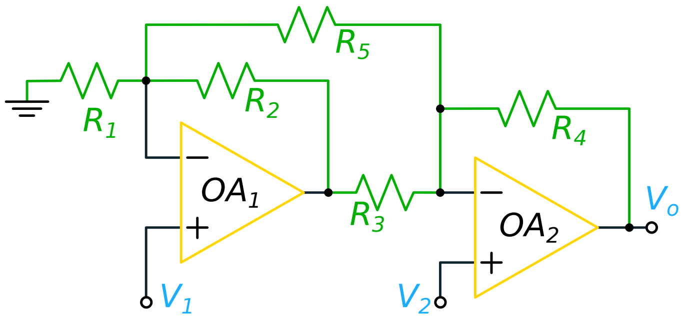
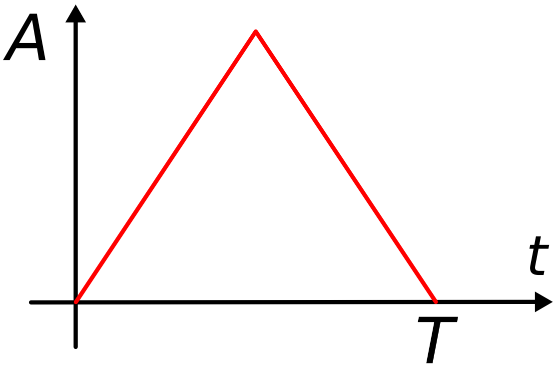
Electronics February 14 2025 Prof. A. Spinelli

For a correct evaluation, please write your answers in a readable way; thank you





Solving six points correctly gives you 30/30

Problem 1

The scheme in the left figure is a two-OA instrumentation amplifier. Parameter values are $R_1 = 20$ kΩ, $R_2 = R_3 = 2$ kΩ, $R_5 = 450$ Ω. The OAs have $A_0 = 100$ dB and $GBWP = 1$ MHz.

1. Find the value of R_4 that allows to reject the common mode.
2. Evaluate the differential gain.
3. Compute the output rms noise voltage due to the equivalent current noise source of one of the amplifiers, $\sqrt{S_I} = 2$ pA/$\sqrt{\text{Hz}}$. Consider the other OA as ideal.
4. Evaluate the CMRR due to the finite OA gain (hit: consider only the most critical one).

Problem 2

A sensor outputs triangular current signals of amplitude $A \approx 10$ nA and width $T = 1$ ms, on top of a white noise with unilateral PSD $\sqrt{S_I} = 40$ pA/$\sqrt{\text{Hz}}$ (right figure).

1. Evaluate the maximum achievable S/N.
2. Consider now a gated integrator. Find the optimum integration window and compute the new S/N.
3. A sinusoidal interference $I_d \sin(\omega_d t + \phi)$ with $I_d = 10$ mA and $f_d = 300$ kHz is present at the GI input. Design an LPF to be placed before the GI to lower the interference output to 1/10 of the signal output.
4. Besides S_I, both signal and interference are also subjected to shot noise. Provide an estimate of the new value of S/N.

Allowed time: 2 hours 45 minutes – Do a good job!

Results will be posted by February 19th Mark registration: Friday, February 21st

Solution

Problem 1

1.1

Under common-mode bias, no current flows in R_5, that can be neglected. So, V_1 sees a non-inverting followed by an inverting amplification, while V_2 experiences a non-inverting transfer:

$$V_o = -V_1 \left(1 + \frac{R_2}{R_1}\right) \frac{R_4}{R_3} + V_2 \left(1 + \frac{R_4}{R_3}\right).$$

Setting $V_1 = V_2$ and $V_o = 0$ we obtain

$$\frac{R_2 R_4}{R_1 R_3} = 1 \Rightarrow R_4 = \frac{R_1 R_3}{R_2} = R_1 = 20 \text{ k}\Omega.$$

1.2

We now consider $V_2 = -V_1 = V_d/2$ and write the KCL at the inverting input pin of OA1:

$$\frac{V_d}{2R_1} + \frac{V_{o1} + V_d/2}{R_2} + \frac{V_d}{R_5} = 0 \Rightarrow V_{o1} = -V_d \left(\frac{1}{2} + \frac{R_2}{R_5} + \frac{R_2}{2R_1}\right),$$

where V_{o1} is the output voltage of OA1. The KCL at the inverting input of OA2 gives then the result (remember the condition set in 1.1):

$$\frac{V_o - V_d/2}{R_4} - \frac{V_d}{R_5} + \frac{V_{o1} - V_d/2}{R_3} = 0 \Rightarrow \frac{V_o}{V_d} = 1 + \frac{R_4}{R_3} + \frac{2R_4}{R_5} = 100.$$

1.3

The scheme for noise calculations is reported in Fig. 1 (left). We begin with OA1 and the noise current source placed at its NI input (which is at zero voltage). No current can flow in R_1 and R_5, and

$$V_{o1} = -I_{n1} R_2 \Rightarrow V_o = I_{n1} R_2 \frac{R_4}{R_3} = I_{n1} R_1.$$

To compute the loop gain of OA1, we see that R_1 and R_5 are in parallel, because of the OA2 virtual ground (see Fig. 1, right), leading to:

$$G_{loop} = -A(s) \frac{R_1 \parallel R_5}{R_2 + R_1 \parallel R_5} = -A(s) \frac{1}{5.54} \Rightarrow f_{0dB}^1 = \frac{GBWP}{5.54} = 180 \text{ kHz},$$

leading to

$$\overline{V_o^2} = S_I R_1^2 \frac{\pi}{2} f_{0dB}^1 \approx (21\ \mu\text{V})^2.$$

As for OA2, the ideal transfer is simply:

$$V_o = I_{n2} R_4,$$

as no current flows in R_3 and R_5. For the loop calculation, we must now consider that OA1 acts as an inverting amplifier with gain $-R_2/R_5$ (see scheme in Fig. 1, right), and we can now write the KCL, simpler than linear superposition in this case:

$$\frac{V^- - V_T}{R_4} + \frac{V^-}{R_5} + \frac{V^- + (R_2/R_5)V^-}{R_3} = 0 \Rightarrow V^- = \frac{V_T}{100} \Rightarrow G_{loop} = -\frac{A(s)}{100} \Rightarrow f_{0dB}^2 = \frac{GBWP}{100} = 10 \text{ kHz},$$

and

$$\overline{V_o^2} = S_I R_4^2 \frac{\pi}{2} f_{0dB}^2 \approx (5\ \mu\text{V})^2.$$

1.4

The amplifier to consider here is OA1, whose finite gain will affect V_{o1}, resulting in a non-exact cancellation at the OA2 output. Under common-mode bias, we neglect R_5 and write

$$V_{o1} = V_c \left(1 + \frac{R_2}{R_1}\right) \frac{1}{1 - 1/G_{loop}} = V_c \left(1 + \frac{R_3}{R_4}\right) \frac{1}{1 - 1/G_{loop}},$$

leading to

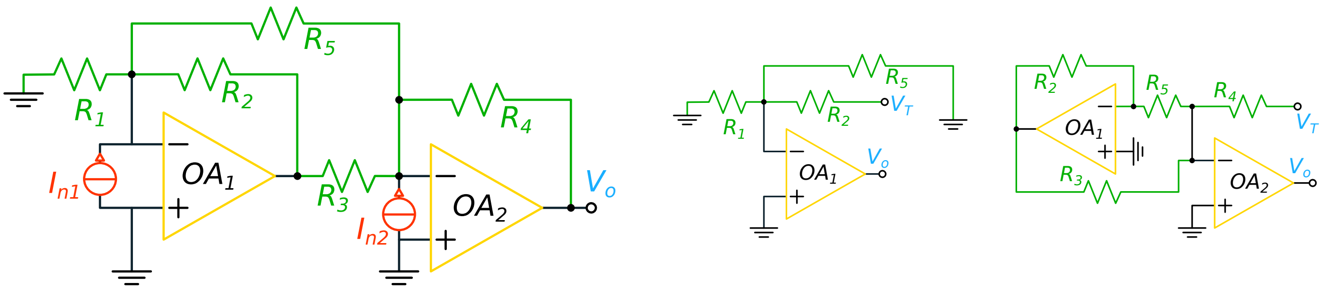


Figure 1: Left = Scheme for current noise calculation. Right = Schemes for loop gain calculations.

$$V_o = -\frac{R_4}{R_3} V_{o1} + V_c \left(1 + \frac{R_4}{R_3}\right) = \left(1 + \frac{R_4}{R_3}\right) \frac{1}{1 - G_{loop}} V_c = \frac{11}{1 - G_{loop}} V_c.$$

$G_{loop} = -A(s)/5.54$ has been already computed in #1.3, while the differential-mode gain is 100. Setting $A(s) = A_0/(1 + s\tau)$, we obtain

$$CMRR = \frac{A_{dm}}{A_{cm}} = \frac{100}{A_{cm}} = \frac{100}{11} \left(1 + \frac{A(s)}{5.54}\right) \approx 1.64 \frac{A_0 + 5.54 s\tau}{1 + s\tau},$$

which is shown in Fig. 2 (left). Note that the pole of $A(s)$ degrades the high-frequency CMRR even with perfect resistor matching.

Problem 2

2.1

The maximum S/N is obtained with an optimum filter, that in this case mimics the shape of the signal, *i.e.*,

$$w(t, \tau) \propto 1 - \frac{2|t|}{T} \qquad \forall |t| \leq T,$$

where the zero time reference has been moved to correspond to the signal maximum. We have then

$$\left(\frac{S}{N}\right)_{opt} = \frac{A}{\sqrt{S_I/2}} \sqrt{\int_{-T/2}^{T/2} \left(1 - \frac{2|t|}{T}\right)^2 dt} = A \sqrt{\frac{T}{S_I}} \sqrt{\frac{2}{3}} = 6.46.$$

2.2

We know from theory that the optimum integration time for the case of a right triangular signal is equal to 2/3 of the pulse width. The same is true here, as the signal is symmetric with respect to its maximum value, meaning that we can consider only the $t \geq 0$ part and multiply signal and mean square value of the noise by a factor of two. We then get:

$$V_o = 2A \int_0^{T/3} \left(1 - \frac{2t}{T}\right) dt = \frac{4}{9} GAT,$$

where G is the GI gain, and

$$\overline{V_o^2} = 2G^2 \frac{S_I}{2} \frac{T}{3} = G^2 S_I \frac{T}{3},$$

from which

$$\left(\frac{S}{N}\right) = A \sqrt{\frac{T}{S_I}} \frac{4}{3\sqrt{3}} = 6.09.$$

Please note that S/N is degraded by about 6% when moving from an optimum filter to a much simpler gated integrator.

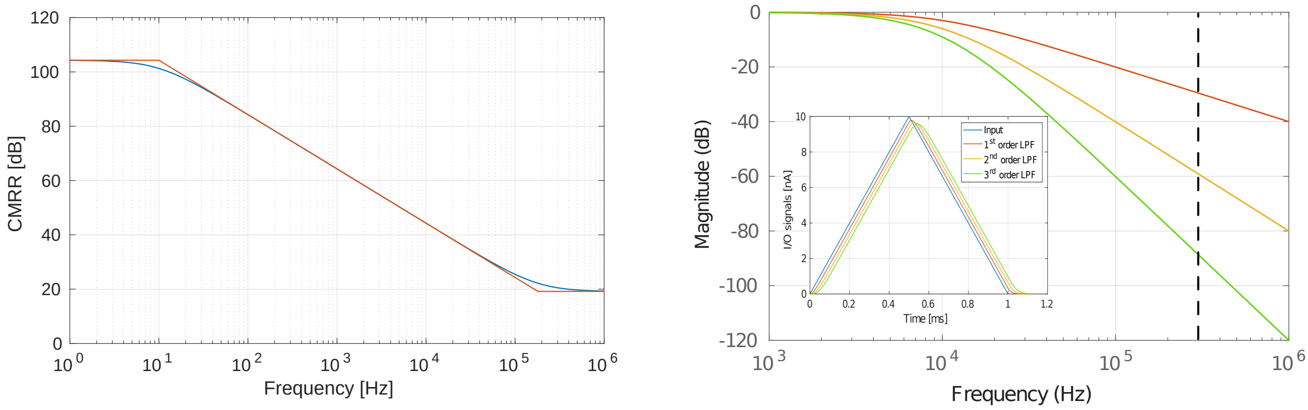


Figure 2: Left = CMRR as a function of frequenct. Right = Transfer function of a $1, 2$, and 3-pole LPF at 10 kHz and the attenuation at the interfering frequency. The inset shows the output signal for the cases considered.

2.3

The GI output signal due to the interference is

$$D_o = GI_d \int_{-T/3}^{T/3} \sin(\omega_d t + \phi) dt = I_d \frac{G}{\omega_d} \left(\cos(-\omega_d T/3 + \phi) - \cos(\omega_d T/3 + \phi)\right) < 2I_d \frac{G}{\omega_d},$$

where we have considered the worst case in which the sinusoidal terms of unit amplitude add in phase. This is much larger than the signal:

$$\frac{V_o}{D} = \frac{4\pi}{9} \frac{A}{I_d} f_d T = 4.2 \times 10^{-4}.$$

Given that the signal BW is about 1 kHz, we can set the LPF frequency to $f_p = 10$ kHz. However, this leads to a reduction in the interference amplitude by a factor of 30, which is not enough. Since we cannot lower f_p without affecting the signal, the only option is to increase the order of the filter! A second-order filter provides $30^2 = 900$, still not good enough! We need a *third-order* filter, giving an attenuation of $30^3 = 27000$, bringing the interference down to about 13% of the signal! The filter transfer function is shown in Fig. 2 (right); the inset shows the effect on the signal, which is largely unaffected (the peak reduction is about 5%).

2.4

The signal shot noise PSD has a maximum value of

$$S_s = 2qA = \left(56 \text{ fA}/\sqrt{\text{Hz}}\right)^2 \ll S_I,$$

and can be neglected. The maximum interference shot noise is instead

$$S_d = 2qI_d = \left(56 \text{ pA}/\sqrt{\text{Hz}}\right)^2,$$

and must be accounted for. We can consider again the worst case, taking the maximum value of the shot noise PSD, obtaining:

$$\left(\frac{S}{N}\right) = A\sqrt{\frac{T}{S_I + S_d} \frac{4}{3\sqrt{3}}} = 3.54.$$