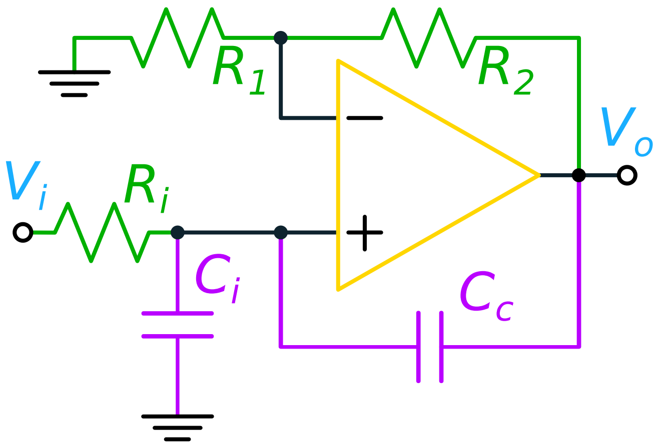
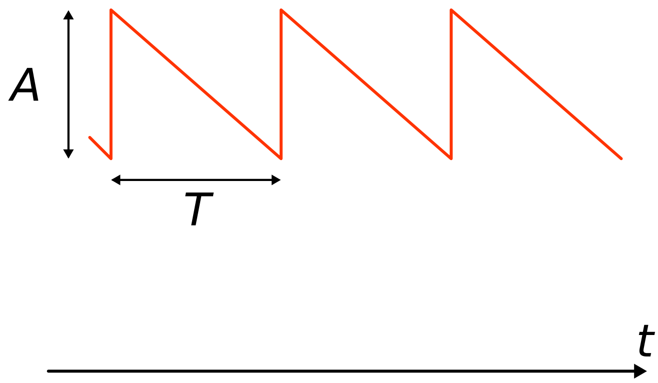
Electronics February 18 2026 Prof. A. Spinelli

For a correct evaluation, please write your answers in a readable way; thank you





Solving six points correctly gives you 30/30

Problem 1

The scheme in the left figure is an amplifier for high-impedance signals. The OA has $A_0 = 100$ dB and $GBWP = 1$ MHz. Other parameter values are $R_1 = R_2 = 10\ \text{k}\Omega$, $R_i = 1\ \text{M}\Omega$, $C_i = 10$ pF.

1. Compute the ideal gain. What condition must C_c satisfy?
2. Compute the loop gain.
3. Pick a (non-zero) value of C_c that grants a phase margin of at least 45°.
4. Compute the output noise voltage PSD due to the equivalent voltage noise source of the OA. Comment on the resulting transfer function.

Problem 2

We want to measure the amplitude $A \approx 10\ \mu$V of a sawtooth-like signal shown in the right figure. The period of the triangular wave is $T = 0.5$ s, and the signal is affected by a low-frequency offset with amplitude of about 2 V and by a white noise with unilateral PSD $S_V = 2 \times 10^{-12}\ \text{V}^2/\text{Hz}$.

1. A single sample is taken, having ideal samplers and filters (LP and HP) at our disposal. Find the sampling time and frequencies of the filters (if needed), and estimate the resulting S/N.
2. A single BA (plus filters, if needed) is now used to recover the signal. Find the relevant parameters to achieve $S/N = 10$.
3. A noise having triangular autocorrelation with $T_n = 0.1$ s is present *at the BA input*. Estimate the output noise. *Hint: the solution is easier if you consider a short* T_C.
4. The offset is now an interfering sinusoidal signal at 1 Hz. Provide an (approximated) estimate of its effect on S/N of the previous filters.

Allowed time: 2 hours 45 minutes – Do a good job!

Results will be posted by February 20th Mark registration: February 25th

Solution

Problem 1

1.1

The inverting input bias is simply

$$V^- = V_o \frac{R_1}{R_1 + R_2} = \frac{V_o}{k} = V^+,$$

where $k = 2$ is the NI stage gain. The NI input bias can be written via superposition between V_i and V_o:

$$V^+ = V_i \frac{\dfrac{1}{s(C_i + C_c)}}{\dfrac{1}{s(C_i + C_c)} + R_i} + V_o \frac{\dfrac{R_i}{1 + sC_iR_i}}{\dfrac{R_i}{1 + sC_iR_i} + \dfrac{1}{sC_c}}.$$

Equalling the expressions, we obtain

$$\frac{V_o}{V_i} = \frac{k}{1 + sR_i\left(C_i - (k-1)C_c\right)} = \frac{2}{1 + sR_i\left(C_i - C_c\right)}.$$

For the pole to be in the LHP, we must then have

$$C_i - (k-1)C_c > 0 \Rightarrow C_c < C_i \frac{R_1}{R_2} = C_i.$$

1.2

We cut the loop at the OA output and apply a test voltage V_T. We have then $V^- = V_T/k$ and

$$V^+ = V_T \frac{\dfrac{R_i}{1 + sC_iR_i}}{\dfrac{R_i}{1 + sC_iR_i} + \dfrac{1}{sC_c}} = V_s \frac{sC_cR_i}{1 + s(C_c + C_i)R_i},$$

from which

$$G_{loop} = -\frac{A(s)}{k} \frac{1 + s(C_i - (k-1)C_c)R_i}{1 + s(C_i + C_c)R_i} = -\frac{A(s)}{2} \frac{1 + s(C_i - C_c)R_i}{1 + s(C_i + C_c)R_i}.$$

1.3

We follow the approach discussed in Drill #4. As the zero will be located at f_{0dB}, we can neglect it for lower frequencies and obtain

$$G_{loop} \approx -\frac{A_0}{2} \frac{1}{(1 + s\tau)(1 + s(C_i + C_c)R_i)},$$

which means

$$f_{0dB} = \frac{1}{2\pi} \sqrt{\frac{A_0}{2\tau(C_i + C_c)R_i}} = \sqrt{\frac{GBWP}{4\pi(C_i + C_c)R_i}}.$$

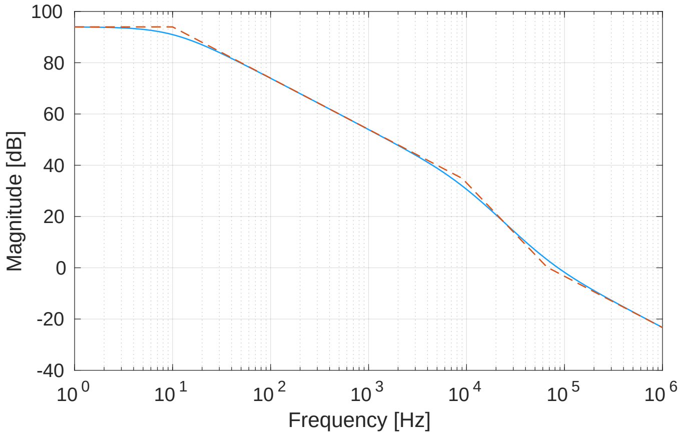
We now start with $C_c = 0$ and obtain $f_{0dB} \approx 89$ kHz. Setting the zero to this frequency, we get

$$\frac{1}{2\pi R_i(C_i - C_c)} = f_{0dB} \Rightarrow C_c = C_i - \frac{1}{2\pi R_i f_{0dB}} = 8.2 \text{ pF}.$$

Plugging this value in the expression for f_{0dB} we get about 66 kHz, which returns $C_c = 7.6$ pF, close enough to the first guess. Of course, we could have instead solved the full equation:

$$\frac{1}{2\pi} \sqrt{\frac{A_0}{2\tau(C_i + C_c)R_i}} = \frac{1}{2\pi} \frac{1}{R_i(C_i - C_c)} \Rightarrow C_c^2 - C_c\left(2C_i + \frac{2\tau}{A_0R_i}\right) + C_i^2 - \frac{2\tau}{A_0R_i}C_i = 0,$$

which gives $C_c = 7.63$ pF. The resulting loop gain is reported in Fig. 1 (left).



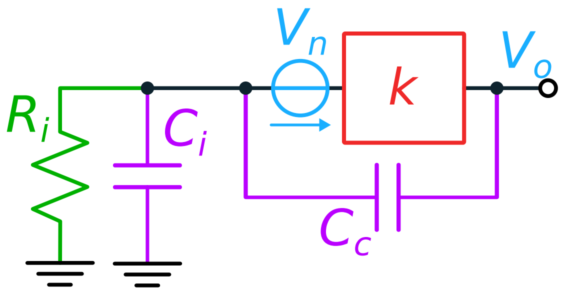


Figure 1: Left = Loop gain after compensation. Right = Scheme for noise calculation.

1.4

The scheme for noise calculation is shown in Fig. 1 (right), where the OA and $R_1 - R_2$ feedback is replaced by a stage with gain $k = 2$. At the input of the stage we have therefore a voltage V_o/k, which becomes $V_o/k - V_n$ at the $R_i - C_i$ node. We can then write:

$$\frac{V_o}{k} - V_n = V_o \frac{R_i \parallel \dfrac{1}{sC_i}}{R_i \parallel \dfrac{1}{sC_i} + \dfrac{1}{sC_c}} \Rightarrow V_o = V_n k \frac{1 + s(C_i + C_c)R_i}{1 + s(C_i - (k-1)C_c)R_i} = 2V_n \frac{1 + s(C_i + C_c)R_i}{1 + s(C_i - C_c)R_i}.$$

The TF has gain of 2 (6 dB), a zero at $f_z \approx 9$ kHz and a pole at $f_p \approx 66$ kHz, where the gain is about 15 (23 dB). However, f_p is exactly equal to f_{0dB}, meaning that we will have an additional pole there. The exact behavior is discussed in the Appendix.

Problem 2

2.1

Before sampling, we obviously need an HPF to remove the offset. As the frequency of the signal is 2 Hz, we can place the HPF pole at 0.2 Hz (for example, with $R = 350$ kΩ, $C = 2.2$ μF, so that $4k_BTR \ll S_V$). At its output, we have then the triangular waveform *with zero average value, i.e.*, with amplitude $\pm A/2$, which is the signal to be sampled.
Besides, we also need an LPF to filter out the white noise. For example, a time constant $T_F = T/10 = 5$ ms (*i.e.*, $BW \approx 32$ Hz, with $R = 5$ kΩ, $C = 1$ μF, so that $4k_BTR \ll S_V$) would give

$$\left(\frac{S}{N}\right) = \frac{A}{\sqrt{2\pi S_V BW}} \approx 0.5.$$

Note also that this expression is overestimated, because the LPF will also filter the leading edge of the signal, resulting in a difference smaller than A (actually equal to about $0.67A$ in this case). However, as a detailed calculation is not required, we will stick with this approximation.

2.2

With a single BA, we still need the HPF designed in #2.1. We can now make several choices for the gate time T_C. If we pick $T_C \ll 250$ ms in order to sample the maximum amplitude $A/2$, we can write

$$\left(\frac{S}{N}\right) = \frac{A}{2}\sqrt{\frac{4T_F}{S_V}} = 10 \Rightarrow T_F = \frac{100 S_V}{A^2} = 2 \text{ s}.$$

For example, for $T_C = 2$ ms, this means $N_{eq} = 2T_F/T_C = 2000$. If instead we choose to integrate over the entire time $T/2$ where the signal is positive, we must consider that the BA behaves like an LPF in the

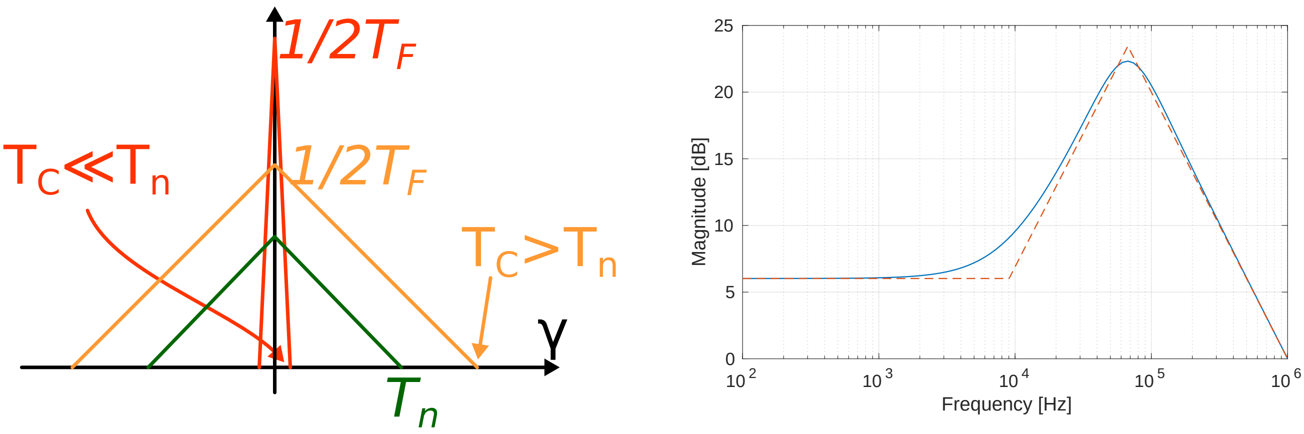


Figure 2: Left = BA WF (first peak) and noise autocorrelation. Right = Actual noise transfer (see Appendix).

equivalent time, and the output signal is the average input signal, *i.e.*, $A/4$, which leads to $T_F = 8$ s and $N_{eq} = 64$.

2.3

The time correlation of the BA weighting function is made up of a series of (nearly) triangular pulses with base of $2T_C$ and separated by T (see class notes). As $T > T_n$, only the first of such peaks (with amplitude $1/2T_F$, apart from the gain) gives a contribution. If $T_C \ll T_n$ the noise autocorrelation can be assumed as constant, and we have

$$\overline{n_o^2} = \int R_{xx}(\gamma) k_{w_{tt}}(\gamma) d\gamma \approx \overline{n_x^2} \int_{-T_C}^{T_C} k_{w_{tt}}(\gamma) d\gamma = \overline{n_x^2} \frac{T_C}{2T_F}.$$

If instead we choose – as before – $T_C = T/2 > T_n$, we need to evaluate the integral over the first peak of k_{ww}:

$$\overline{n_o^2} = \int R_{xx}(\gamma) k_{w_{tt}}(\gamma) d\gamma \approx \overline{n_x^2} \frac{1}{T_F} \int_0^{T_n} \left(1 - \frac{\gamma}{T_n}\right) \left(1 - \frac{\gamma}{T_C}\right) d\gamma = \overline{n_x^2} \frac{T_n}{2T_F} \left(1 - \frac{T_n}{3T_C}\right).$$

With the previous numbers, we get $\overline{n_o^2} = \overline{n_x^2}/2000$ for $T_C = 2$ ms ($T_F = 2$ s), and $\overline{n_o^2} = \overline{n_x^2}/200$ for $T_C = 250$ ms ($T_F = 8$ s).

2.4

We write the interfering signal as $D\cos(\omega t + \phi)$. When a single sample is taken, we have obviously, in the worst case:

$$\left(\frac{S}{N}\right) \approx \frac{A}{D}.$$

Another approximation could be to compute the *rms* value of D_{out}, which leads to a factor of $\sqrt{2}$; both are fine at this stage.

As for the BA case, we can note that the interference is synchronous with the sampling, that takes place every 0.5 s. This means that the samples have alternate sign. For small values of T_C, we can treat the disturb as constant, resulting in (with $x = T_C/T_F$):

$$D_{out} = D\cos(\omega t + \phi) x \left(1 - e^{-x} + e^{-2x} - e^{-3x} \ldots\right) = D\cos(\omega t + \phi) \frac{x}{1 + e^{-x}} \approx D\cos(\omega t + \phi) \frac{x}{2} \leq D\frac{x}{2}.$$

With the previous number, this means an output disturb amplitude $D\ x/2 = 5 \times 10^{-4} D$.

Appendix

To spice up the noise calculations, we discuss the actual transfer of the noise voltage source. With reference to Fig. 1, right, this can be simply achieved by considering the frequency dependence of the gain stage k. Since this is a NI amplifier, its closed-loop bandwidth will by $GBWP/k = 500$ kHz. By calling $\tau = 1/(2\pi \times 5 \times 10^5) \approx 160$ ns, we can then write

$$k(s) = \frac{2}{1+s\tau},$$

and add this to the expression for V_o, obtaining

$$\frac{V_o}{V_n} = k(s)\frac{1+s(C_i+C_c)R_i}{1+sR_i(C_i-(k(s)-1)C_c)} = 2\frac{1+s(C_i+C_c)R}{1+s(\tau+R(C_i-C_c))+s^2\tau R(C_i+C_c)},$$

which is shown in Fig. 2 (right), and where the existence of two poles can be seen. From Drill #1, the denominator can be written as

$$\frac{s^2}{\omega_p^2}+\frac{s}{Q\omega_p}+1 \Rightarrow \begin{cases} f_p = \dfrac{1}{2\pi}\dfrac{1}{\sqrt{\tau R(C_i+C_c)}} \approx 67 \text{ kHz} \\ Q = \dfrac{\sqrt{\tau R(C_i+C_c)}}{R(C_i-C_c)} \approx 0.87. \end{cases}$$

Since the Q factor is larger than 0.5, the poles at 67 kHz are indeed complex. To compute the rms noise voltage, we need just one more quantity, *i.e.*, the transfer function T at $s = j\omega_p$. Rather than proceeding with brute force calculation, we write T as

$$T(s) = 2\frac{(1+s\tau_z)}{1+\dfrac{s}{Q\omega_p}+\dfrac{s^2}{\omega_p^2}} \Rightarrow T(j\omega_p) = -2jQ(1+j\omega_p\tau_z) \Rightarrow |T(j\omega_p)| \approx 2Q\omega_p\tau_z = 13 \text{ (22 dB)}.$$

The -3 dB BW is then f_p/Q (see Appendix of Drill #6), leading to

$$\overline{V_o^2} = S_V\frac{\pi}{2}\frac{f_p}{Q}|T(j\omega_p)|^2.$$

Of course, we are neglecting the white contribution up to 10 kHz.