Electronics – 96032

POLITECNICO DI MILANO





Active filters

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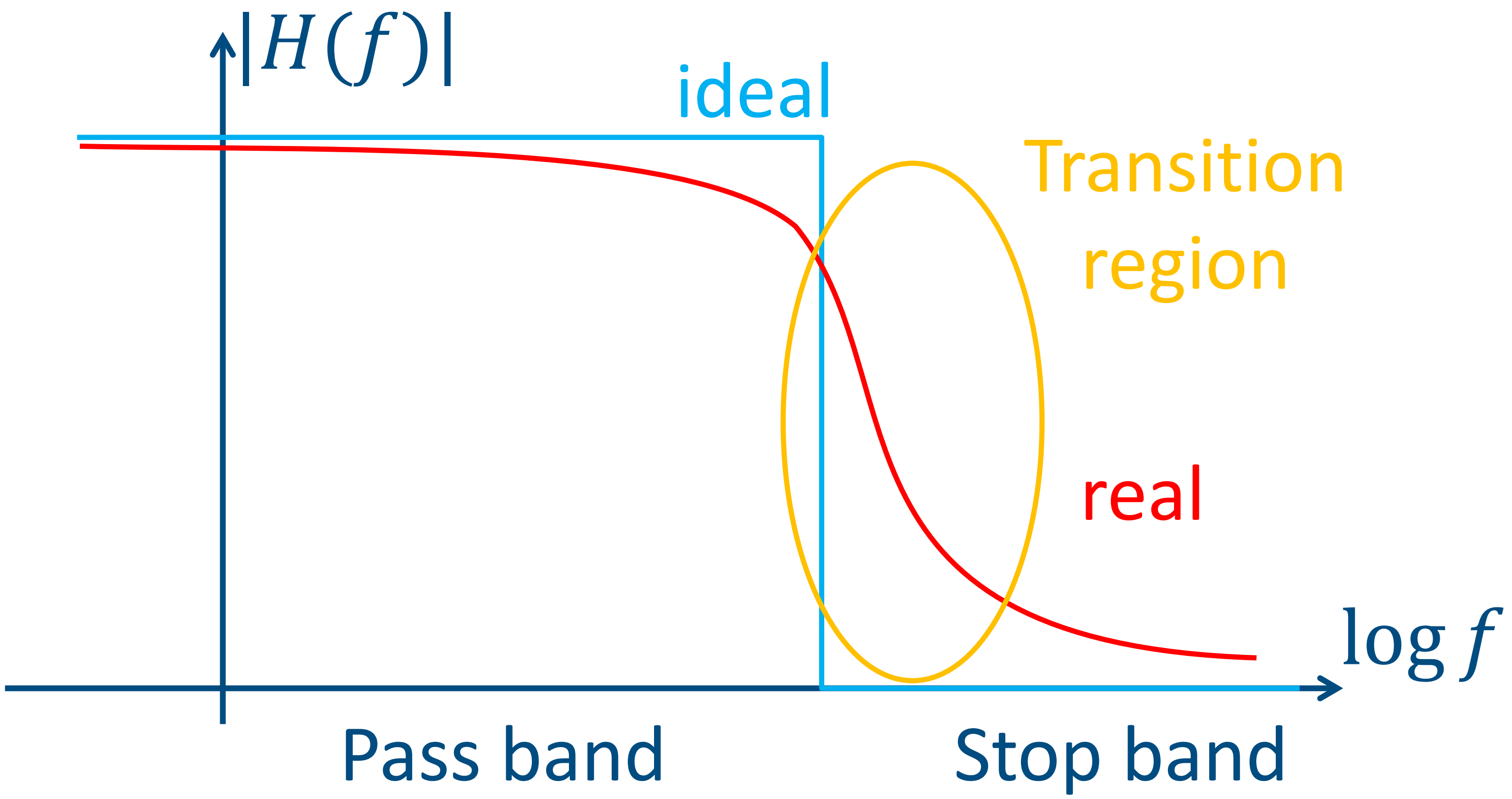
Disclaimer

Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes

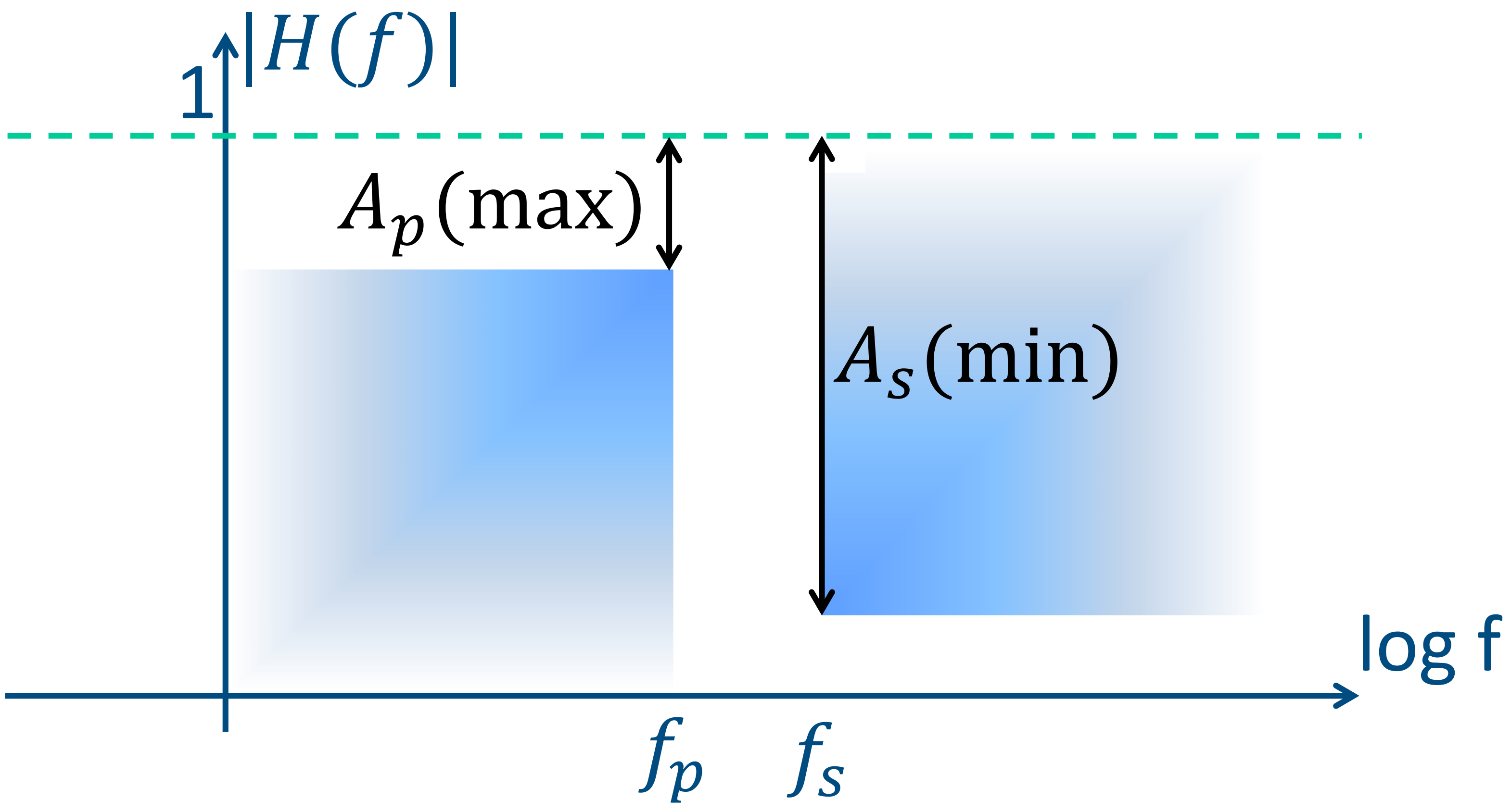
Active filters

- Filter networks made with active elements
- No need to use inductors or large resistors
- Easier to implement high-order filters
- Improved performance

Transfer function (LP case)



Typical design parameters



Butterworth LP filters (1930)

- Single-pole filter:

$$H = \frac{1}{1+s\tau} \Rightarrow |H| = \frac{1}{\sqrt{1+(s\tau)^2}} = \frac{1}{\sqrt{1+\left(\frac{f}{f_0}\right)^2}}$$

- Extension to n poles:

$$|H| = \frac{1}{\sqrt{1+\left(\frac{f}{f_0}\right)^{2n}}}$$

Poles and zeroes

$$1+\left(\frac{\omega}{\omega_0}\right)^{2n}=0$$

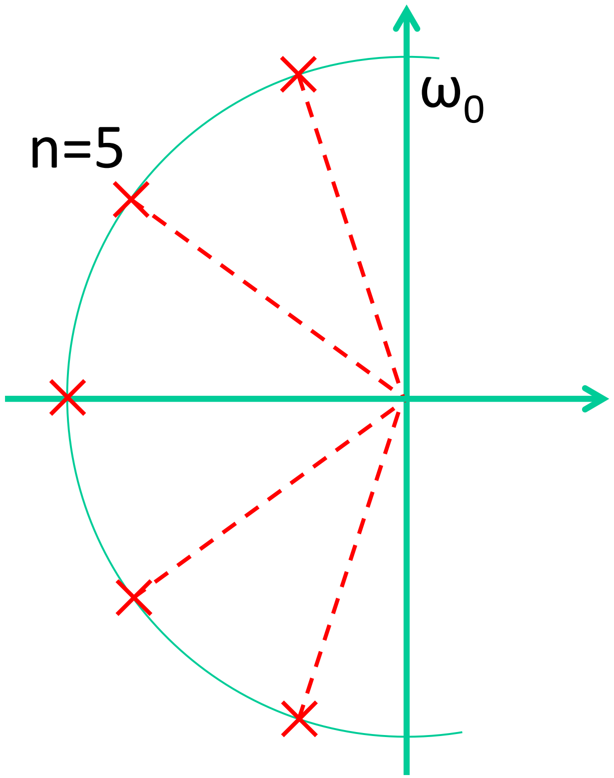
$$\omega=-js\Rightarrow s^{2n}=-1^{n-1}\omega_0^{2n}=\rho^{2n}e^{jn\theta}$$

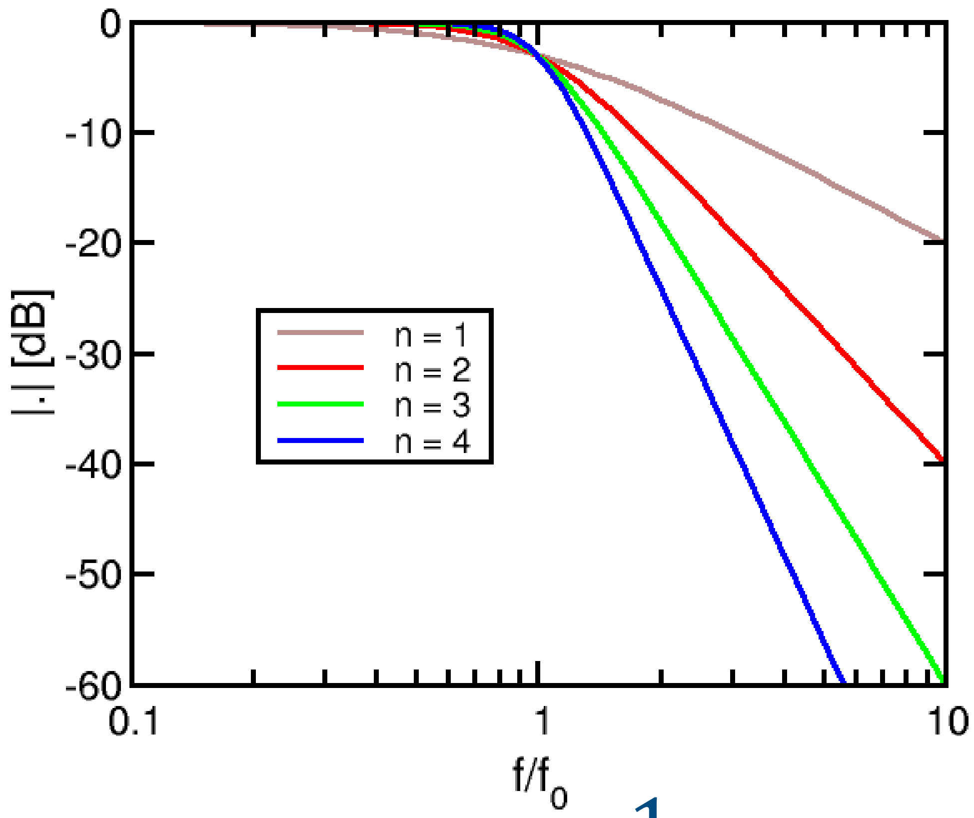
$$\rho=\omega_0$$

$$\theta=\frac{\pi}{2}+\frac{2k-1}{2n}\pi$$

where $k=1,\dots,n$. The remaining n poles are in RHP and solution only of the $|\cdot|^2$

Butterworth LP filters (1930)



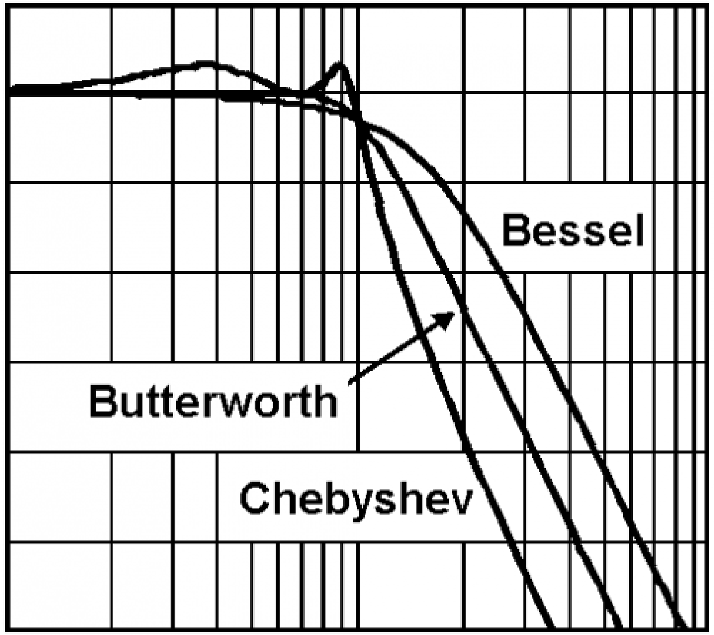


$$\theta = \frac{\pi}{2} + \frac{2k-1}{2n}\pi$$

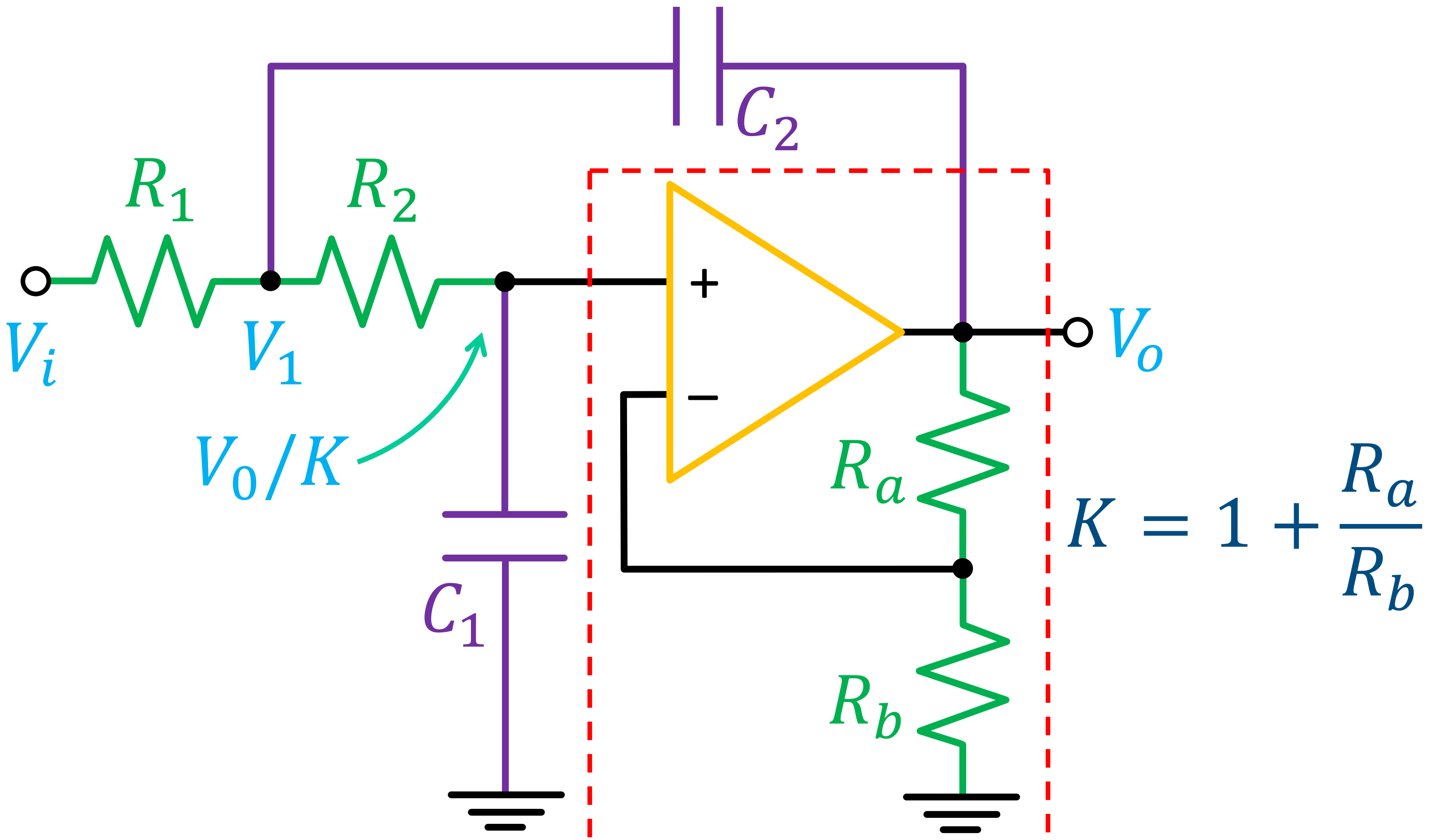
$$|H| = \frac{1}{\sqrt{1+(f/f_0)^{2n}}}$$

Notes

- Buttherworth filters correspond to a «maximally flat» approximation, with no ripples in the frequency response
- Attenuation at f_0 is 3 dB, independent of n
- Other solutions exist, that optimize attenuation or phase response



Sallen-Key LP filter (1955)



Transfer function

$$\begin{cases} sC_2(V_o - V_1) + \dfrac{V_i - V_1}{R_1} = V_1 \dfrac{sC_1}{1 + sC_1 R_2} \\ \dfrac{V_1}{1 + sC_1 R_2} = \dfrac{V_o}{K} \end{cases}$$

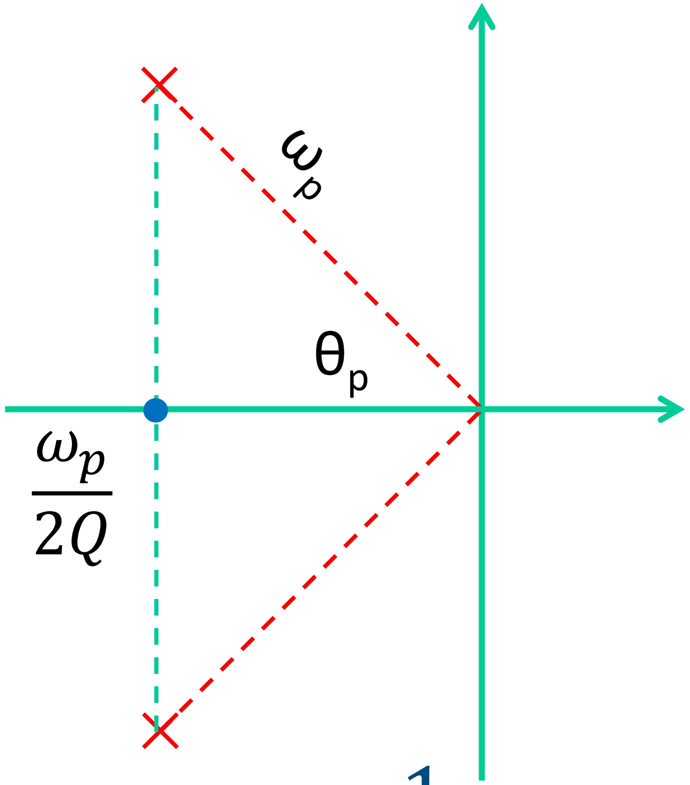
$$R_1 = R_2 = R$$
$$C_1 = C_2 = C$$

$$\frac{V_o}{V_i} = \frac{K}{(sRC)^2 + sRC(3-K) + 1} = \frac{K/(RC)^2}{s^2 + s\dfrac{3-K}{RC} + \left(\dfrac{1}{RC}\right)^2}$$

Filter design

$$RC = \frac{1}{\omega_p}; \quad K = 3 - \frac{1}{Q}$$

- Since $K \geq 1, Q \geq 0.5 \Rightarrow$ poles are always complex (or real coincident)
- K must be less than 3 to avoid instability

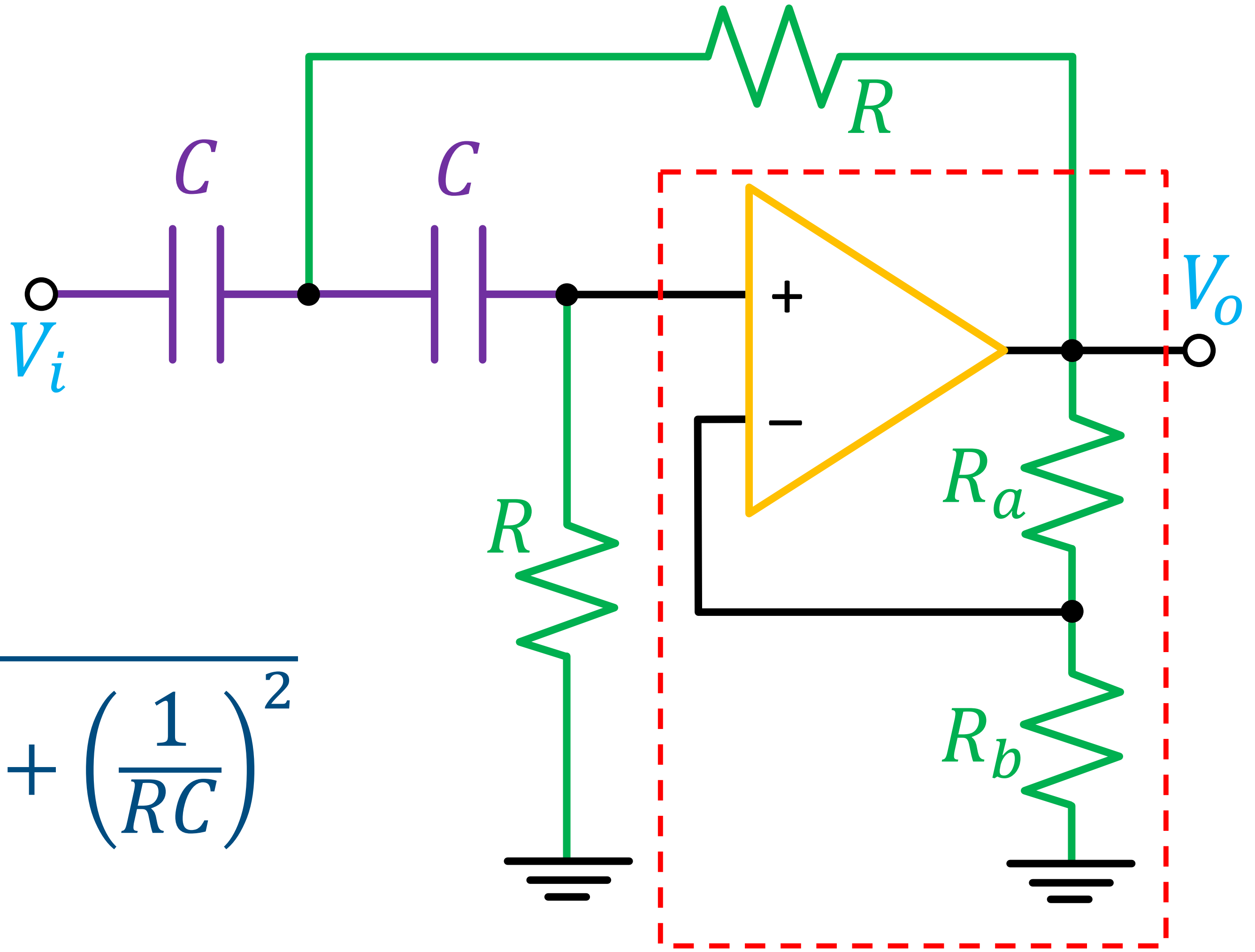


$$Q = \frac{1}{2\cos\theta_p}$$

Comments

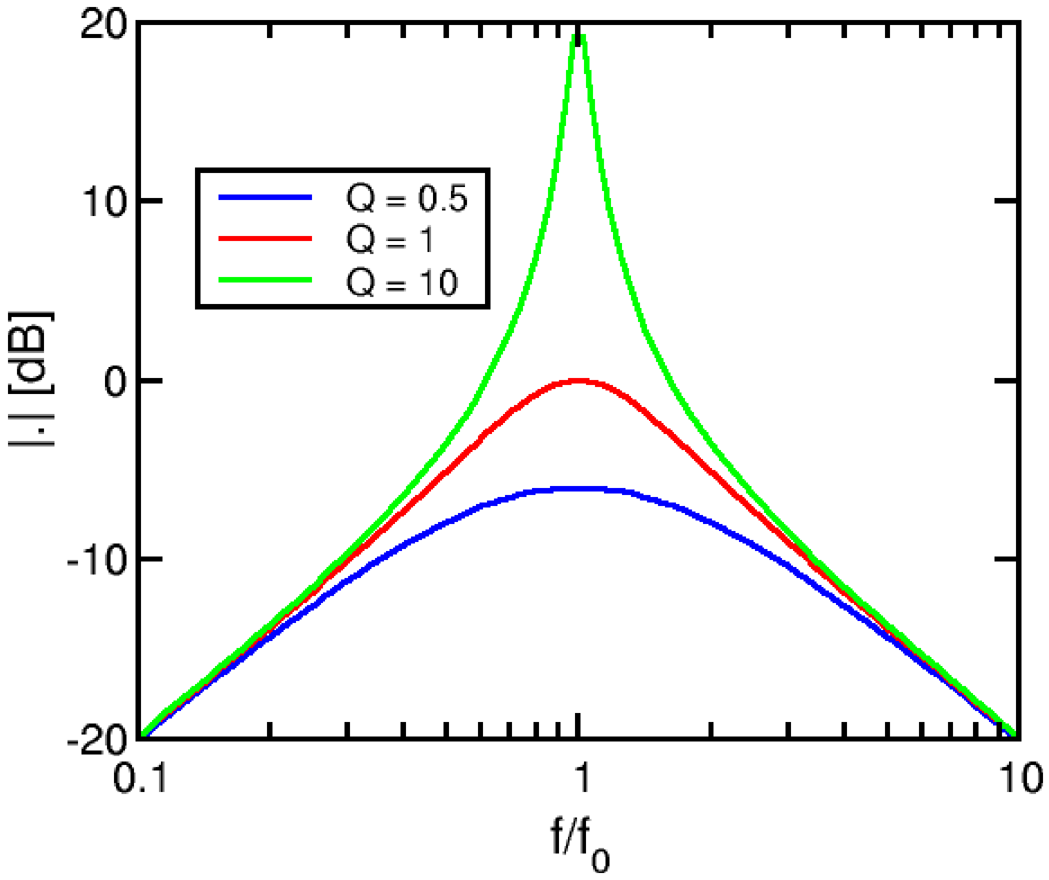
- SK cells can be cascaded for high-order filters
- Use OA with $GBWP \approx 10 - 30\ f_p$ for not affecting the filter response
- Also check the slew rate at f_p
- R_o will limit the attenuation in the stop band

High-pass SK filter ($R \leftrightarrow C$)



$$\frac{V_o}{V_i} = \frac{Ks^2}{s^2 + s\frac{3-K}{RC} + \left(\frac{1}{RC}\right)^2}$$

Band-pass filters



$$f_0 = \sqrt{f_H f_L}$$

$$Q = \frac{f_0}{BW} = \frac{f_0}{f_H - f_L}$$

Band-pass SK filter

$$f_0 = \frac{1}{2\pi RC}$$

$$Q = \frac{1}{3-k}$$

$$G_0 = \frac{k}{3-k}$$