Reflections on transmission lines

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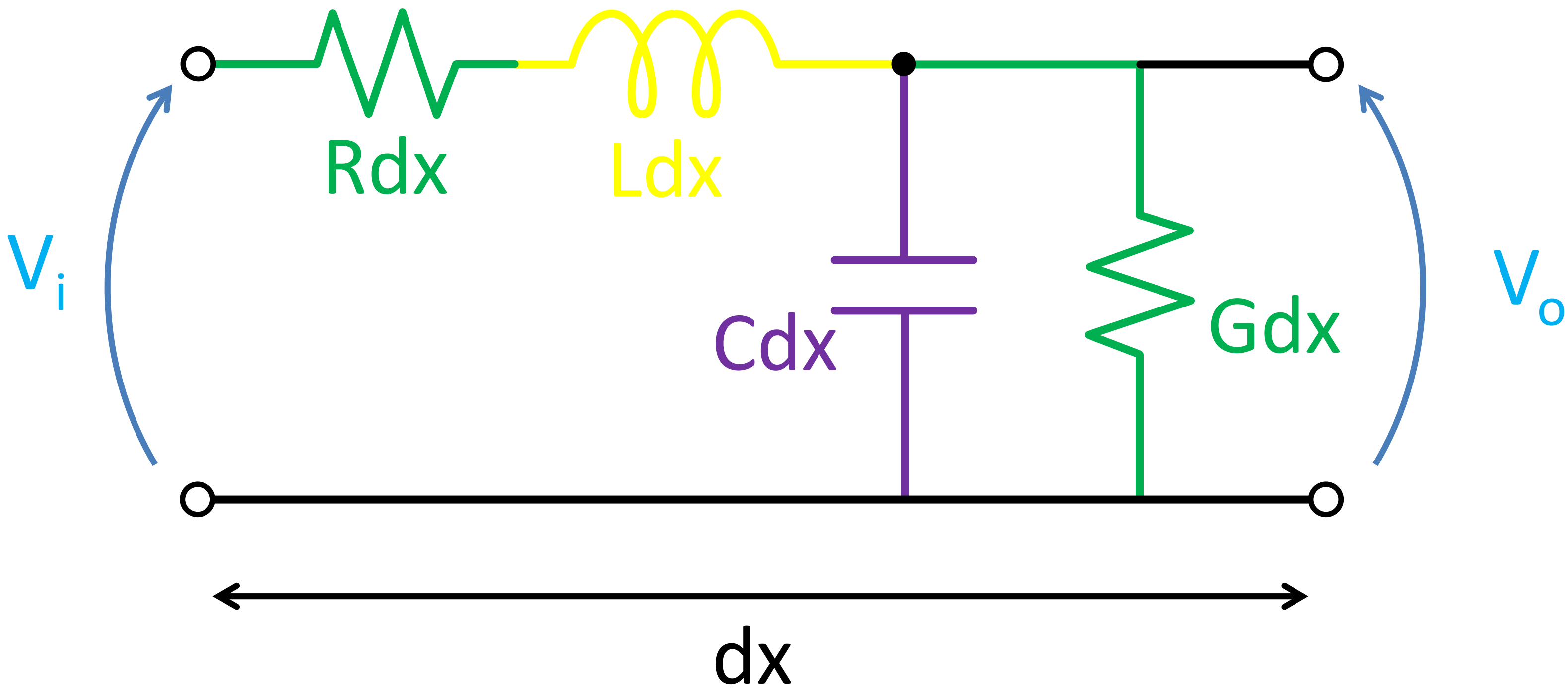
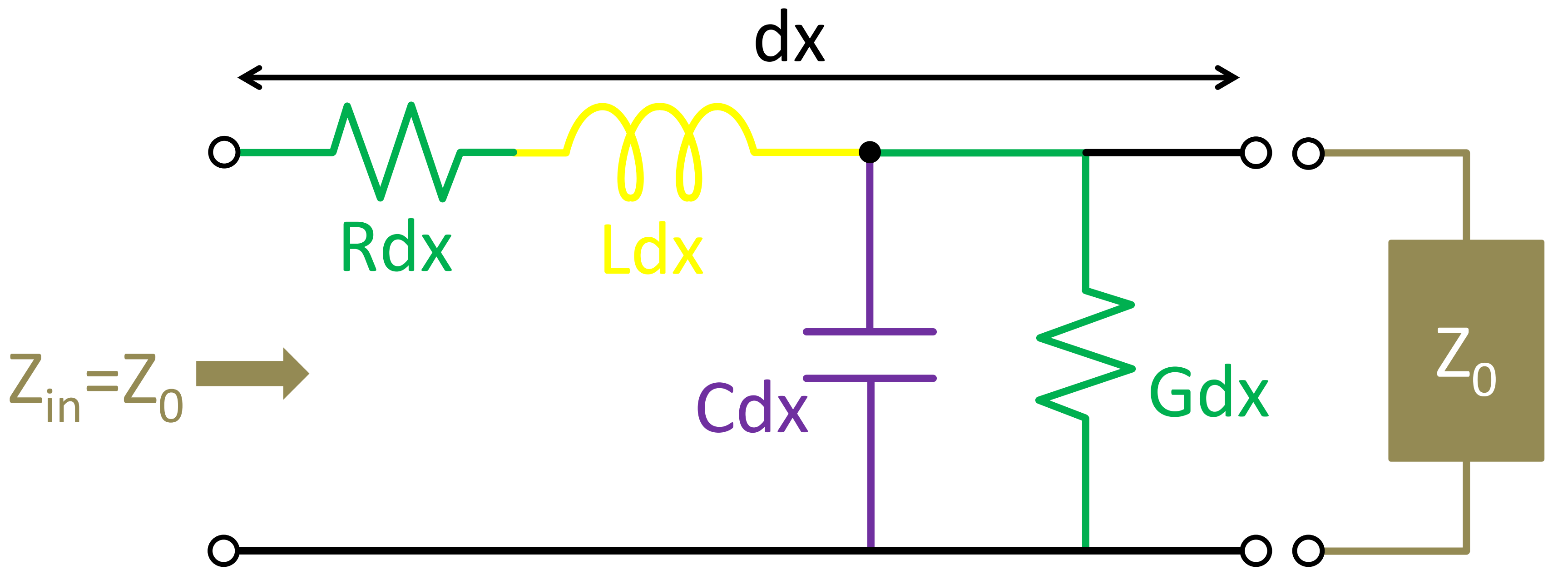
Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes

Transmission-line model



Characteristic impedance



$$Z_0 = Rdx + j\omega Ldx + Z_0 \| \left(\frac{1}{Gdx + j\omega Cdx} \right)$$

Calculations…

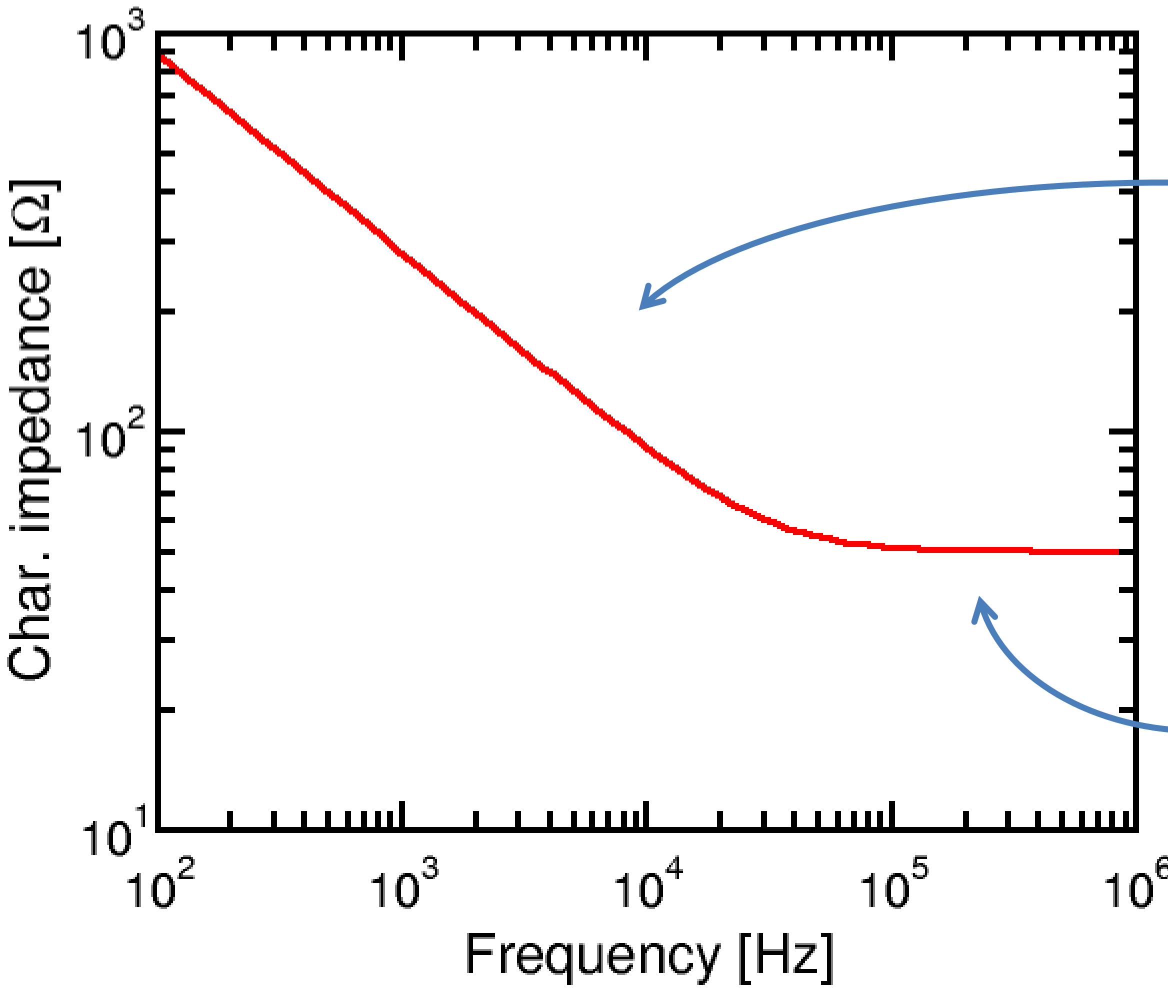
$$Z_0 = (R + j\omega L)dx + \frac{Z_0}{1 + Z_0(G + j\omega C)dx}$$

$$\cancel{Z_0} + Z_o^2(G + j\omega C)dx = (R + j\omega L)dx + Z_0(R + j\omega L)(G + j\omega C)dx^2 + \cancel{Z_0}$$

neglected

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

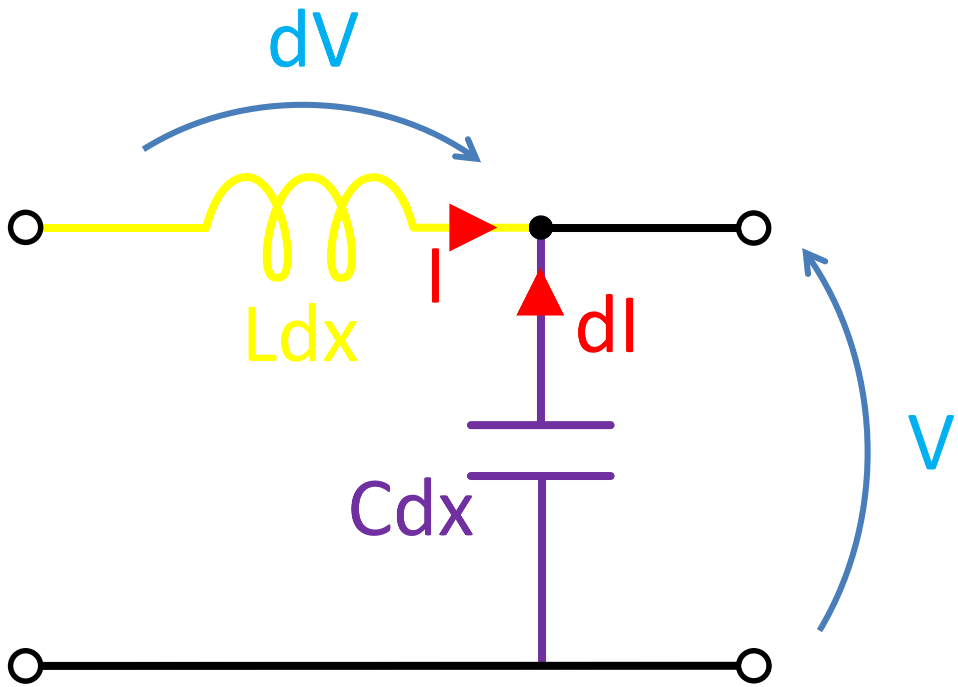
Typical results



$$Z_0 = \sqrt{\frac{R}{j\omega C}}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

Non-dissipative line (R=G=0)



$$dV = -Ldx\frac{dI}{dt}$$

$$dI = -Cdx\frac{dV}{dt}$$

$$\frac{d^2V}{dx^2} = LC\frac{d^2V}{dt^2} \qquad \frac{d^2I}{dx^2} = LC\frac{d^2I}{dt^2}$$

Travelling-wave equations

$$V(x,t) = V^{+}\left(t - \frac{x}{u}\right) + V^{-}\left(t + \frac{x}{u}\right)$$

$$I(x,t) = I^{+}\left(t - \frac{x}{u}\right) + I^{-}\left(t + \frac{x}{u}\right)$$

forward backward

$u = \frac{1}{\sqrt{LC}}$ = propagation speed

Z_0 revisited – 1

We consider the forward wave. From

$$\frac{dV^+(t - x/u)}{dx} = -L\frac{dI^+(t - x/u)}{dt}$$

we take $z = t - x/u$, i.e.

$$\frac{dV^+\left(t - \frac{x}{u}\right)}{dx} = \frac{dV^+(z)}{dz}\frac{dz}{dx} = -\frac{1}{u}\frac{dV^+(z)}{dz}$$

$$\frac{dI^+\left(t - \frac{x}{u}\right)}{dt} = \frac{dI^+(z)}{dz}\frac{dz}{dt} = \frac{dI^+(z)}{dz}$$

Z_0 revisited – 2

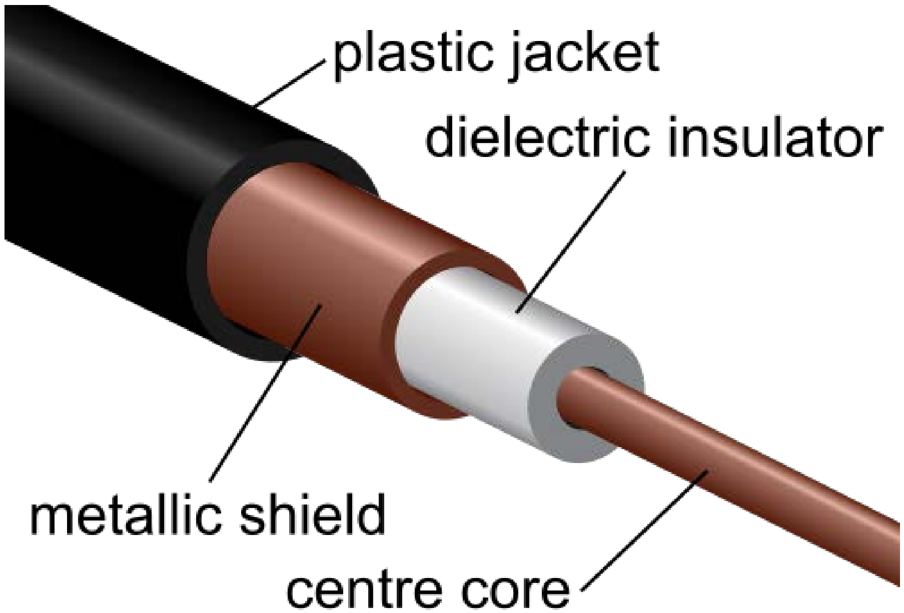
- Replacing, we get

$$\frac{dV^+}{dz}\sqrt{LC} = L\frac{dI^+}{dz} \rightarrow V^+ = Z_0 I^+$$

and analogously $V^- = -Z_0 I^-$

which is the result already obtained

Example: coaxial cables



$$C = \frac{2\pi\varepsilon}{\ln(D/d)}$$

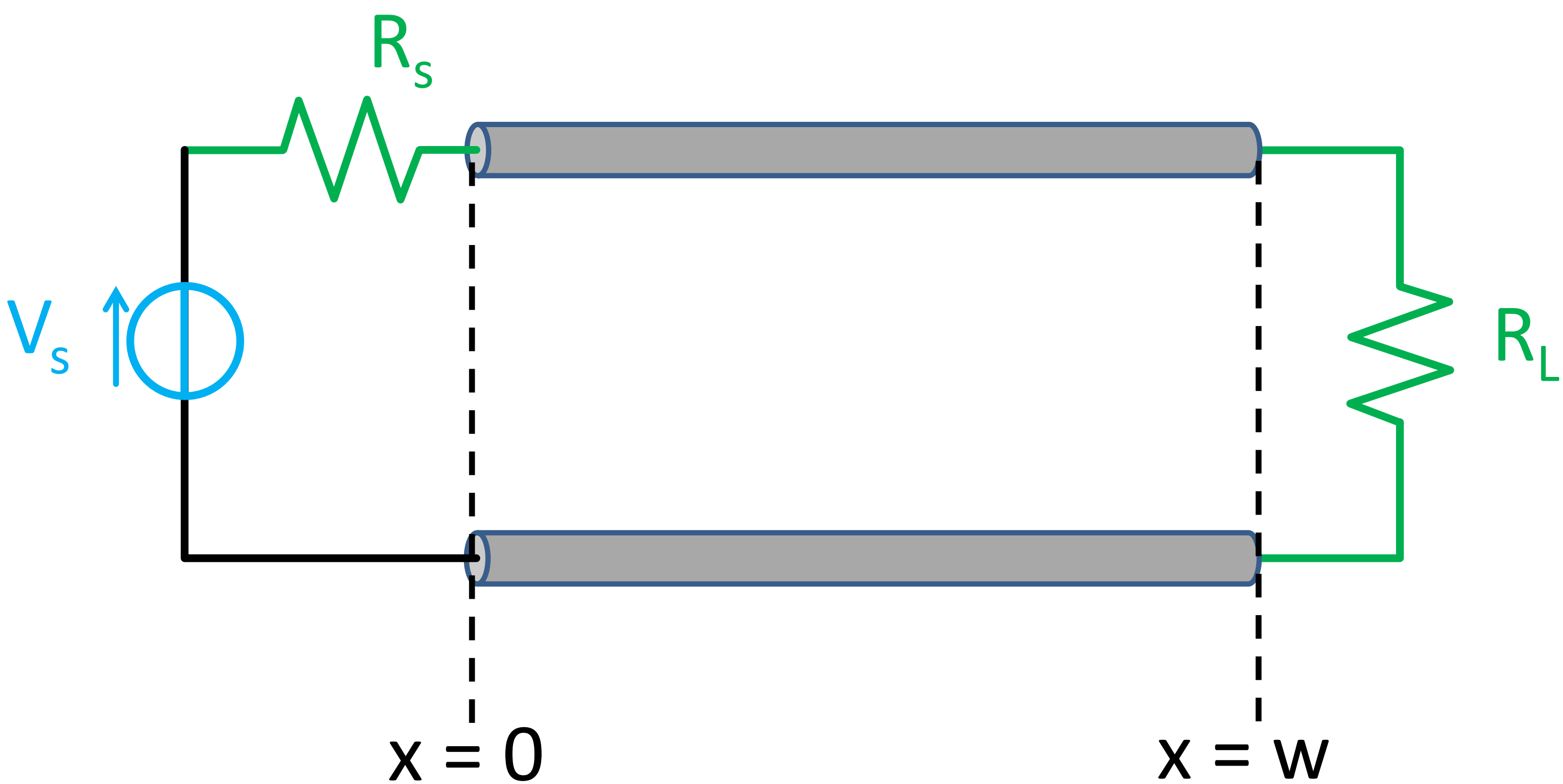
$$L = \frac{\mu}{2\pi}\ln(D/d)$$

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\left(\frac{D}{d}\right) = \frac{138}{\sqrt{\varepsilon_r}}\ln\left(\frac{D}{d}\right)\ [\Omega]$$

50 Ω cables

- Chosen as compromise between power-handling capabilities (maximum at around 30 Ω) and attenuation (minimum around 60-70 Ω)
- Diameters of inner and outer conductors have «good» values and are easy to manufacture
- Other values can be used in special applications (e.g. 93 Ω for digital transmission, 25 Ω for RF,...)

Loaded line



Boundary condition

- Load resistor forces

$$V(w,t) = R_L I(w,t)$$

where $V = V^+ + V^-$ and $I = I^+ + I^-$

- Recalling that $I^\pm = \pm V^\pm / Z_0$, we get

$$V^+ + V^- = \frac{R_L}{Z_0}(V^+ - V^-)$$

Matched line ($R_L = Z_0$)

$$V^+ + V^- = V^+ - V^- \Longrightarrow V^- = 0$$

- No backward wave
- The line behaves as of infinite length (recall the calculation of Z_0)

The case of $R_L \neq Z_0$

- Waves are partially reflected from the mismatch at the load: $V^- \neq 0$
- We define a voltage reflection coefficient at the load, Γ_w, as

$$\Gamma_w = \frac{V^-\left(t + \frac{w}{u}\right)}{V^+\left(t - \frac{w}{u}\right)}$$

$$V^+(1 + \Gamma_w) = \frac{R_L}{Z_0} V^+(1 - \Gamma_w)$$

Reflection coefficient

$$\frac{R_L}{Z_0} = \frac{1+\Gamma_w}{1-\Gamma_w}$$

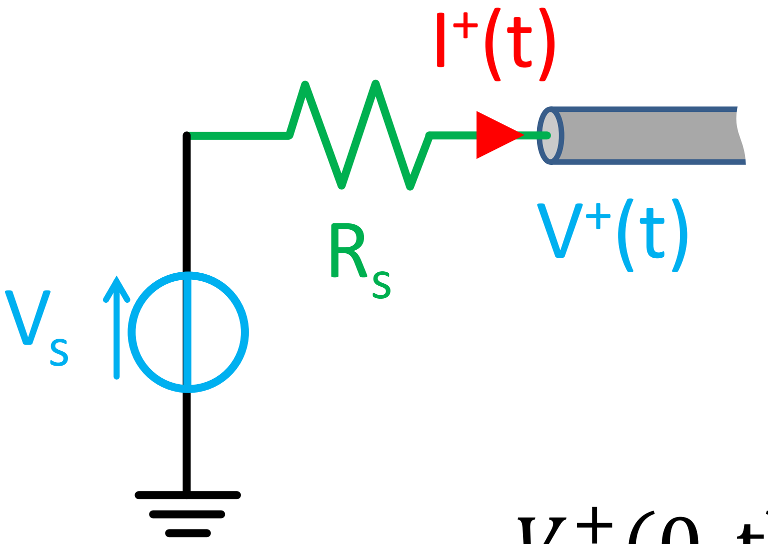
$$\Gamma_w = \frac{R_L - Z_0}{R_L + Z_0}$$

$$-1 \le \Gamma_w \le 1$$

$R_L = 0$ $\qquad\qquad$ $R_L = \infty$

Reflection at the source

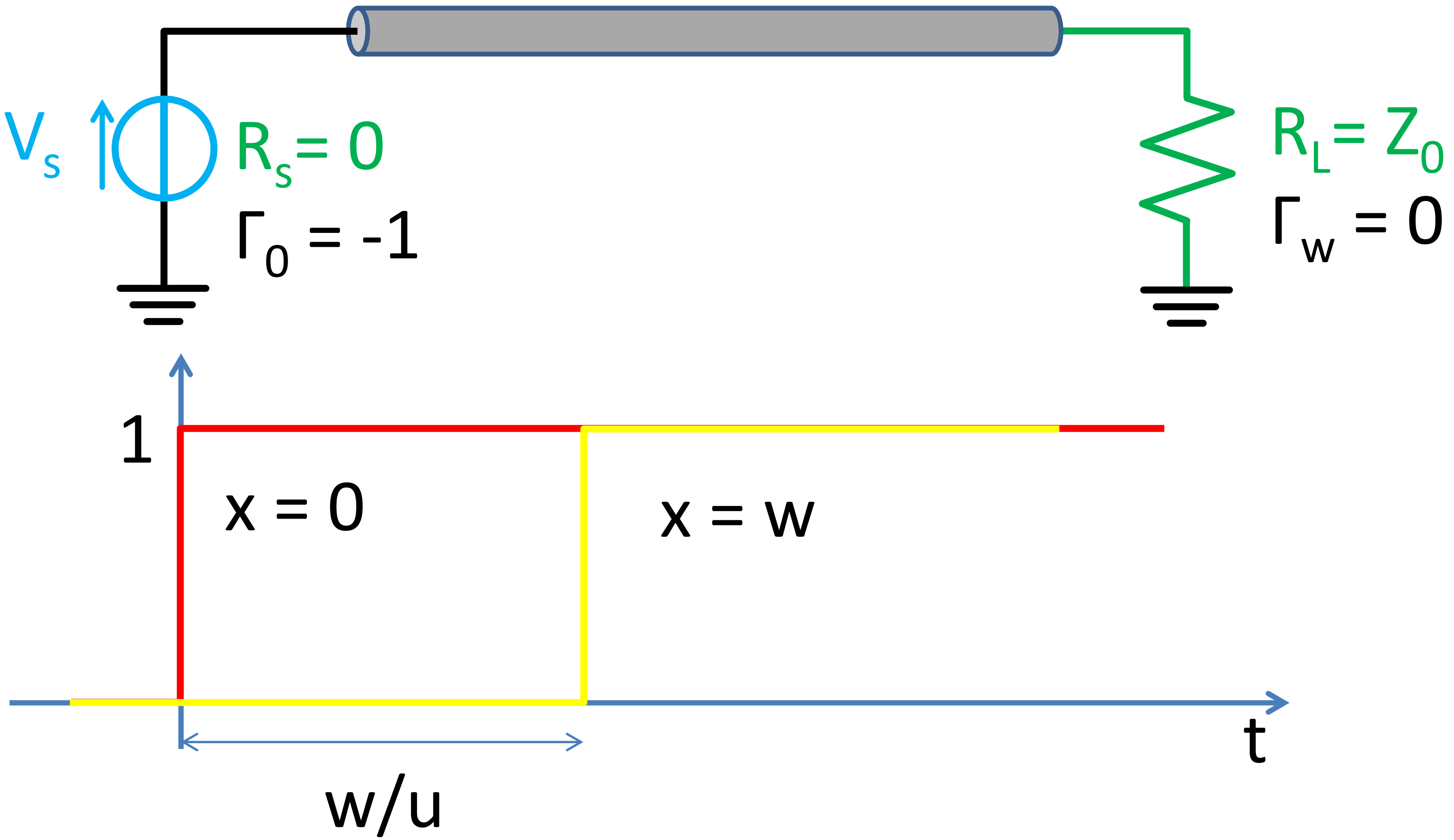
$$\Gamma_0 = \frac{V^+(t)}{V^-(t)} = \frac{R_S - Z_0}{R_S + Z_0} \Rightarrow \frac{R_S}{Z_0} = \frac{1+\Gamma_0}{1-\Gamma_0}$$



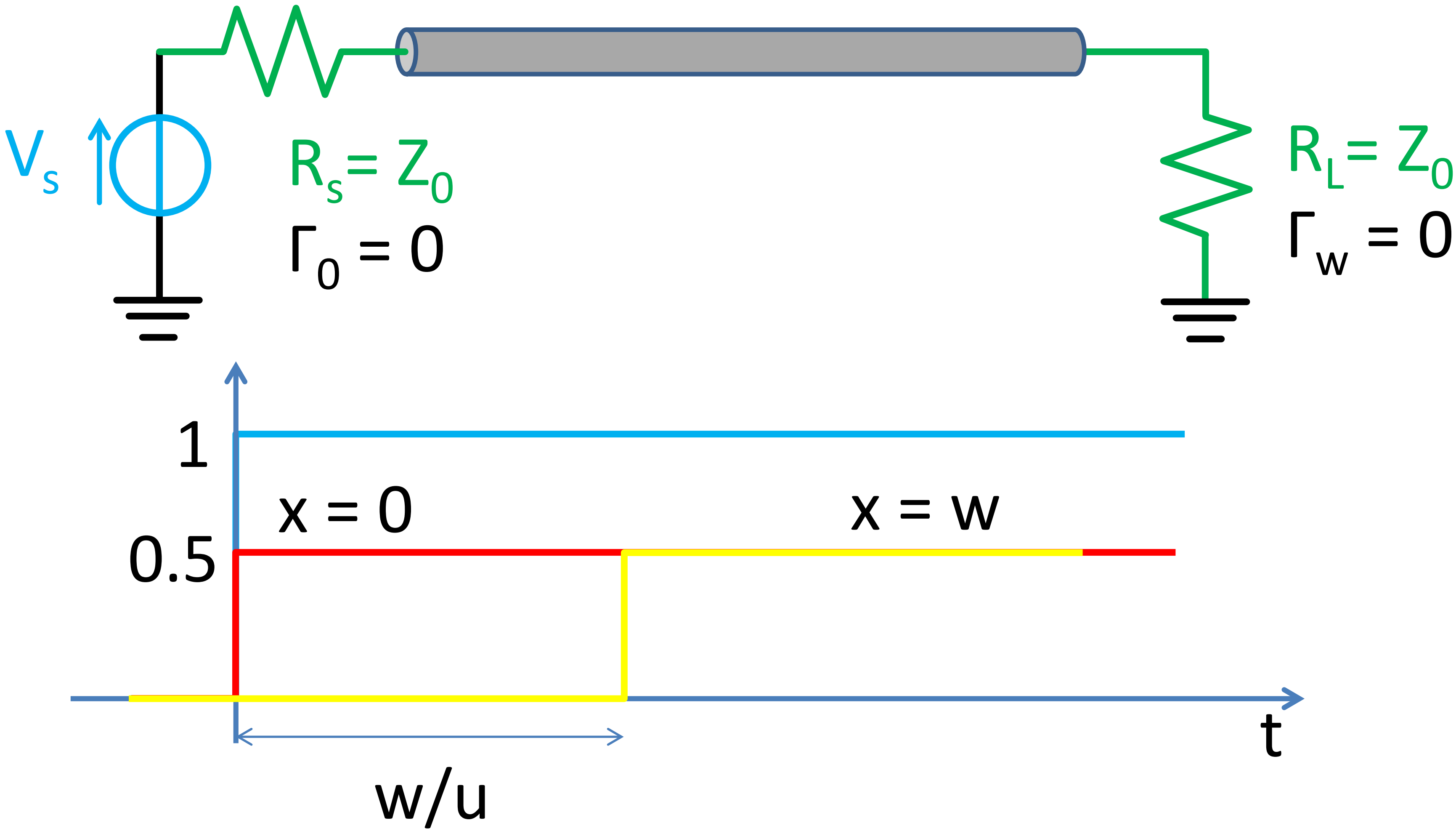
$$V_s = V^+ + I^+ R_s$$
$$= V^+\left(1 + \frac{R_s}{Z_0}\right)$$

$$V^+(0,\mathrm{t}) = V_s \frac{Z_0}{Z_0 + R_s} = V_s \frac{1-\Gamma_0}{2}$$

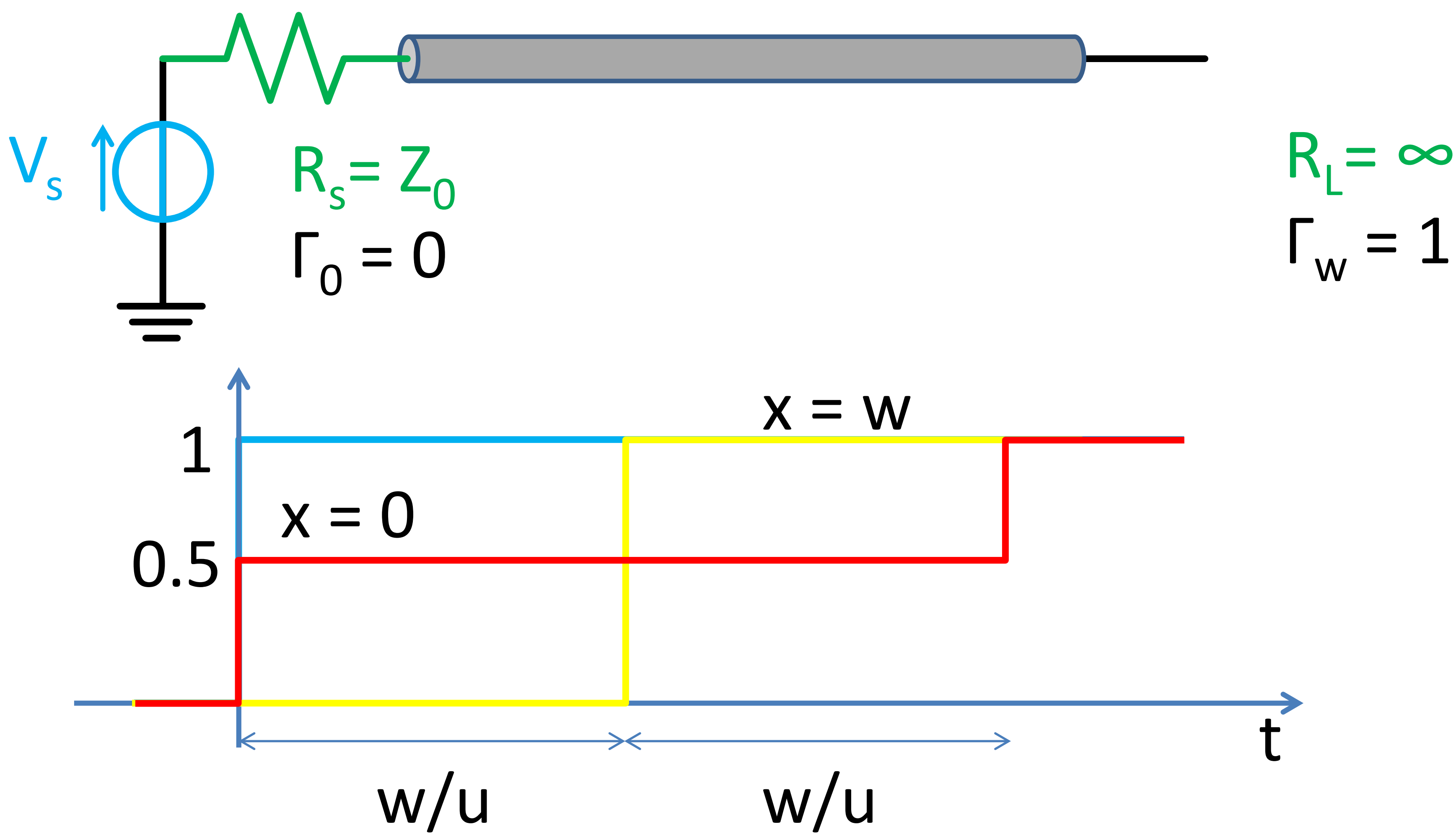
Line matched at load side



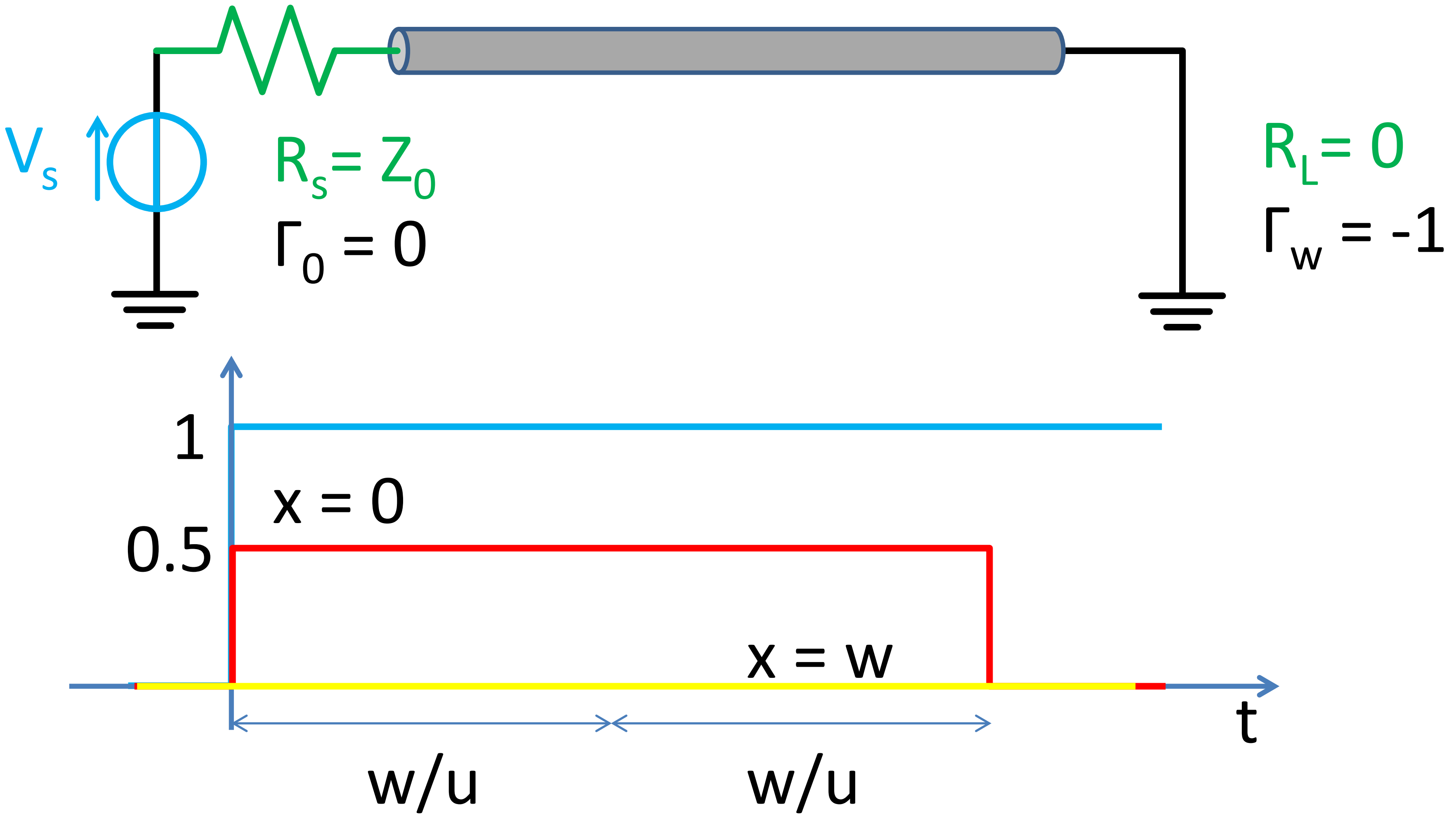
Line matched at both sides



Line open-circuited at load



Line short-circuited at load



General case

$$\frac{1-\Gamma_0}{2} \times \Big(\quad 1$$

$$+ \qquad 1+\Gamma_w$$

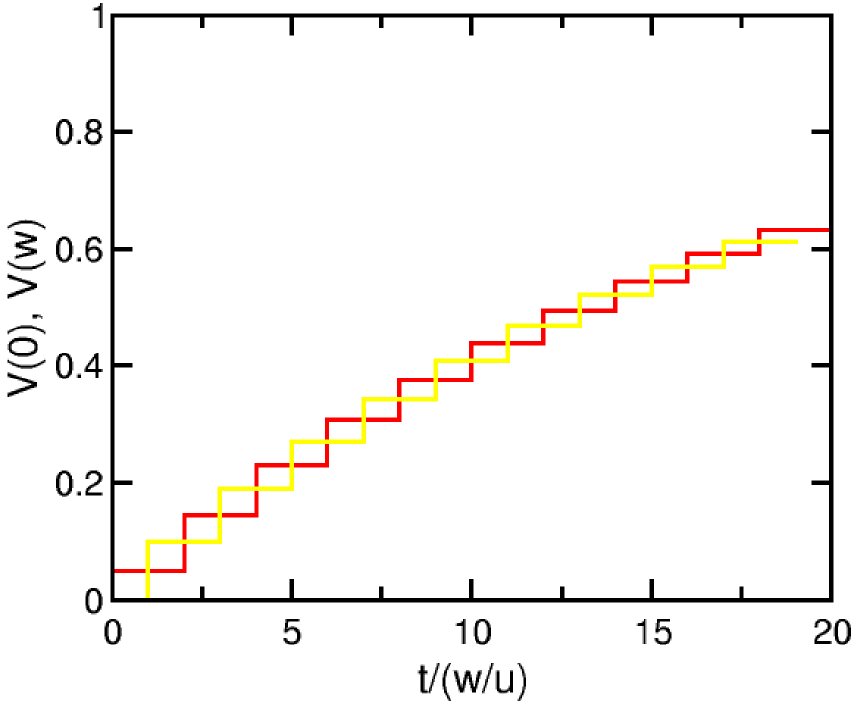
$$\Gamma_w(1+\Gamma_0) \qquad +$$

$$+ \qquad \Gamma_w\Gamma_0(1+\Gamma_w)$$

$$\Gamma_0\Gamma_w^2(1+\Gamma_0) \qquad +$$

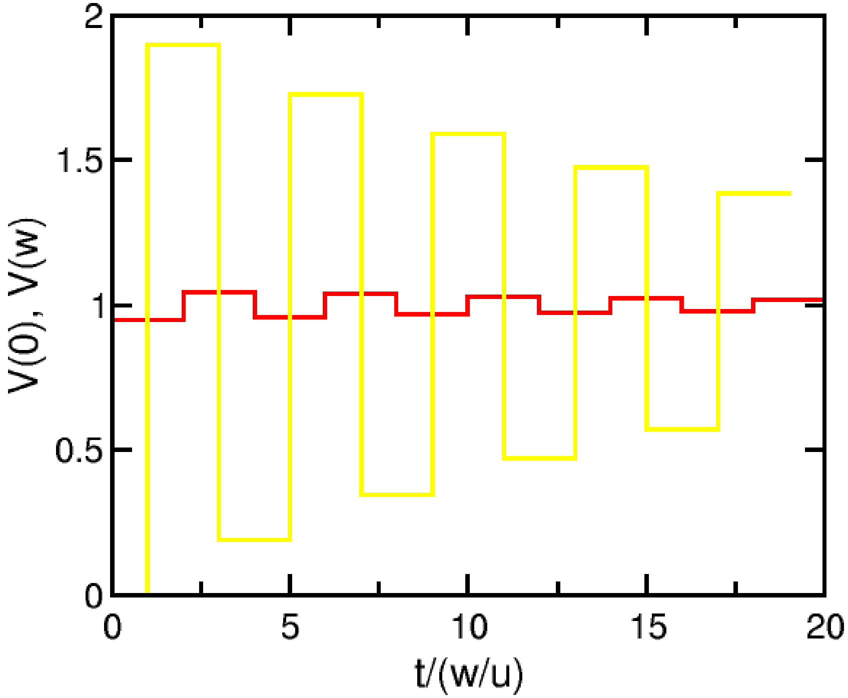
$$\vdots \qquad \vdots$$

Examples (strong mismatch)



$\Gamma_0 = 0.9, \Gamma_w = 1$

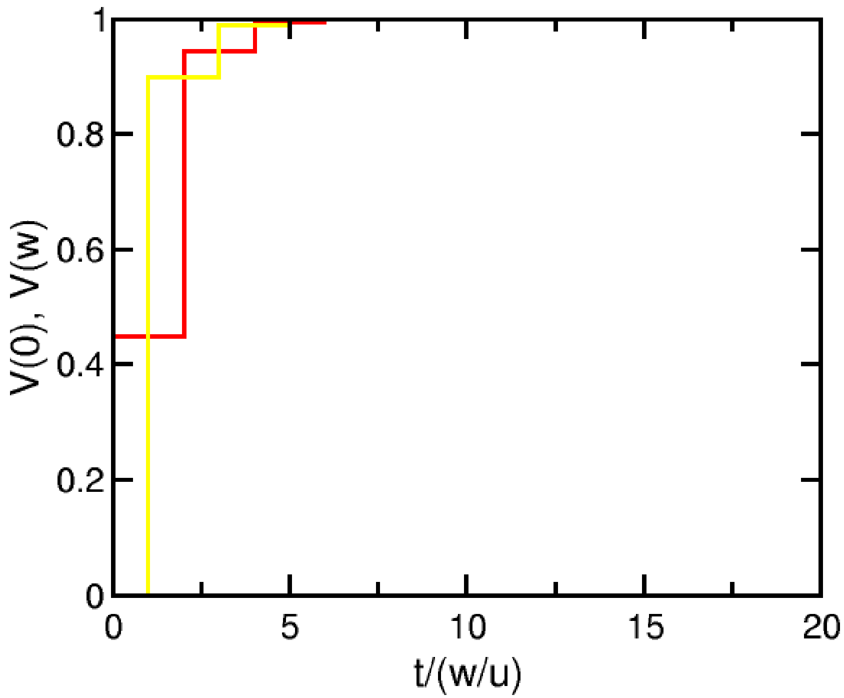
$R_s = 19\ Z_0$



$\Gamma_0 = -0.9, \Gamma_w = 1$

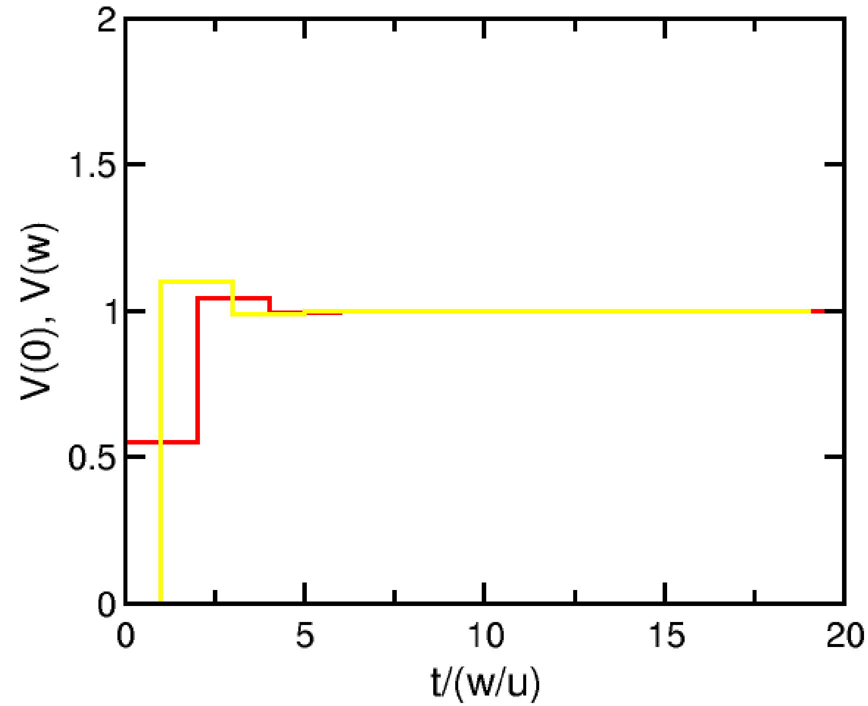
$R_s \approx 0.05\ Z_0$

Examples (almost matched)



$\Gamma_0 = 0.1, \Gamma_w = 1$

$R_s \approx 1.22\ Z_0$



$\Gamma_0 = -0.1, \Gamma_w = 1$

$R_s \approx 0.82\ Z_0$