rightarrow \frac{V_o}{V_i} = \frac{1 + sC_c(R_c + R_o)}{1 + sC_c(R_c + R_o) + s^2 C_c C_L R_o R_c}$$

and the closed-loop poles might be complex. To ensure this is not the case, we must have

$$C_c^2 (R_c + R_o)^2 > 4C_c C_L R_o R_c \Rightarrow C_L < C_c \frac{(R_c + R_o)^2}{4R_o R_c} \approx C_c \frac{R_c}{4R_o} = 83 \text{ nF}.$$

If a small amount of oscillation might be tolerated (quality factor of the poles equal to one), the requirement becomes the one already discussed in #1.2, *i.e.*

$$C_L < C_c \frac{R_c}{R_o} = 0.33\ \mu\text{F}.$$