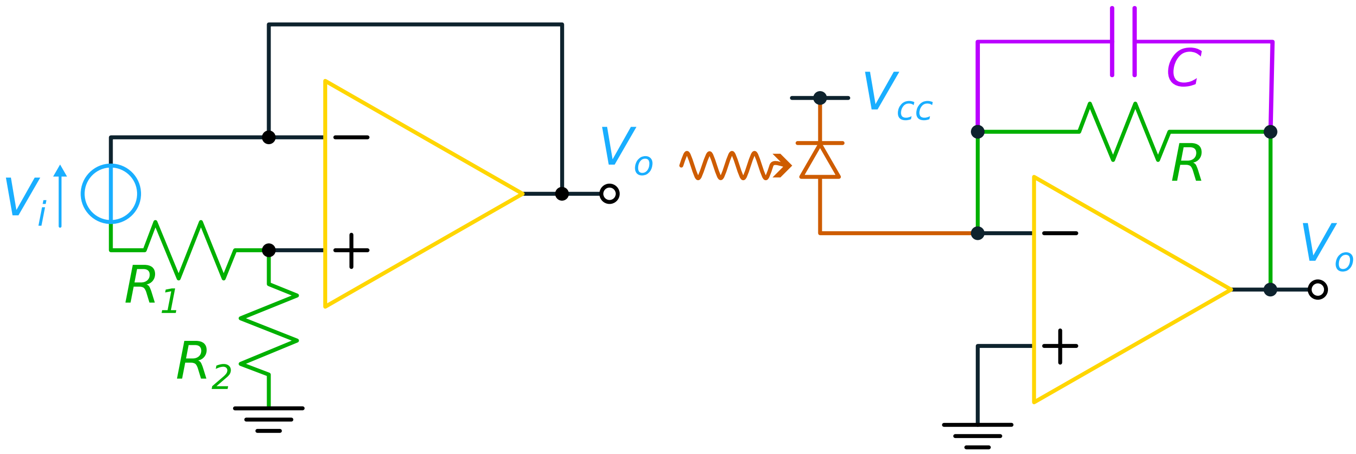
Electronics   January 14 2026   Prof. A. Spinelli

For a correct evaluation, please write your answers in a readable way; thank you



Solving six points correctly gives you 30/30

Problem 1

The scheme in the left figure is an amplifier for floating signals. The OA has $GBWP = 50$ MHz. Other parameter values are $R_1 = 20$ kΩ, $R_2 = 200$ kΩ.

1. Compute the ideal and loop gains.
2. The OA has an input impedance represented by the parallel of $R_i = 10$ MΩ and $C_i = 5$ pF. Check the stability and compensate if necessary, *without changing the value of the resistors.*
3. Compute the output rms noise voltage due to the equivalent noise sources of the OA, $\sqrt{S_V} = 20\ \text{nV}/\sqrt{\text{Hz}}$, $\sqrt{S_I} = 2\ \text{pA}/\sqrt{\text{Hz}}$ and resistors ($4k_BT \approx 1.646 \times 10^{-20}$ J).
4. How can we change the circuit so that the output voltage is also floating rather than being referred to ground?

Problem 2

The scheme in the right figure is a TI stage used to amplify the (current) signal coming from a photodiode. The OA has $\sqrt{S_V} = 10\ \text{nV}/\sqrt{\text{Hz}}$ and $\sqrt{S_I} = 1\ \text{pA}/\sqrt{\text{Hz}}$. The photodiode can be represented as a current source with pulses $I = Q\delta(t)$ at a maximum frequency of 10 MHz.

1. Evaluate the (single-pulse) output signal and find values of R, C, and $GBWP$ in order to get a gain of 80 dB.
2. A single sampling of the output is taken. What is the MDS (*i.e.*, the minimum detectable value of Q)?
3. A GI is now used after the amplifier. Evaluate the improvement in the MDS with respect to the previous case. Consider the limiting cases of very short and long T_G.
4. Derive the *correct* expression for the MDS in the previous case. As usual, remember that $\int_0^x te^{-t}dt = 1 - e^{-x} - xe^{-x}$.

Allowed time: 2 hours 45 minutes – Do a good job!

Results will be posted by January 16$^{\text{th}}$ Mark registration: January 21$^{\text{st}}$

Solution

Problem 1

1.1

As the differential input voltage of the OA is zero, the current flowing in R_1 is

$$I = \frac{V_i}{R_1},$$

that also flows in R_2. It follows that

$$V_o = V^+ = V^- = -IR_2 = -\frac{R_2}{R_1}V_i.$$

To evaluate G_{loop}, we turn off V_i and cut the loop at the OA output, obtaining

$$G_{loop} = -A(s)\frac{R_1}{R_1 + R_2},$$

meaning that $f_{0dB} = GBWP\ R_1/(R_1+R_2) = GBWP/11 = 4.5$ MHz. Note that the circuit behaves exactly as an inverting amplifier with a gain of -10 for signals referred to ground.

1.2

Resistor R_i is now in parallel with R_1 and can be neglected. The effect of C_i is to add a pole given by the resistance "seen" by the capacitor, *i.e.*

$$G_{loop} = -A(s)\frac{R_1}{R_1 + R_2}\frac{1}{1 + sC_i(R_1 \parallel R_2)}.$$

The additional pole is located at 1.75 MHz, which is lower than f_{0dB}, meaning that the circuit is unstable. A simple solution for the compensation is to act similarly to the standard amplifier case, and add a compensation capacitor in parallel to R_2. As discussed in the class, if we pick $C_cR_2 = C_iR_1 \Rightarrow C_c = 0.5$ pF we end up with a pole-zero cancellation and G_{loop} remains the same as in #1.1. This of course limits the signal bandwidth to $1/(2\pi C_cR_2) \approx 1.6$ MHz.

1.3

The voltage noise of the OA induces again a current V_n/R_1 in R_1 and R_2, leading to an output voltage

$$V_o = V_n\frac{R_1 + R_2}{R_1}.$$

As for the current, it is easy to see that we can group all contributions into a single current source I_n connected in parallel to R_2 (see Fig. 1, left), having PSD

$$S_{I_n} = S_I + \frac{4k_BT}{R_1} + \frac{4k_BT}{R_2} \approx 4.9 \times 10^{-24}\ \mathrm{A^2/Hz}.$$

Such a current can flow only in R_2, leading to $V_o = I_nR_2$ and

$$S_{V_o} = S_V\left(\frac{R_1 + R_2}{R_1}\right)^2 + S_{I_n}R_2^2 = 4.84 \times 10^{-14} + 1.9 \times 10^{-13} = 2.44 \times 10^{-13}\ \mathrm{V^2/Hz}.$$

With the compensation just discussed, we have $f_{0dB} = 4.5$ MHz, which means

$$\overline{V_o^2} = S_{V_o}\frac{\pi}{2}f_{0dB} = (1.32\ \mathrm{mV})^2.$$

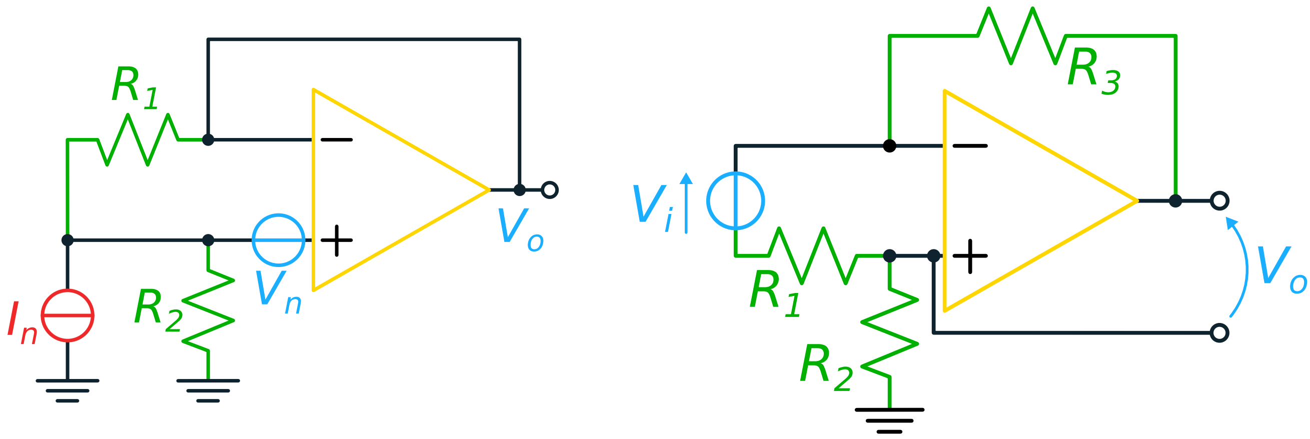


Figure 1: Left = Scheme for noise calculations. Right = Scheme for floating input and output signals.

1.4

If the output must float, we need to find a reference other than ground. The only available nodes here are the input pins of the OA. However, it V_o is measured between the output and the NI input of the OA, the result is zero, because of the short-circuit between them. We need then to add an extra resistor as in Fig. 1, right, that gives a gain $-R_3/R_1$.

However, please note that now the output impedance is no longer zero, and care must be applied if stages are cascaded.

Problem 2

2.1

The transfer function of the stage is simply

$$\frac{V_o(s)}{I(s)} = \frac{R}{1+sCR} \Rightarrow V_o(t) = \frac{Q}{C} e^{-t/\tau} u(t),$$

where $\tau = RC$. If the maximum pulse repetition rate is 10 MHz, the minimum time between consecutive pulses is 100 ns. We can then set, as an example, $RC = 10$ ns to avoid pile-up errors, *i.e.*, $R = 10$ kΩ (to set a gain of 80 dB) and $C = 1$ pF. The pole of the transfer function is now at $f_p = 1/(2\pi RC) = 16$ MHz.

It is easy to see that the loop gain is $G_{loop} = -A(s)$, so $f_{0dB} = GBWP$. We can now set $GBWP \geq 10 f_p =$ 160 MHz to avoid the effect of the extra pole.

2.2

With a single sample taken at $t = 0$ we get an output signal equal to Q/C. The output noise PSD is instead (see Fig. 2, left)

$$S_{Vo} = \left(S_I + \frac{4k_BT}{R}\right)\left|\frac{R}{1+sCR}\right|^2 + S_V,$$

leading to an rms value

$$\sqrt{\overline{V_o^2}} = \sqrt{\left(S_I + \frac{4k_BT}{R}\right)\frac{R^2}{4RC} + S_V\frac{\pi}{2}GBWP} \approx 178\ \mu\text{V}.$$

The MDS is then $Q_{min} = C\sqrt{\overline{V_o^2}} = 178$ aC, corresponding to about a thousand electrons.

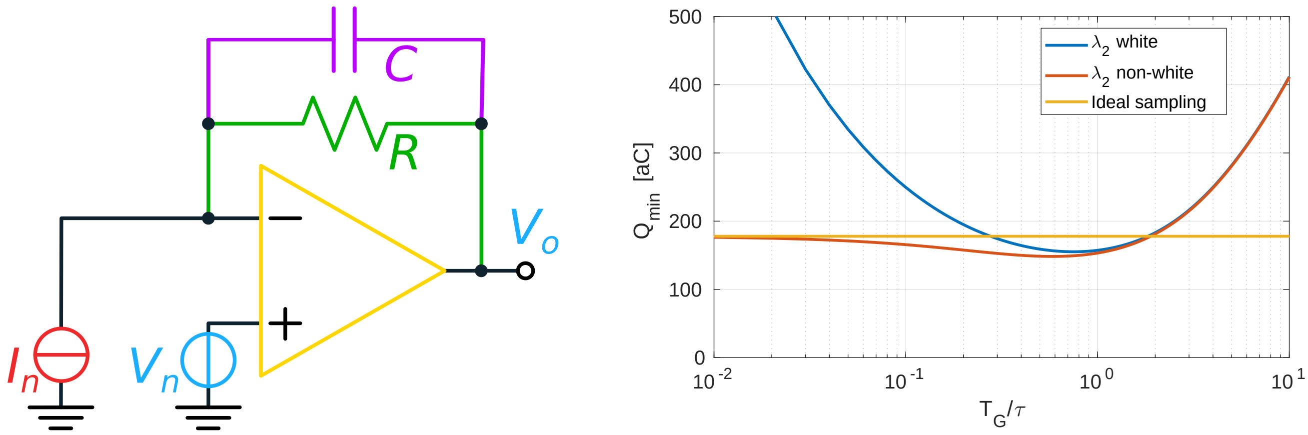


Figure 2: Left = Scheme for noise calculations. Right = Q_{min} as a function of T_G/τ.

2.3

For very short values of the integration time T_G, we obviously recover the previous result. At the other extreme, if T_G is much longer than $\tau = RC = 10$ ns, we can consider both noise contributions as white, and write

$$V_o = G\int_0^{T_G} \frac{Q}{C} e^{-t/\tau} dt \approx G\frac{Q\tau}{C} \qquad \sqrt{\overline{V_o^2}} = G\sqrt{(S_I R^2 + 4k_B TR + S_V)\frac{T_G}{2}},$$

from which, for $T_G = 100$ ns, we obtain

$$Q_{min} = \frac{C}{\tau}\sqrt{(S_I R^2 + 4k_B TR + S_V)\frac{T_G}{2}} \approx 428 \text{ aC}.$$

2.4

The noise S_V has a bandwidth equal to $GBWP = 160$ MHz and can be regarded as white. The other contributions are instead filtered by the RC LP filter and result in an autocorrelation

$$R_{nn}(\gamma) = \frac{\lambda_1}{2\tau} e^{-|\gamma|/\tau},$$

where $\lambda_1 = (S_I R^2 + 4k_B TR)/2 \approx 1.33 \times 10^{-13}$ V$^2/\sqrt{\text{Hz}}$. Recalling that the time correlation of the GI WF is a triangular function (see class notes), we get

$$\overline{V_o^2} = 2\lambda_1 \int_0^{T_G} G^2 T_G \left(1 - \frac{\gamma}{T_G}\right) \frac{e^{-\gamma/\tau}}{2\tau} d\gamma = \lambda_1 \frac{G^2 T_G}{\tau}\left[\tau\left(1 - e^{-T_G/\tau}\right) - \frac{\tau^2}{T_G}\int_0^{T_G/\tau} te^{-t} dt\right] =$$

$$= \lambda_1 G^2 T_G \left[1 - \frac{1 - e^{-T_G/\tau}}{T_G/\tau}\right] = \lambda_1 G^2 \tau \left[x - 1 + e^{-x}\right],$$

where $x = T_G/\tau$. The output signal is now $(GQ\tau/C)\,(1 - e^{-x})$, from which

$$Q_{min} = \frac{C}{\sqrt{\tau}} \frac{\sqrt{\lambda_1(x - 1 + e^{-x}) + \lambda_2 x}}{1 - e^{-x}},$$

where $\lambda_2 = S_V/2 = 5 \times 10^{-17}$ V^2/Hz. These results are shown in Fig. 2, right (blue curve). Note that there is a small interval of values $x < 2$ where Q_{min} is improved up to about 13%. Note also that as $x \to 0$ we get $Q_{min} \to \infty$ rather than converging to the sampling case. This happens because for very small T_G the GI bandwidth is very large and even λ_2 cannot be regarded as white. This means that its contribution is no longer $\lambda_2 \tau x$ but (see previous calculations)

$$\lambda_2 \tau \left(x - \frac{1 - e^{-10x}}{10}\right).$$

Plugging in this value, we obtain the correct limit (red curve).