Electronics – 96032

POLITECNICO DI MILANO





Lock-in Amplifiers (LIAs)

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Disclaimer

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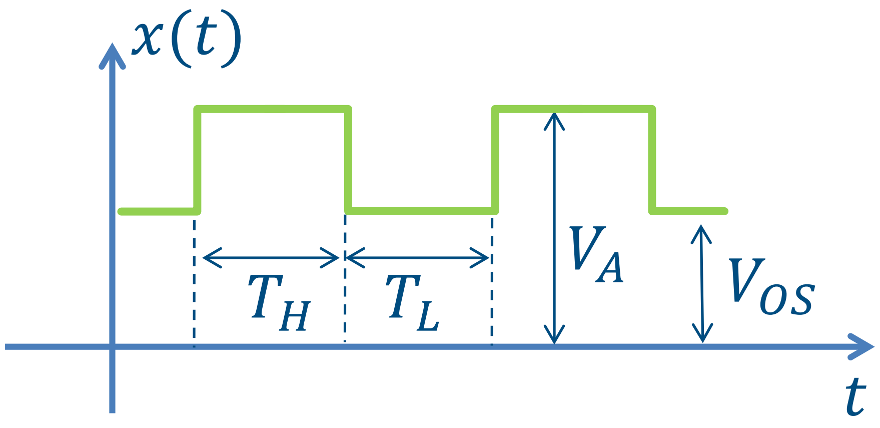
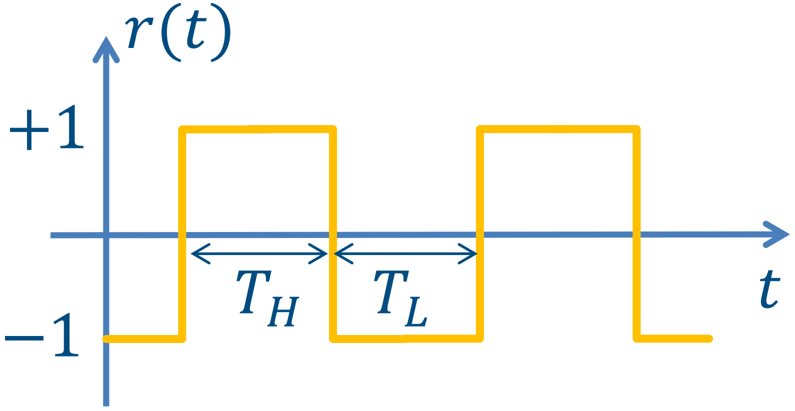
Problem: Square wave detection with LIA



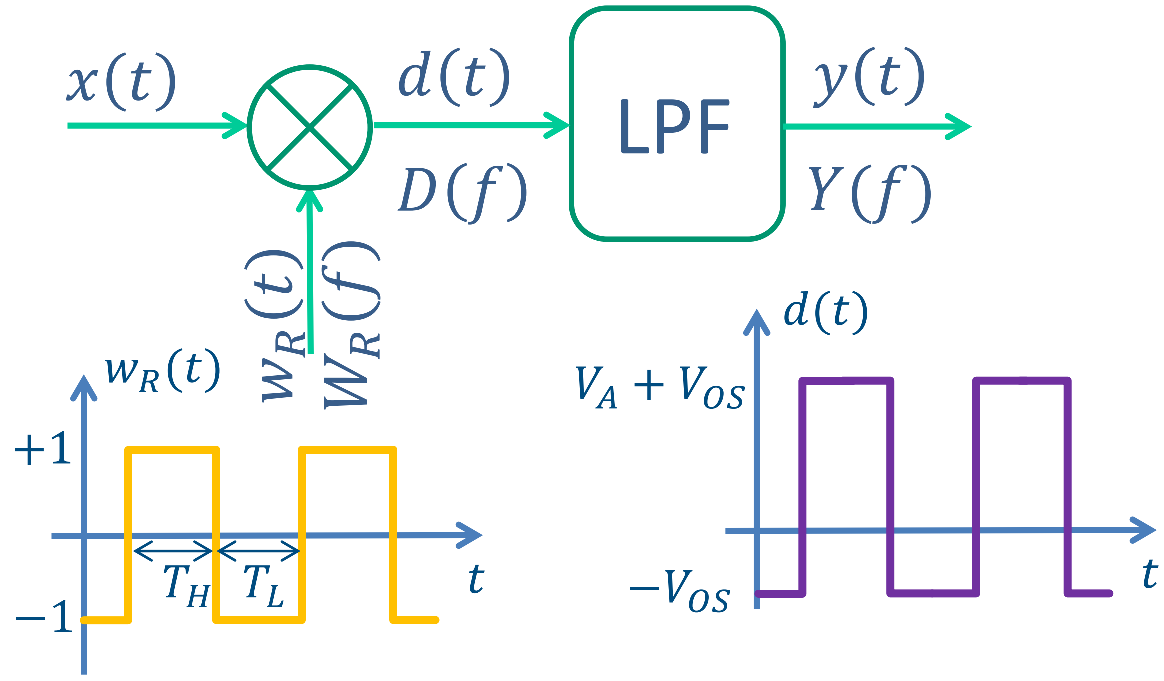
$$f_0 = \frac{1}{T_H + T_L}$$

WN PSD: $S_V = 50\text{nV}/\sqrt{\text{Hz}}$
Flicker noise (unknown)

1. Use a LIA to get S/N = 10 using as a reference signal $r(t)$



LIA output: signal

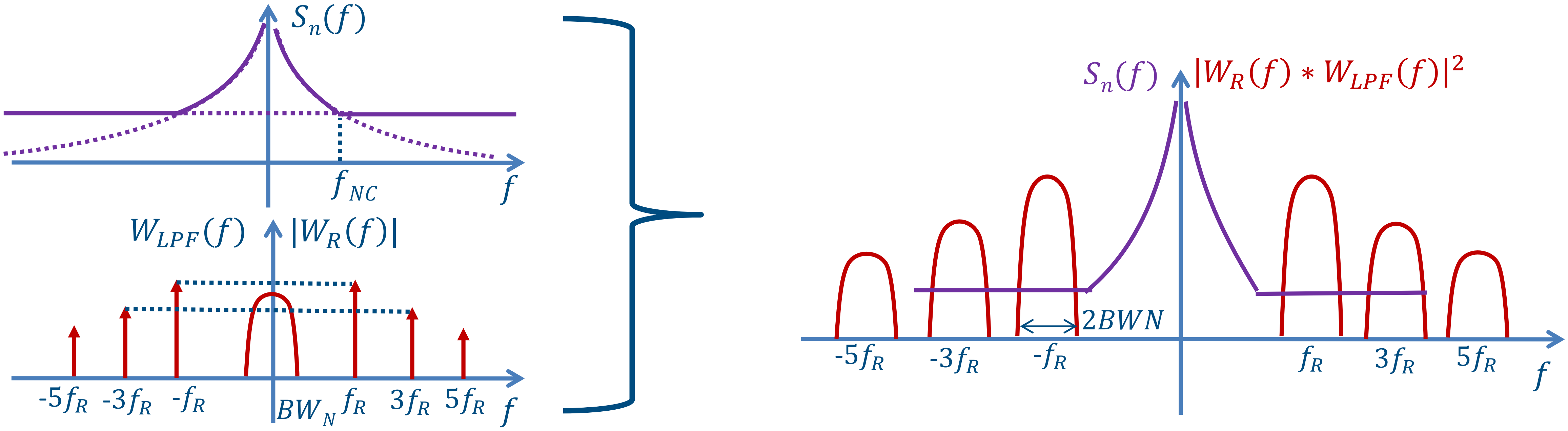


- $x(t)$ is already "modulated" (it is a square wave)
- It is easier to work in the time domain to avoid to deal with sinc (if LPF is a GI)

- $y(t)$ is the output of the LPF, i.e. the DC component of $d(t)$

$$y = \frac{V_A + V_{OS} + (-V_{OS})}{2} = \frac{V_A}{2}$$

LIA output: noise



- This is easier in the frequency domain:

$$\overline{n_o^2} = \int S_n(f)|W_R * W_{LPF}|^2 df = 2\sum_k \left(\frac{2}{\pi}\right)^2 \frac{1}{(2k+1)^2} S_n[(2k+1)f_R] \cdot 2BW_N$$

LIA output: S/N ratio

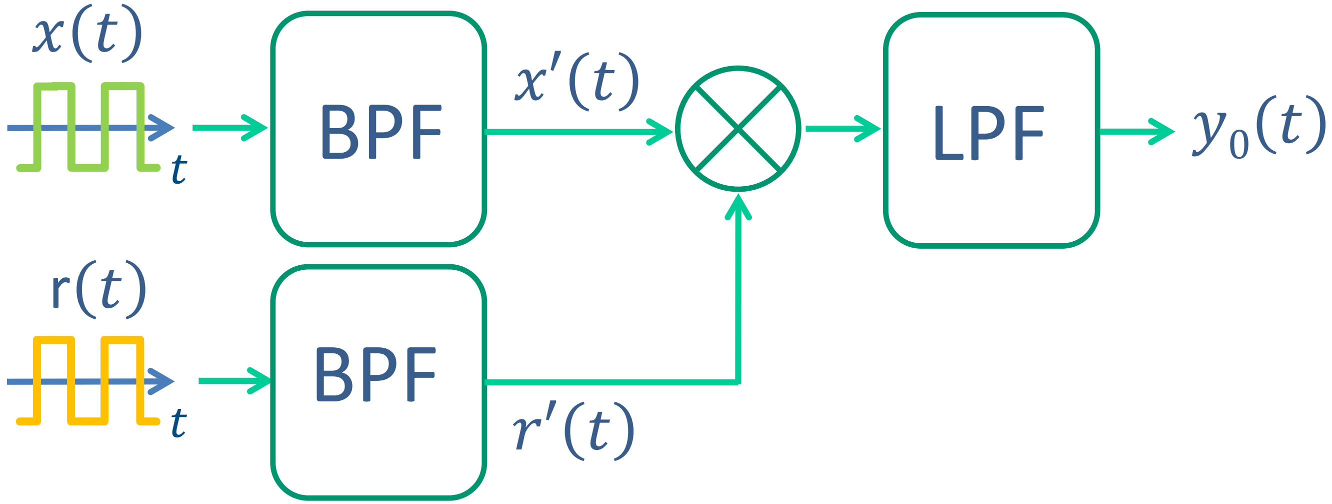
- Choosing $f_R > f_{NC}$ allows to neglect 1/f noise: $S_n[(2k+1)f_R] \simeq \frac{S_n^u}{2}$

$$\overline{n_o^2} = 2\left(\frac{2}{\pi}\right)^2 \cdot \frac{\pi^2}{8} \cdot \frac{S_n^u}{2} \cdot 2BW_N = S_n^u \cdot BWN$$

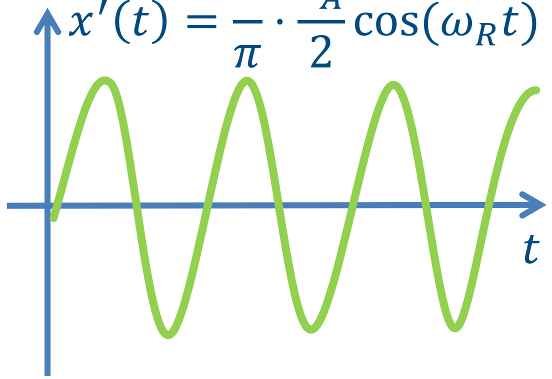
- From S/N expression we derive the LPF bandwidth:

$$\left(\frac{S}{N}\right) = \frac{\frac{V_A}{2}}{\sqrt{S_n^u \cdot BW_{LPF}}} = 10 \rightarrow BWN\text{=100Hz}$$

Alternative solution: BPF + demodulation



$$x'(t) = \frac{4}{\pi} \cdot \frac{V_A}{2} \cos(\omega_R t)$$

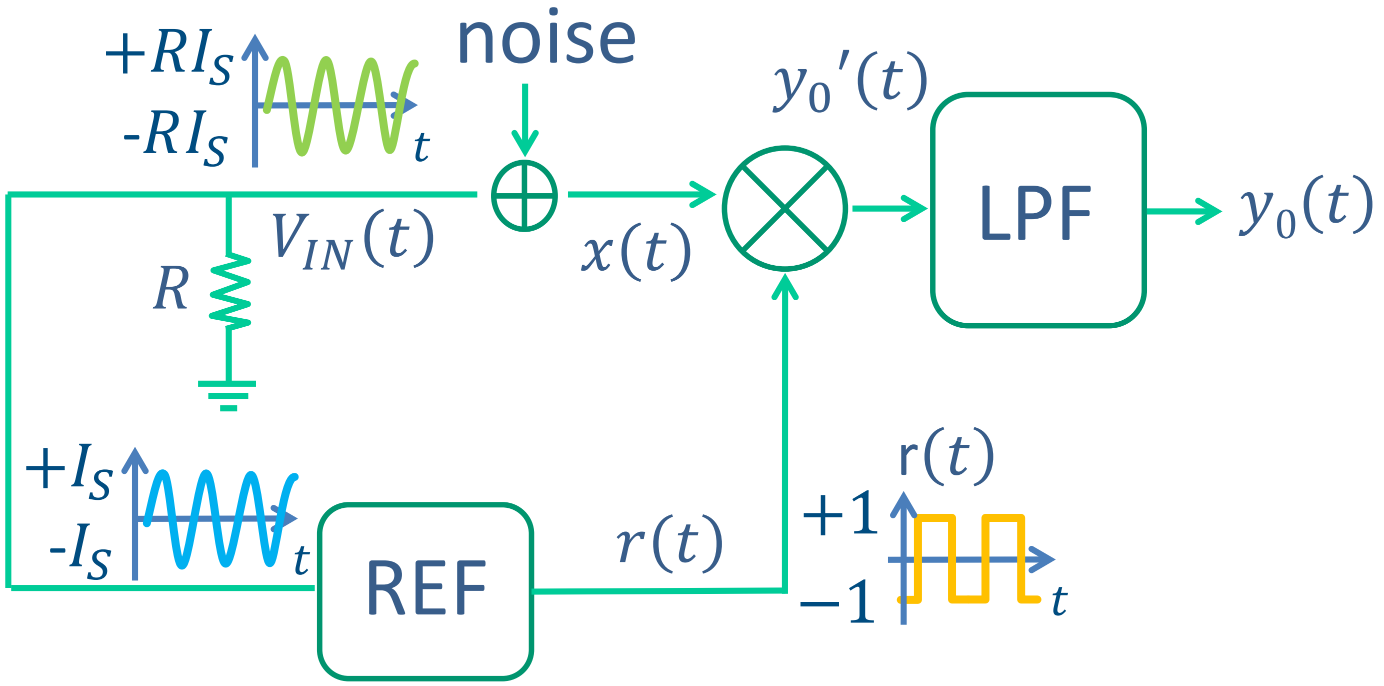


$$r'(t) = \frac{4}{\pi} \cdot \cos(\omega_R t)$$

- With two BPFs (for signal and reference) demodulation is done with pure sine waves:

$$\frac{S}{N} = \frac{\left(\frac{4}{\pi}\right)^2 \cdot \frac{V_A}{4}}{\sqrt{\left(\frac{4}{\pi}\right)^2 S_n^u \cdot \frac{BW_{LPF}}{2}}} = 10 \rightarrow BW_{LPF} = 81\text{Hz} = \frac{100}{\pi^2/8}\text{Hz}$$

Problem: sin-square wave modem



$R \simeq 1k\Omega$

$BW_R = 5Hz$

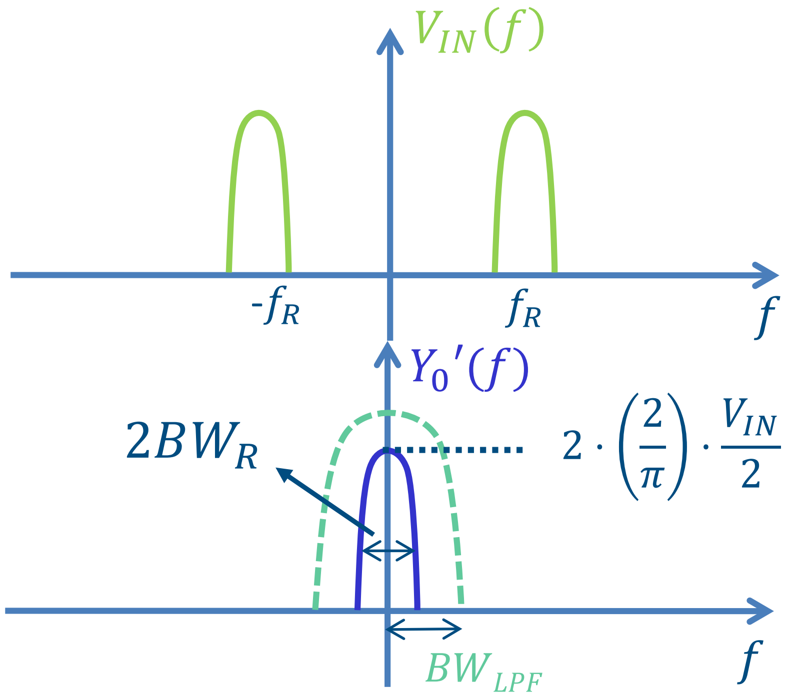
$S_n = \frac{k_n}{f}$, $k_n = 6 \cdot 10^{-8} V^2$

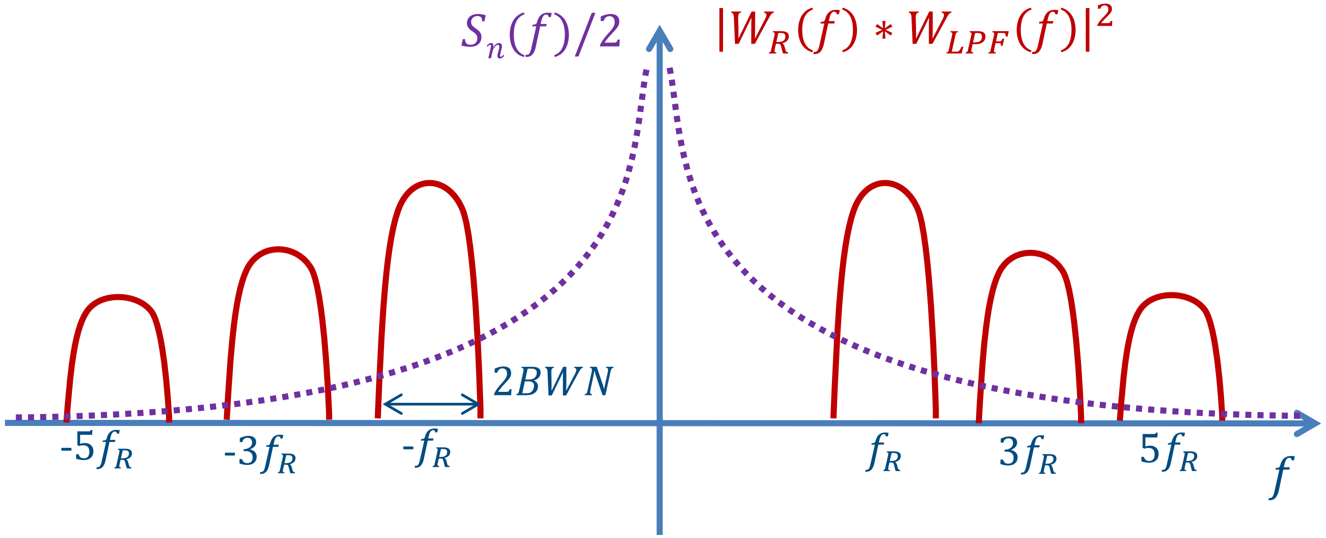
(unilateral)

$I_S = 10\mu A$

1. Size f_R and BW_{LPF} to have S/N=10 on the measurement of R
2. Suggest a way to measure a complex impedance (such as $Z = R + 1/SC$)

Signal and noise





$$2BW_{LPF} \gg 2BW_R$$
$$\rightarrow BW_{LPF} = 10BW_R$$
$$= 50Hz$$

$$\overline{n_o^2} = \sum_k \left(\frac{2}{\pi}\right)^2 \frac{1}{(2k+1)^2} S_n[(2k+1)f_R]\, 2BW_N$$

unilateral!

f_R sizing

- The difference vs. previous case is that we have no white noise. We can approximate $S_n[(2k+1)f_R] = S_n[f_R]$ (worst-case) and also $BW_N = BW_{LPF}$

$$\frac{S}{N} = \frac{\frac{2}{\pi} I_S R}{\sqrt{\left(\frac{2}{pi}\right)^2 \cdot 2BW_{LPF} \cdot S_n(f_R) \cdot \frac{\pi^2}{8}}} = 10 \ \rightarrow f_R = \left(\frac{100\pi^2}{8}\right) \mathrm{kHz}$$

Complex impedance measurement

- Measure separately real and imaginary part of Z (phase shift):

$$V_{IN} = I_S\left(R + \frac{1}{SC}\right) = I_S\cos(\omega_R t) + \frac{I_S}{\omega_R C}\cos\left(\omega_R t + \frac{\pi}{2}\right)$$