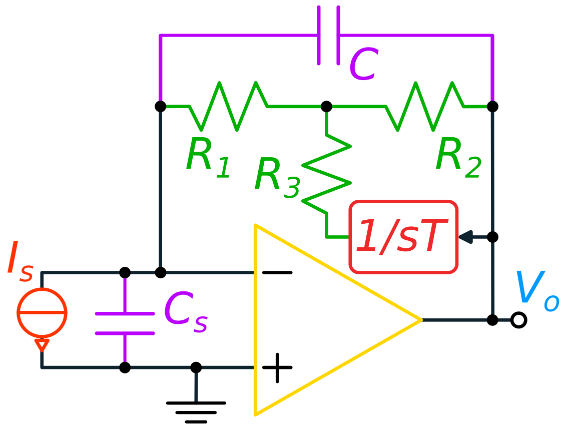
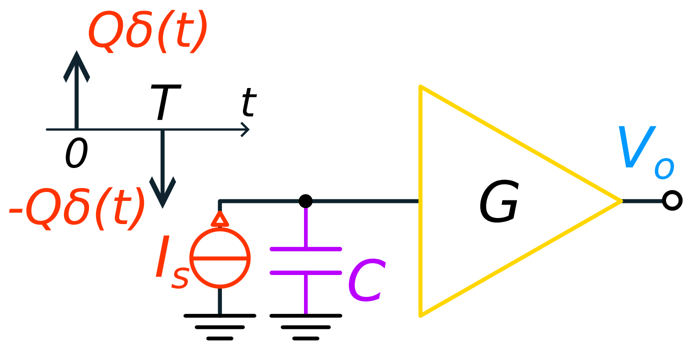
Electronics – July 19 2024 Prof. A. Spinelli

For a correct evaluation, please write your answers in a readable way; thank you





Solving six points correctly gives you 30/30

Problem 1

The scheme in the left figure is a preamplifier for an accelerometer, schematized by a current source and its capacitance $C_s = 1.2$ nF. Parameters are $R_1 = 100$ MΩ, $R_2 = 17$ kΩ, $R_3 = 2$ kΩ, $C = 1.25$ nF, $T = 40$ s. The OA has $A_0 = 120$ dB and $GBWP = 4$ MHz.

1. Neglect the integrator block and R_3 (for this point only) and evaluate the ideal gain.
2. Repeat the previous point for the full circuit (consider $R_1 \gg R_2, R_3$). What is the purpose of the integrator block?
3. Compute the high-frequency output noise voltage PSD and rms value considering the equivalent voltage noise source of the OA $\sqrt{S_V} = 20$ nV/$\sqrt{\text{Hz}}$.
4. Suggest a realization for the integrator block in the above scheme using only one OA. *Hint: think of the standard integrator and tweak its input.*

Problem 2

A sensor outputs a current signal made of two delta pulses with charge $Q \approx 5$ fC separated by $T = 80$ ms. The sensor, with a capacitance of $C = 500$ pF is connected to a voltage amplifier with gain $G = 10$ and equivalent input noise $\sqrt{S_I} = 1$ pA/$\sqrt{\text{Hz}}$, $\sqrt{S_V} = 4$ nV/$\sqrt{\text{Hz}}$.

1. Compute the output signal and noise PSD.
2. Compute the weighting function of the optimum filter.
3. Compute the optimum S/N.
4. S_V and S_I have a flicker component with the same noise corner frequency f_{nc}. What is the maximum value of f_{nc} that allows to neglect the flicker noise (in the previous optimum filter case)? *Hint: consider the noise sources after the whitening filters and find a suitable approximation for* $|W(f)|^2$ *in the FN case.*

Allowed time: 2 hours 45 minutes – Do a good job!

Results will be posted by July 24th Mark registration: Friday, July 26th

Solution

Problem 1

1.1

Without the integrator block and R_3, the feedback network is the parallel of C and $R_1 + R_2$. The output voltage is then

$$\frac{V_o}{I_s} = \frac{R_1 + R_2}{1 + sC(R_1 + R_2)} \approx \frac{R_1}{1 + sCR_1}.$$

1.2

We label V_m the node between R_1 and R_2 and write the KCL:

$$\frac{V_m}{R_1} + \frac{V_m - V_o}{R_2} + \frac{V_m - \dfrac{V_o}{sT}}{R_3} = 0 \Rightarrow V_o\left(\frac{1}{R_2} + \frac{1}{sTR_3}\right) = V_m\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \approx V_m\left(\frac{1}{R_2} + \frac{1}{R_3}\right),$$

from which

$$V_m = V_o\frac{sTR_3 + R_2}{sT(R_2 + R_3)}.$$

The KCL at the input node of the OA gives then

$$I_s = sCV_o + \frac{V_m}{R_1} \Rightarrow \frac{V_o}{I_s} = \frac{sTR_1(R_2 + R_3)}{s^2TCR_1(R_2 + R_3) + sTR_3 + R_2}.$$

We now have a zero in the origin and two coincident poles (the quality factor is 0.5023) at 67 mHz, beyond which the gain follows the result in #1.1. A Bode plot of the gain is reported in Fig. 1 (left). The zero in the origin cancels any effect related to offset and bias currents of the OA.

1.3

At HF, the impedances of the capcitors is small and the feedback network is dominated by C (note that the output of the integrator is almost zero). We have then a partition between C_s and C, i.e.,

$$V_o = V_n\frac{C + C_s}{C} \Rightarrow S_{V_o} = S_V\frac{(C + C_s)^2}{C^2} \approx \left(39\ \mathrm{nV}/\sqrt{\mathrm{Hz}}\right)^2$$

and

$$G_{loop} = -A(s)\frac{\dfrac{1}{sC_s}}{\dfrac{1}{sC_s} + \dfrac{1}{sC}} = -A(s)\frac{C}{C + C_s},$$

i.e., $f_{0dB} = GBWP\ C/(C + C_s) \approx 2$ MHz, leading to

$$\overline{V_o^2} \approx S_V\frac{(C + C_s)^2}{C^2}\frac{\pi}{2}f_{0dB} \approx (69\ \mu\mathrm{V})^2.$$

Full calculation (still considering $R_1 \gg R_2, R_3$) leads to

$$V_o = V_n\frac{sT(R_2 + R_3)(1 + s(C + C_s)R_1)}{s^2TCR_1(R_2 + R_3) + sTR_3 + R_2},$$

from where the HF limit can be recovered.

1.4

The standard integrator is obviously inverting, so to achieve the desired result we should apply the input at the NI input of the OA. But this result in

$$\frac{V_o}{V_i} = 1 + \frac{1}{sCR} = \frac{1 + sCR}{sCR},$$

which is good apart from the zero. To cancel it, we can apply an LPF at the input, resulting in the scheme of Fig. 1 (right), that gives the desired result.

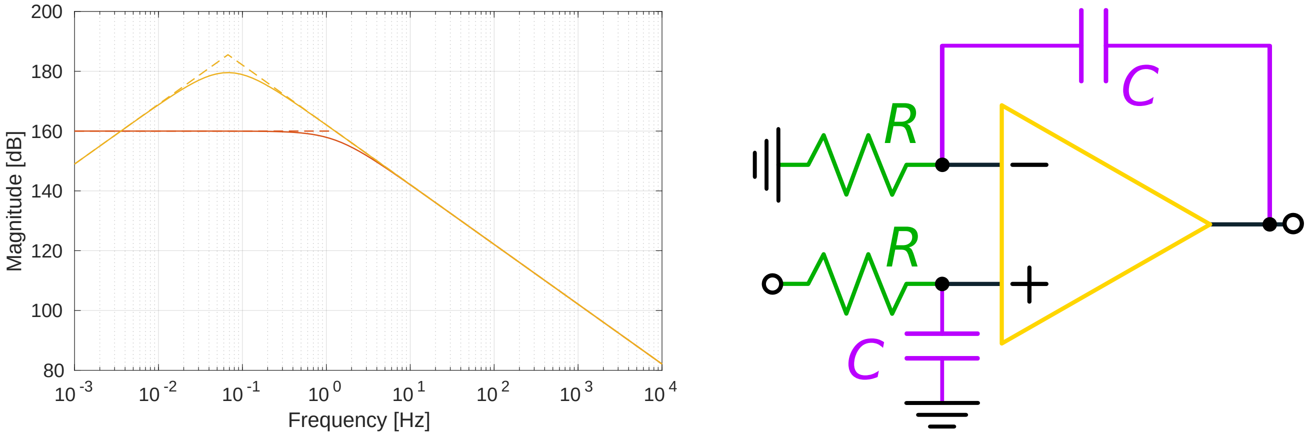


Figure 1: Left = Circuit gain with (yellow) and without (red) the integrator stage. Right = scheme for a NI integrator.

Problem 2

2.1

The current signal is integrated by the capacitor C, resulting in a rectangular signal at the amplifier output:

$$V_o = I_s \frac{1}{sC} G \Rightarrow V_o(t) = \frac{QG}{C} \int (\delta(t) - \delta(t-T))\, dt = \frac{QG}{C} \mathrm{rect}(0,T),$$

where rect is the unit-amplitude rectangular function. The output noise is instead

$$V_o = \left(V_n + \frac{I_n}{sC} \right) \Rightarrow S_{V_o} = G^2 \left(S_V + \frac{S_I}{(\omega C)^2} \right) = G^2 S_V \frac{1 + \omega^2 C^2 \frac{S_V}{S_I}}{\omega^2 C^2 \frac{S_V}{S_I}} = G^2 S_V \frac{1+\omega^2 T_n^2}{\omega^2 T_n^2},$$

where $T_n = C\sqrt{S_V/S_I} \approx 2\ \mu$s.

2.2

As the noise PSD is not white, a whitening filter is needed, whose TF H_w is given by

$$S_{V_o} |H_w|^2 = \text{const} \Rightarrow |H_w| \propto \frac{1}{\sqrt{S_{V_o}}} = K \frac{\omega T_n}{\sqrt{1+(\omega T_n)^2}} \Rightarrow H_w(s) = \frac{sT_n}{1+sT_n},$$

i.e., an LTI HPF. This filter obviously affects the signal as well, that now becomes (remember the step response of the HPF):

$$V_w(s) = V_o(s) H_w(s) = \frac{QG}{sC} \left(1 - e^{-sT}\right) \frac{sT_n}{1+sT_n} \Leftrightarrow V_w(t) = \frac{QG}{C} \left(e^{-t/T_n} u(t) - e^{-(t-T)/T_n} u(t-T) \right).$$

The second stage W_2 of the optimum filter is now a matched filter with weighting function proportional to $V_w(t)$. In the frequency domain, we have then

$$W_2(T,f) = \frac{1 - e^{-j\omega T}}{1+\omega T_n} \qquad |W(T,f)| \propto |H_w(f)||W_2(T,f)| = \left(1 - e^{-j\omega T}\right) \frac{\omega T_n}{1+(\omega T_n)^2}.$$

Note that the expression of W is equal to $V_o(f)|H_w(f)|^2$, and could have been obtained as $V_o(f)/S_{V_o}(f)$.

2.3

To compute S/N, we can look at the whitening filter output and remember that

$$\left(\frac{S}{N} \right)_{opt} = \frac{Q}{C\sqrt{S_V/2}} \sqrt{2 \int_0^\infty e^{-2t/T_n} dt} = \frac{Q}{C} \sqrt{\frac{2T_n}{S_V}} = 5,$$

where the factor of 2 before the integral accounts for the two equal exponential signals that do not overlap. Of course, the same result can be obtained from

$$\left(\frac{S}{N}\right)_{opt} = \frac{Q}{C}\sqrt{\int \frac{|X(f)|^2}{S_V(f)/2} df},$$

considering again only the first delta function (and multiplying by two), obtaining

$$\left(\frac{S}{N}\right)_{opt} = \frac{Q}{C}\sqrt{2\int \frac{\left|\frac{1}{\omega}\right|^2}{\frac{S_V}{2}\frac{1+(\omega T_n)^2}{(\omega T_n)^2}}\frac{d\omega}{2\pi}} = \frac{Q}{C}\sqrt{\frac{2T_n}{\pi S_V}\int \frac{d\omega T_n}{1+(\omega T_n)^2}} = \frac{Q}{C}\sqrt{\frac{2T_n}{S_V}}.$$

2.4

Beyond the whitening filter, the noise PSD is equal to $G^2 S_V(f)$, that now contains both white and flicker components. The WN contribution is then

$$\overline{n_{WN}^2} = G^2 S_V \int w_2^2(t,\tau)d\tau = 2G^2 S_V \frac{1}{4T_n} = G^2 \frac{S_V}{2T_n},$$

where we have considered the matched filter amplitude $1/T_n$. For FN we need to switch to the frequency domain, where the input noise is $S_V f_{nc}/f$ and

$$\overline{n_{FN}^2} = G^2 S_V f_{nc} \int_0^{\infty} \frac{|W_2^2(t,f)|^2}{f} df = G^2 S_V f_{nc} \int_0^{\infty} \frac{|1-e^{-j\omega T}|^2}{|1+j\omega T_n|^2}\frac{d\omega}{\omega} = 2G^2 S_V f_{nc} \int_0^{\infty} \frac{1-\cos\omega T}{1+(\omega T_n)^2}\frac{d\omega}{\omega}.$$

The HF limit is set by $f_H = 1/(2\pi T_n)$. At low frequencies, where the denominator equals 1, the numerator goes to zero as $f \to 0$ and then oscillates between 0 and 2, with average value equal to one, behaving like an HPF. A nice guess for f_L could be as follows: at low frequencies we have

$$1-\cos\omega T \approx \frac{(\omega T)^2}{2},$$

while an HPF with time constant T_L has a square modulus of the transfer function equal to $(\omega T_L)^2$. Equalling the terms, we get

$$T_L = \frac{T}{\sqrt{2}} \Rightarrow f_L = \frac{1}{2\pi T_L} = \frac{1}{\sqrt{2}\pi T} \approx \frac{1}{4.4\ T}$$

from which we get

$$\overline{n_{FN}^2} \approx 2G^2 S_V f_{nc} \log\left(\frac{f_H}{f_L}\right) \approx 2G^2 S_V f_{nc} \log\left(\frac{2.2\ T}{\pi T_n}\right) \approx 20.5\ G^2 S_V f_{nc}.$$

To get equal contributions, we must have

$$20.5\ G^2 S_V f_{nc} = G^2 \frac{S_V}{2T_n} \Rightarrow f_{nc} = \frac{1}{41T_n} = 12.2\ \text{kHz}.$$

This value is quite high, meaning that our filter is effective in reducing LF noise.
Please note that a simpler estimate of f_L based on

$$1-\cos\omega T = 1 \Rightarrow \omega_L T = \frac{\pi}{2} \Rightarrow f_L = \frac{1}{4T}$$

returns a very similar value (12.5 kHz) for f_{nc}.