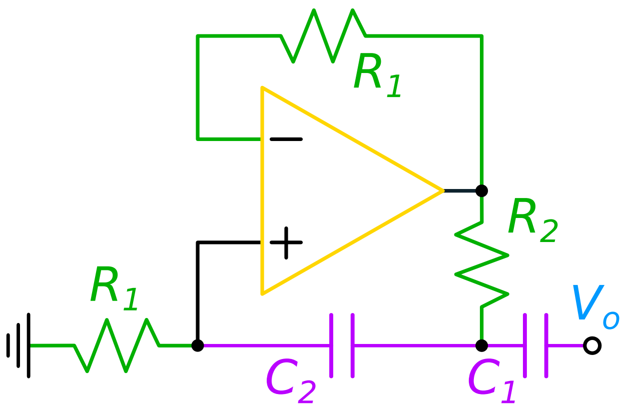
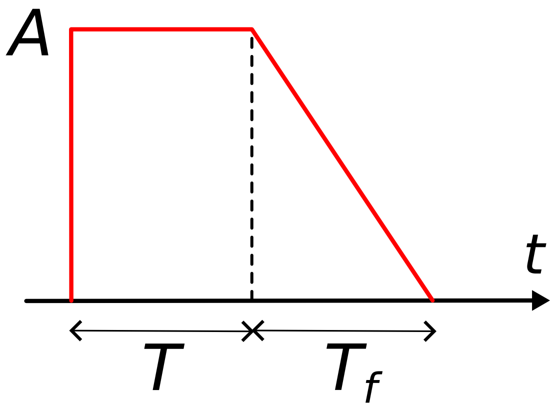
Electronics – June 19 2024 Prof. A. Spinelli

For a correct evaluation, please write your answers in a readable way; thank you





Solving six points correctly gives you 30/30

Problem 1

The scheme in the left figure behaves like an impedance. The OA has $A_0 = 100$ dB and $GBWP = 1$ MHz. Parameters are $R_1 = 68$ kΩ, $R_2 = 1.2$ kΩ, $C_1 = 0.56$ μF, $C_2 = 56$ nF.

1. Evaluate the ideal impedance at the circuit output.
2. Evaluate the loop gain (with open-circuit output) and discuss the stability.
3. Compute the (open-circuit output) noise voltage PSD considering the equivalent voltage noise source of the OA $\sqrt{S_V} = 10\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$.
4. Compute the short-circuit output noise current PSD due to S_V. *Hint: find a smart solution to avoid calculations.*

Problem 2

A current pulse $Ax(t)$ with $A \approx 300$ nA and $T = 5$ μs is shown in the right figure. The signal goes through a current amplifier with gain $G = 300$, bandwidth of 10 MHz, and equivalent input noise $\sqrt{S_I} = 80\ \mathrm{pA}/\sqrt{\mathrm{Hz}}$.

1. Design and sketch a simple LPF that can provide an output signal with amplitude of 1 V and $S/N > 10$. Comment on the result.
2. Evaluate the optimum value of S/N when $T_f = T$.
3. Consider now $T_f = kT$ with $k \gg 1$ and assume (for this point only) an additional current noise source with white unilateral PSD $S_{ns}x(t)$ proportional to the signal. Find a reasonable and justified choice for the integration window of a GI and compute the resulting S/N for $S_{ns} = S_I$.
4. Compute the optimum integration window for $k \ll 1$ (hint: set $T_G = T + T'_G$ and evaluate S/N as a function of T'_G/T_f, correctly neglecting higher-order terms in k).

Allowed time: 2 hours 45 minutes – Do a good job!

Results will be posted by June 26th Mark registration: Friday, June 28th

Solution

Problem 1

1.1

Capacitor C_1 is obviously in series with the rest of the circuit; we can then remove it and consider it later for simplicity. We apply then a test voltage V_T at the node after C_1 and evaluate the current provided. The current flowing through the lower $R_1 - C_2$ series connection is

$$I_1 = V_T \frac{sC_2}{1+sC_2R_1} \Rightarrow V^+ = I_1 R_1 = V_T \frac{sC_2R_1}{1+sC_2R_1} = V^-.$$

But V^- is also equal to the output voltage of the OA, as no current flows through the upper resistor R_1. The current flowing in R_2 is then

$$I_2 = \frac{V_T - V^-}{R_2} = \frac{V_T}{R_2}\left(1 - \frac{sC_2R_1}{1+sC_2R_1}\right) = \frac{V_T}{R_2}\frac{1}{1+sC_2R_1},$$

leading to a total current provided by the voltage source equal to

$$I = I_1 + I_2 = V_T \frac{sC_2}{1+sC_2R_1} + \frac{V_T}{R_2}\frac{1}{1+sC_2R_1} = \frac{V_T}{R_2}\frac{1+sC_2R_2}{1+sC_2R_1}.$$

The total impedance is then

$$Z = \frac{V_T}{I} + \frac{1}{sC_1} = R_2 \frac{1+sC_2R_1}{1+sC_2R_2} + \frac{1}{sC_1} = \frac{1+s(C_1+C_2)R_2 + s^2C_1C_2R_1R_2}{sC_1(1+sC_2R_2)}.$$

The poles are at zero and $f_p = 1/(2\pi C_2R_2) \approx 2.4$ kHz. Note that the zeros are complex! Approximate solutions would return $f_L \approx 1/(2\pi(C_1+C_2)R_2) \approx 215$ Hz and $f_H \approx (C_1+C_2)/(2\pi C_1C_2R_1) \approx 46$ Hz, that cannot be correct! We have then $f_z = 1/(2\pi\sqrt{C_1C_2R_1R_2}) = 100$ Hz. The Bode plot is shown in Fig. 1 (left).

1.2

To compute G_{loop}, we open the circuit at the OA output and apply the test voltage, obtaining:

$$V^+ = V_T \frac{R_1}{R_1 + R_2 + \dfrac{1}{sC_2}} = V_T \frac{sC_2R_1}{1+sC_2(R_1+R_2)}$$

and $V^- = V_T$. This leads to

$$V_o = A(s)(V^+ - V_-) = A(s)\left(\frac{sC_2R_1}{1+sC_2(R_1+R_2)} - 1\right) \Rightarrow G_{loop} = -A(s)\frac{1+sC_2R_2}{1+sC_2(R_1+R_2)},$$

with a pole at 41 Hz and a zero at 2.4 kHz. Beyond those frequencies, $G_{loop} \approx -A(s)R_2/(R_1+R_2)$, meaning that the zero-dB crossing frequency is $f_{0dB} = GBWP\ R_2/(R_1+R_2) \approx 17$ kHz.

1.3

The scheme for the calculation is shown in Fig. 1 (right), where we have again neglected C_1, in which no current flows. The voltage at the OA NI input is

$$V^+ = V_o \frac{sC_2R_1}{1+sC_2R_1} + V_n = V^-,$$

and equal to the OA output voltage. We can then write:

$$V_o = V^- \frac{R_1 + \dfrac{1}{sC_2}}{R_1 + R_2 + \dfrac{1}{sC_2}} = V_n \frac{1+sC_2R_1}{1+sC_2R_2}.$$

The (non requested) rms output noise voltage is ($f_p = 1/2\pi C_2R_2 \approx 2.4$ kHz)

$$\overline{V_o^2} \approx S_V \frac{R_1^2}{R_2^2}(f_{0dB} - f_p) \approx (86\ \mu\text{V})^2.$$

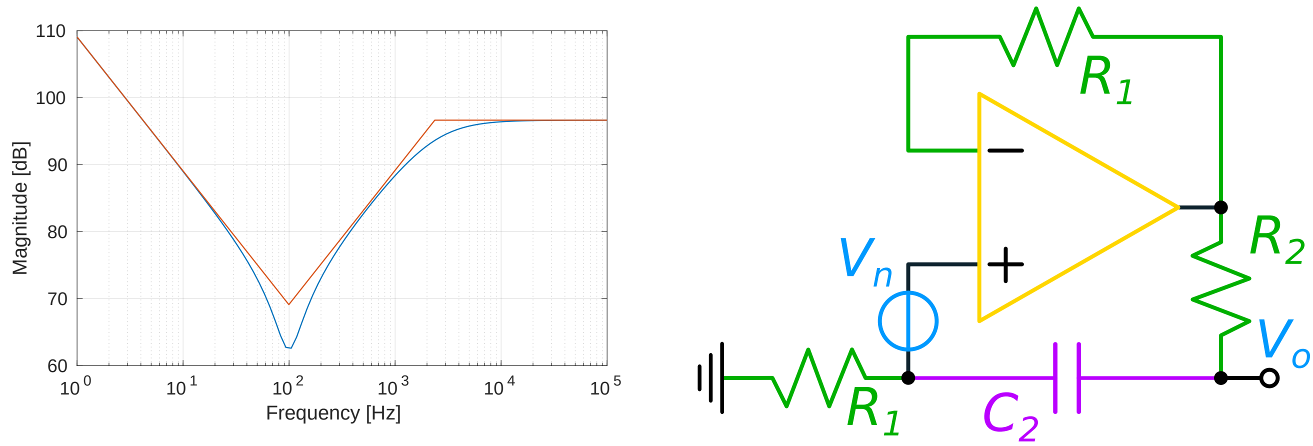


Figure 1: Left = Asymptotic (red) and real (blue) Bode plots of Z. Right = scheme for noise calculation.

1.4

In the previous point we have computed the open-circuit output voltage, while the output impedance Z was evaluated in #1.1. Converting this into a Norton equivalent circuit, we immediately get

$$I_o = \frac{V_o}{Z} = V_n \frac{sC_1(1 + sC_2R_1)}{1 + s(C_1 + C_2)R_2 + s^2C_1C_2R_1R_2}.$$

Problem 2

2.1

The value of S/N at the output of the amplifier is

$$\frac{S}{N} = \frac{A}{\sqrt{S_I(\pi/2)BW}} > 10 \Rightarrow BW < \frac{2A^2}{100\pi S_I} \approx 90 \text{ kHz}.$$

As the signal is a current, the LPF is an $R - C$ *parallel* (not series!) and its component values are:

$$R = \frac{1}{AG} \approx 11 \text{ k}\Omega \qquad C = \frac{1}{2\pi BWR} > 160 \text{ pF}.$$

Note that the associated time constant is $RC \approx 1.7$ μs, which means that in reality the maximum signal will not reach the amplitude A, but rather $A(1 - e^{T/RC}) = 0.95A$ and S/N is slightly degraded. the full solution for $S/N = 10$ leads to $BW \approx 72$ kHz (time constant of about 2.2 μs), but in this case the measured signal is reduced to $0.9A = 270$ nA.

2.2

Labelling $\lambda = S_I/2$ the bilateral PSD, the optimum value of S/N is

$$\frac{S}{N} = \frac{A}{\sqrt{\lambda}}\sqrt{\int x^2(t)dt} = \frac{A}{\sqrt{\lambda}}\sqrt{T + \int_0^{T_f}\left(1 - \frac{t}{T_f}\right)^2 dt} = \frac{A}{\sqrt{\lambda}}\sqrt{T + \frac{T_f}{3}} = A\sqrt{\frac{4T}{3\lambda}} \approx 13.7.$$

2.3

If k is very large, we can neglect the flat top of the pulse and consider a right triangular signal. The non-stationary white noise contribution at the end of the integration window is then

$$\overline{n_{ns}^2(T_G)} = G^2 \int_0^{T_G} \lambda(\alpha) w^2(T_G, \alpha) d\alpha = \lambda_{ns} G^2 \frac{T_G}{2}\left(2 - \frac{T_G}{T_f}\right),$$

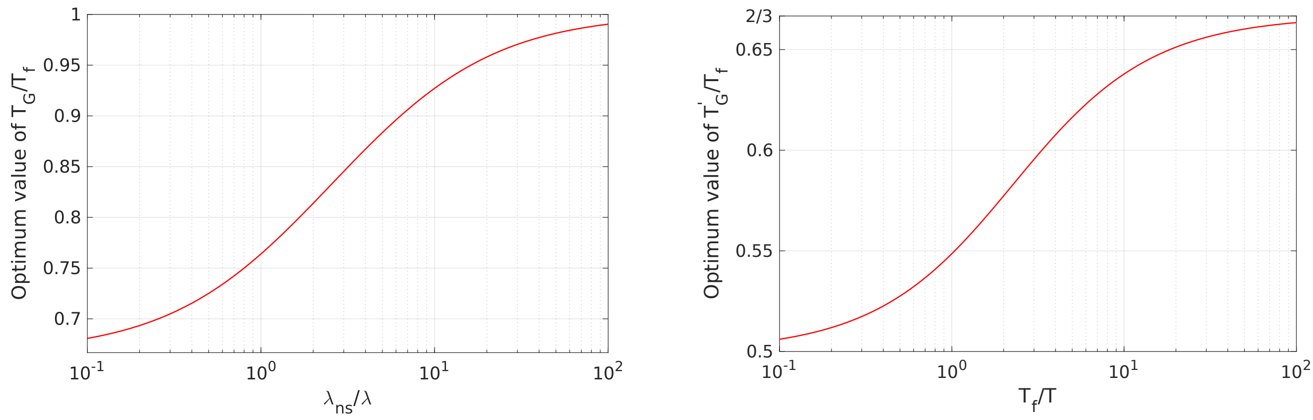


Figure 2: Left = optimum value of T_G/T_f as a function of the ratio of the noise PSDs. Right = optimum integration time over the falling edge as a function of T_f/T.

where G is the GI gain and $\lambda_{ns} = S_{ns}/2$. This leads to an expression for S/N equal to

$$\left(\frac{S}{N}\right) = A\frac{\frac{T_G}{2}\left(2-\frac{T_G}{T_f}\right)}{\sqrt{\lambda T_G + \lambda_{ns}\frac{T_G}{2}\left(2-\frac{T_G}{T_f}\right)}} = A\sqrt{\frac{T_f}{2\lambda}}\frac{x(2-x)}{\sqrt{2x+\frac{\lambda_{ns}}{\lambda}x(2-x)}},$$

where $x = T_G/T_f$. For the case of white stationary noise only, we know that the optimum value of x is $2/3$. We also know from theory that for non-stationary noise with PSD proportional to the signal, the optimum integration window extends over the whole signal, *i.e.*, $x = 1$. In the general case it is then $0.67 < x < 1$. For $\lambda_{ns} = \lambda$ and – say – $x = 0.75$, the second term is about 0.6 and we get $S/N \approx 5\sqrt{k}$. Fig. 2 (left) shows the optimum value of x as a function of λ_{ns}/λ.

2.4

The gate time will extend over the falling part of the pulse, so we label $T_G = T + T'_G$, and obtain

$$\left(\frac{S}{N}\right) = A\frac{T+\frac{T'_G}{2}\left(2-\frac{T'_G}{kT}\right)}{\sqrt{\lambda(T+T'_G)}} \Rightarrow \left(\frac{S}{N}\right)^2 = \frac{A^2T}{\lambda}\frac{\left[1+\frac{kx}{2}(2-x)\right]^2}{1+kx},$$

where $x = T'_G/kT$ lies between 0 and 1. For small values of k we neglect the term in k^2 at the numerator, obtaining

$$\left(\frac{S}{N}\right)^2 \approx \frac{A^2T}{\lambda}\frac{1+kx(2-x)}{1+kx} = \frac{A^2T}{\lambda}\left[1+k\frac{x-x^2}{1+kx}\right].$$

For small values of k we can now neglect the k term at the denominator and note that the maximum of $x - x^2$ is at $x = 0.5$. The optimum integration window extends to one half of the falling edge of the pulse!

As a reference, if one wishes to maximize S/N in the general case, it is wise to express the x-dependent part of it as

$$\frac{\left[1+\frac{kx}{2}(2-x)\right]^2}{1+kx} = \frac{\left[(1+kx)-\frac{k}{2}x^2\right]^2}{1+kx} = 1+kx-kx^2+\frac{k^2}{4}\frac{x^4}{1+kx}.$$

Zeroing its derivative, we obtain

$$1-2x+k\frac{x^3}{1+kx}-\frac{k^2}{4}\frac{x^4}{(1+kx)^2} = 0 \Rightarrow 3k^2x^4+4k(1-2k)x^3+4k(k-4)x^2+8(k-1)x+4 = 0.$$

For $k = 1, x = (\sqrt{7}-1)/3 \approx 0.55$; for $k = 2, x = 1/\sqrt{3} \approx 0.58$. The solution is shown in Fig. 2 (right).