POLITECNICO DI MILANO

Electronics – 96032

Closed-loop gain calculation

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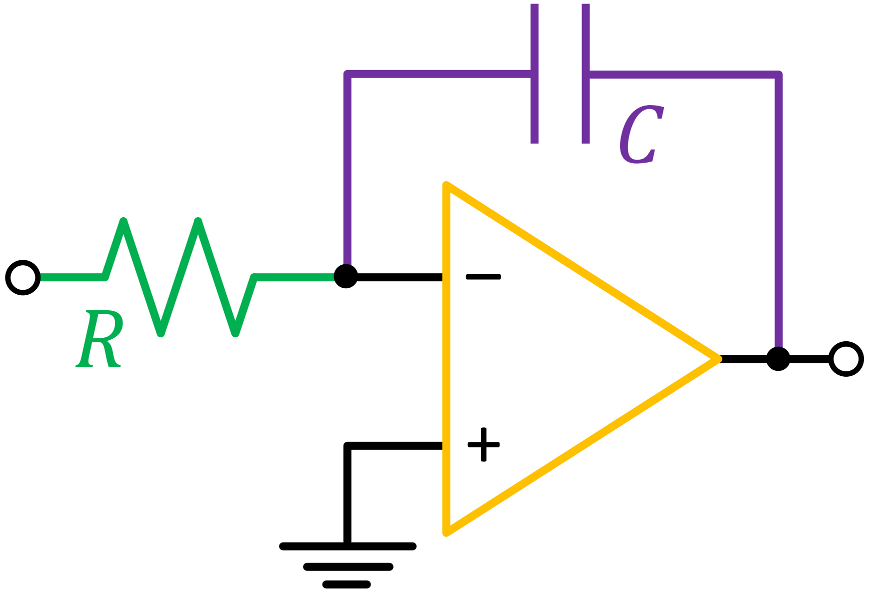
Disclaimer

Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes

Outline

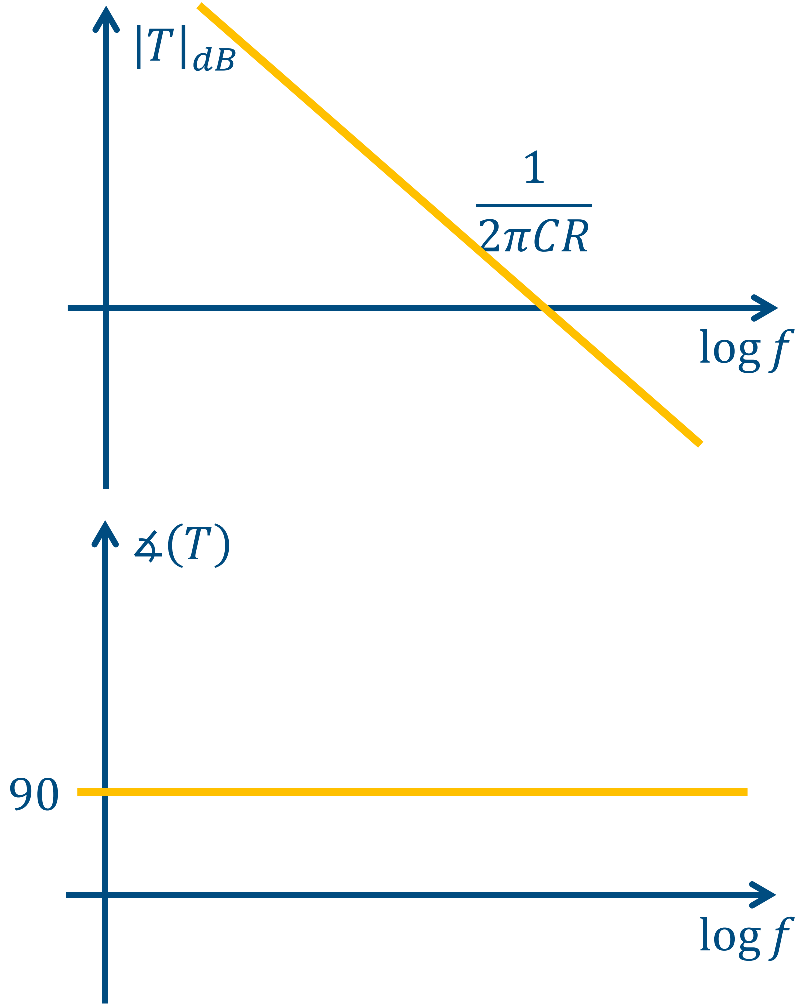
- Integrator stage
- Differentiator stage
- Phase shifter

Ideal integrator

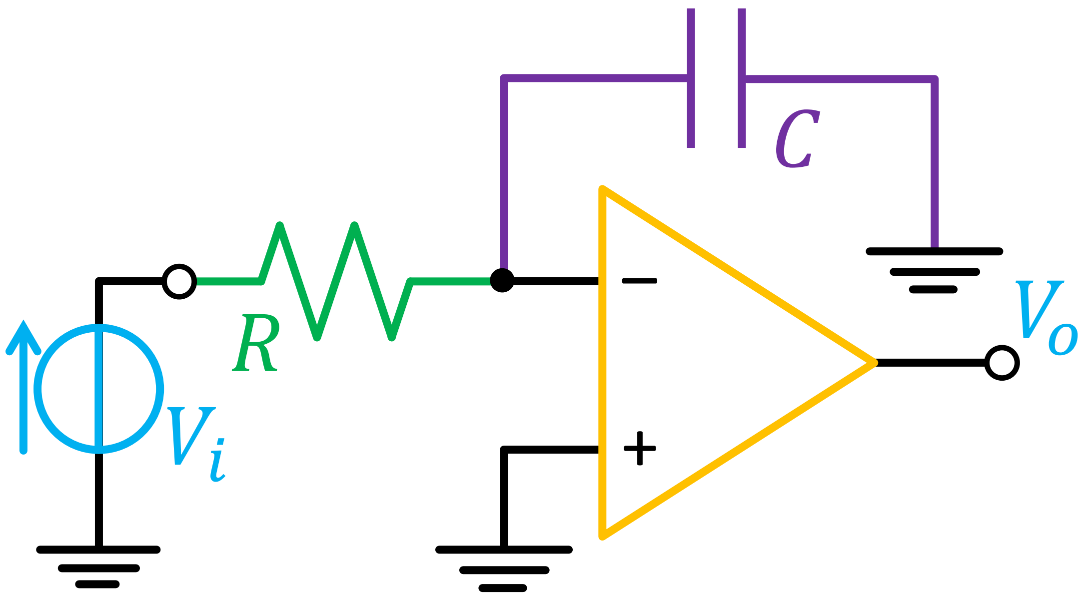


$$\frac{V_o}{V_i} = T(s) = -\frac{1}{sCR}$$

$$T(j\omega) = \frac{j}{\omega CR} = \frac{j}{2\pi f CR}$$



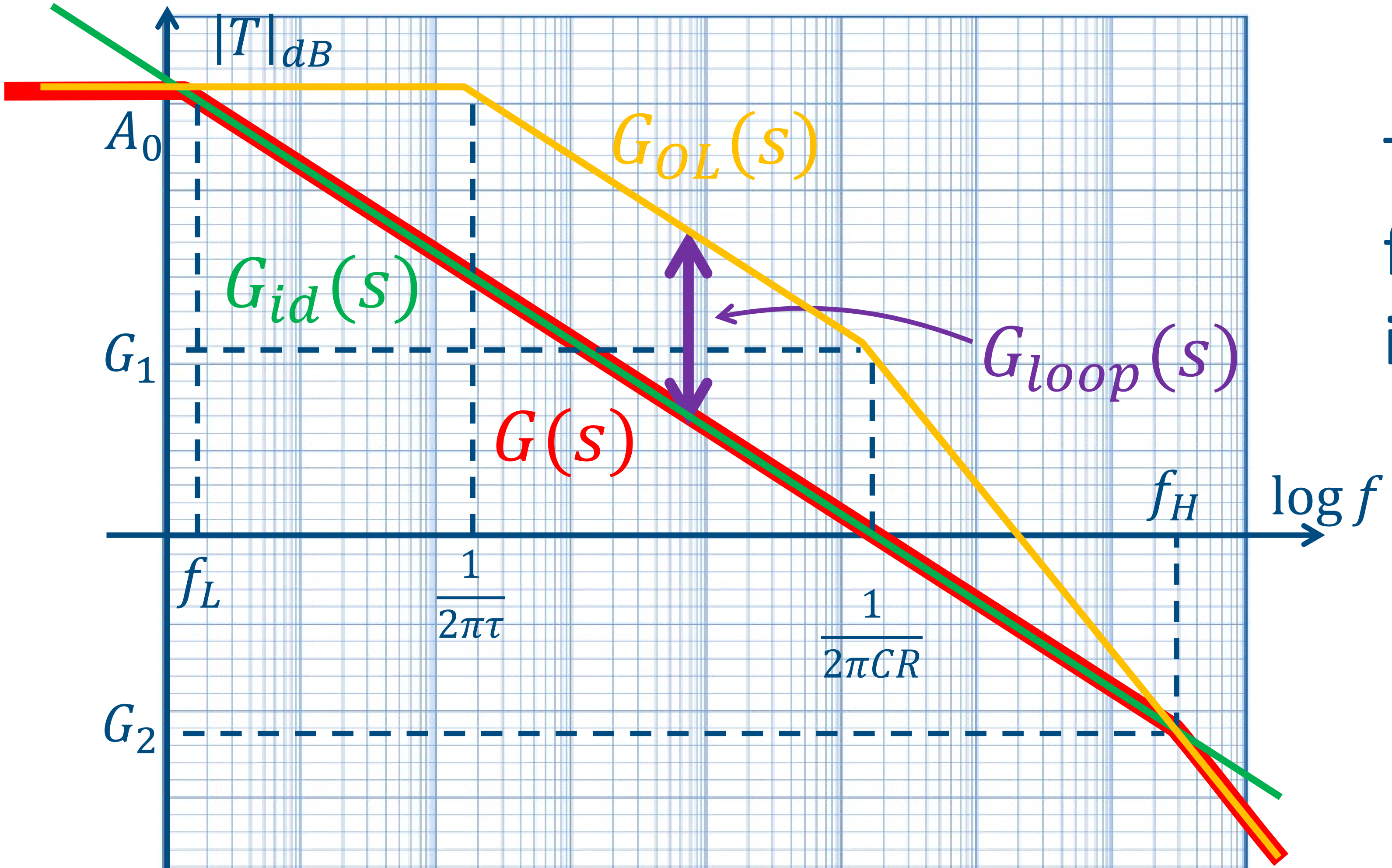
Open-loop gain



$$G_{id} = \frac{1}{sCR}$$

$$G_{OL} = -\frac{A(s)}{1 + sCR} = -\frac{A_0}{1 + s\tau}\frac{1}{1 + sCR}$$

Closed-loop gain



The integration frequency range is $f_L - f_H$

Bode diagram calculations

- Diagram with slope $= -1$:

$$|G| = \frac{K}{f} \Rightarrow |G|f = \text{constant}$$

- Diagram with slope $= -2$:

$$|G| = \frac{K}{f^2} \Rightarrow |G|f^2 = \text{constant}$$

- Diagram with slope $= +1$:

$$|G| = Kf \Rightarrow \frac{|G|}{f} = \text{constant}$$

Calculations

- f_L:

$$|G_{id}| = A_0 \Rightarrow \frac{1}{\omega_L CR} = A_0 \Rightarrow f_L = \frac{1}{2\pi CRA_0}$$

- f_H:

$$\text{slope of } -1 \text{ in } G_{OL} \Rightarrow \frac{A_0}{2\pi\tau} = \frac{G_1}{2\pi CR} \Rightarrow G_1 = A_0 \frac{CR}{\tau}$$

$$\left.\begin{array}{l} \text{slope of } -1 \text{ in } G_{id} \Rightarrow A_0 f_L = G_2 f_H \\ \text{slope of } -2 \text{ in } G_{OL} \Rightarrow G_1 \left(\dfrac{1}{2\pi CR}\right)^2 = G_2 f_H^2 \end{array}\right\} \Rightarrow f_H = \frac{A_0}{2\pi\tau}$$

Calculations – alternative

$$G_{OL} = \frac{A_0}{1+s\tau}\frac{1}{1+sCR} \approx \frac{A_0}{s^2\tau CR}$$ (beyond the 2nd pole)

$$G_{OL} = G_{id} \Rightarrow \frac{A_0}{s^2\tau CR} = \frac{1}{sCR} \Rightarrow f_H = \frac{A_0}{2\pi\tau}$$

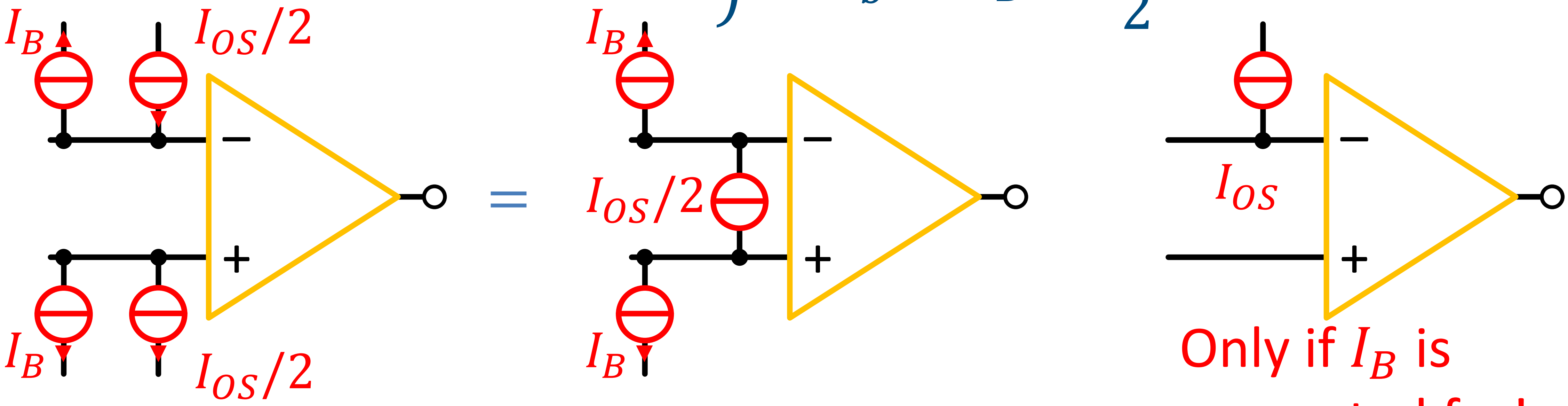
Or via G_{loop}:

$$G_{loop} = -A(s)\frac{R}{R+\frac{1}{sC}} = -A(s)\frac{sCR}{1+sCR}\left(= -\frac{G_{OL}}{G_{id}}\right)$$

$$\left|G_{loop}\right| = 1 \Rightarrow f_L, f_H$$

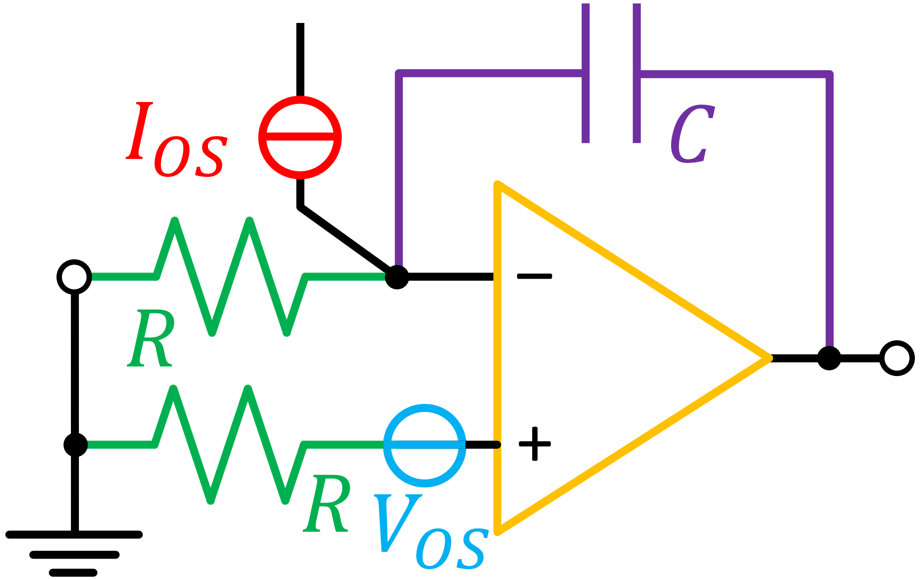
Bias currents representation

$$I_B = \frac{I_b^+ + I_b^-}{2}$$
$$I_{OS} = I_b^+ - I_b^-$$
$$\Rightarrow \quad I_b^+ = I_B + \frac{I_{OS}}{2}, \quad I_b^- = I_B - \frac{I_{OS}}{2}$$



Only if I_B is compensated for!

Bias and offset errors

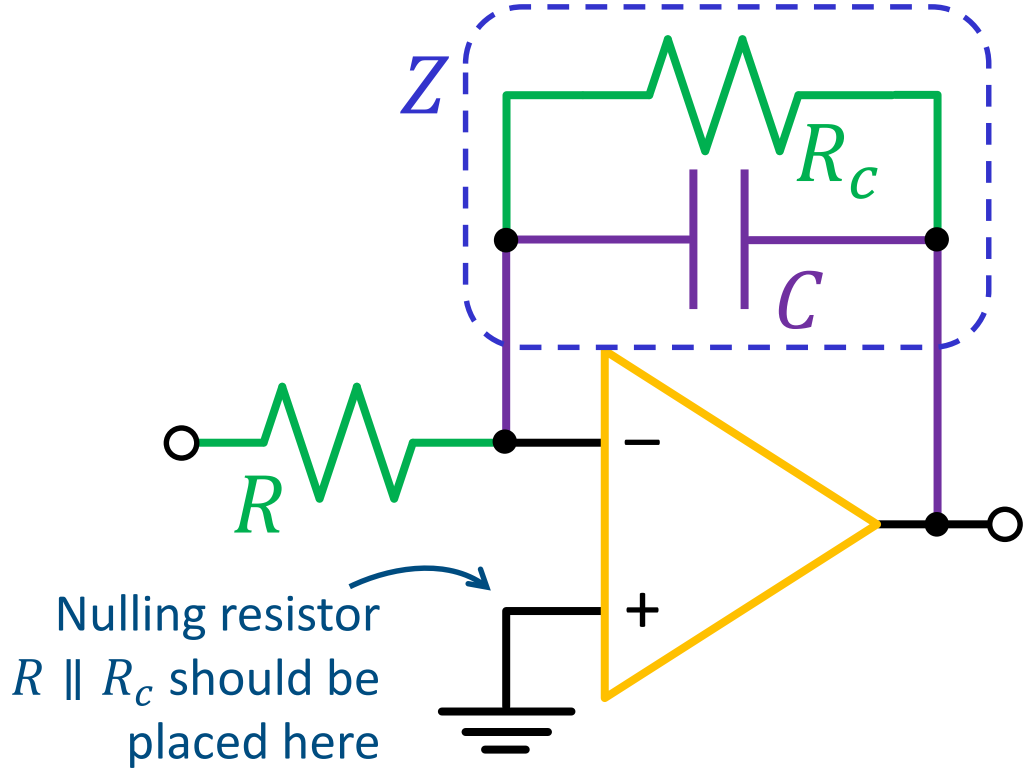


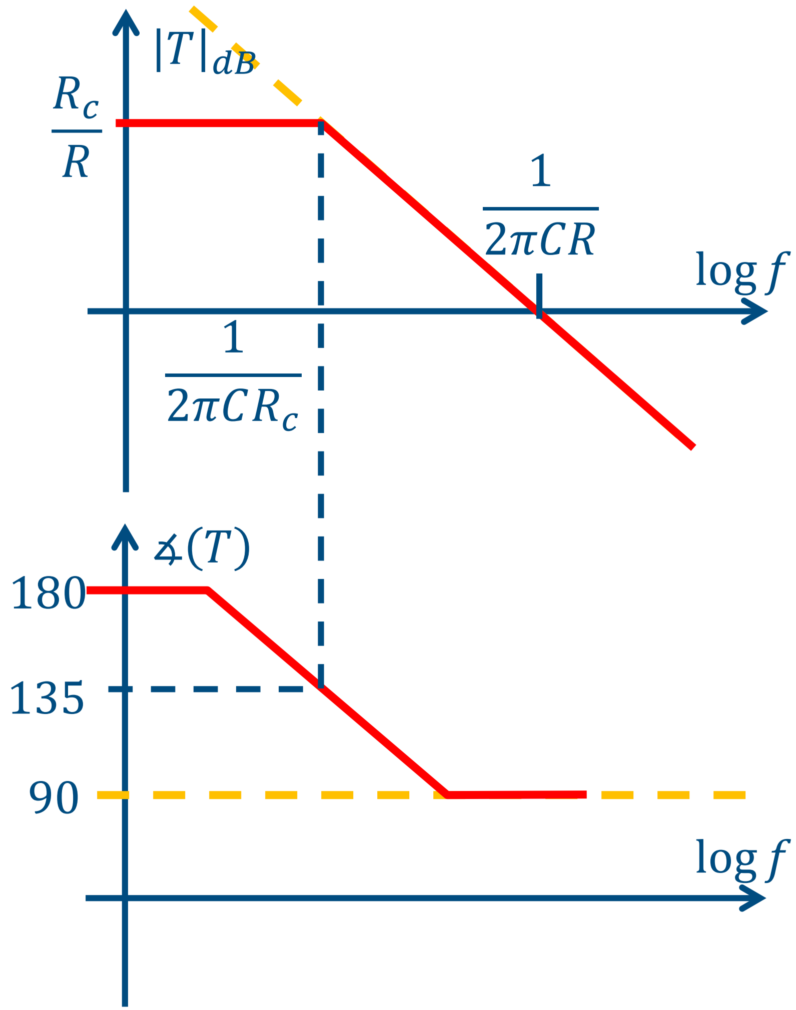
$$V_o = \frac{I_{OS}}{sC} + V_{OS}\left(1 + \frac{1}{sCR}\right)$$

$$v_o(t) = v_{OS} + \frac{1}{C}\int_0^t \left(i_{OS} + \frac{v_{OS}}{R}\right) dt$$

- Nulling resistor can be trimmed for better offset
- Slow drift of the offset will eventually cause saturation of the output $\Rightarrow$ periodic adjustment is needed

Practical integrator



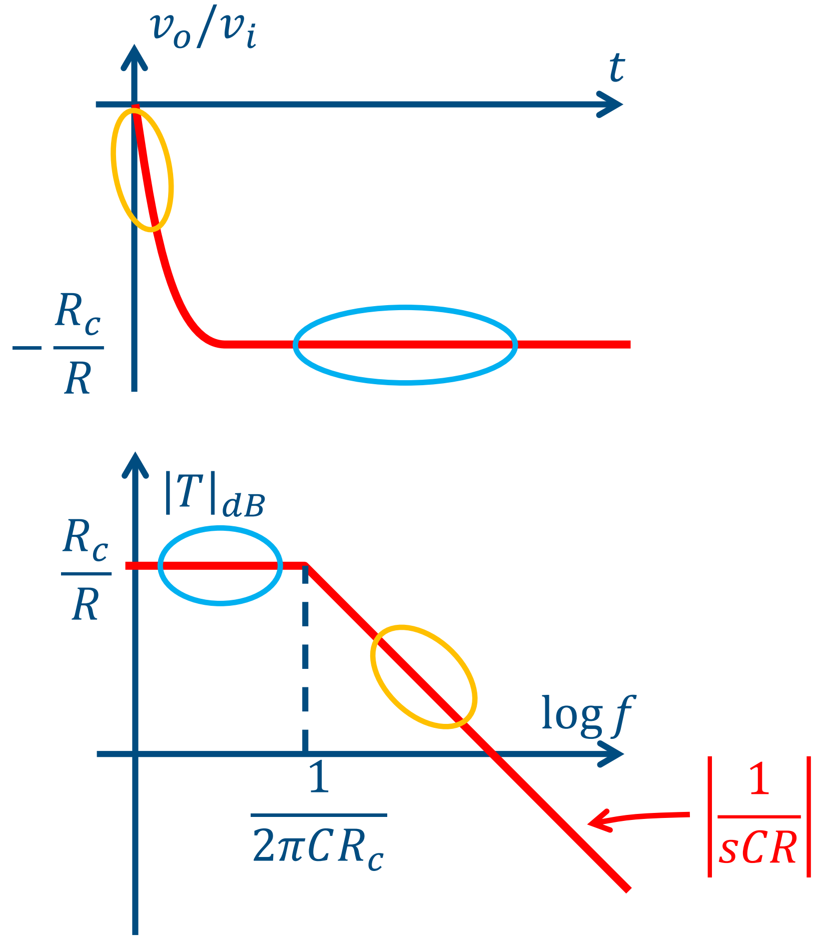


$$\frac{V_o}{V_i} = -\frac{Z}{R} = -\frac{R_c}{R}\frac{1}{1+sCR_c}$$

Step response

$$\frac{v_o(t)}{V_i} = -\frac{R_c}{R}\left(1 - e^{-\frac{t}{CR_c}}\right)$$

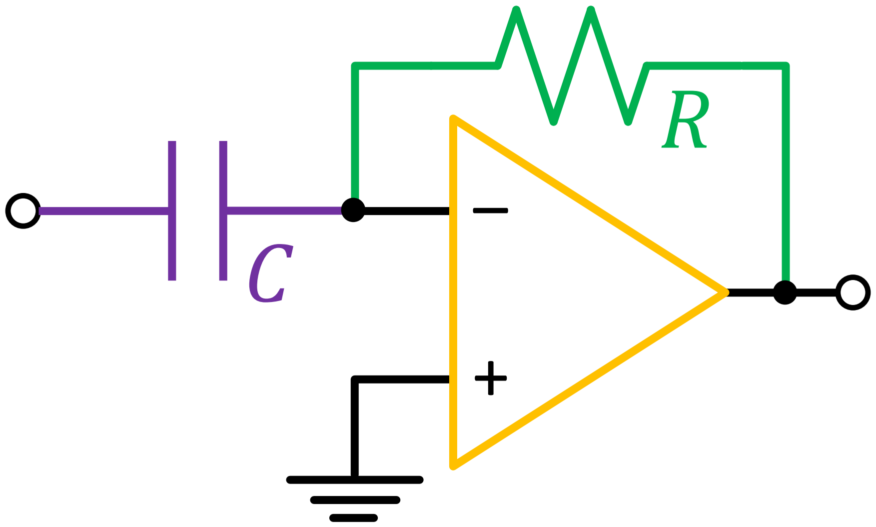
$$\approx \begin{cases} -\dfrac{R_c}{R} & t \gg CR_c \\ -\dfrac{R_c}{R}\dfrac{t}{CR_c} = \dfrac{-t}{CR} & t \ll CR_c \end{cases}$$



Outline

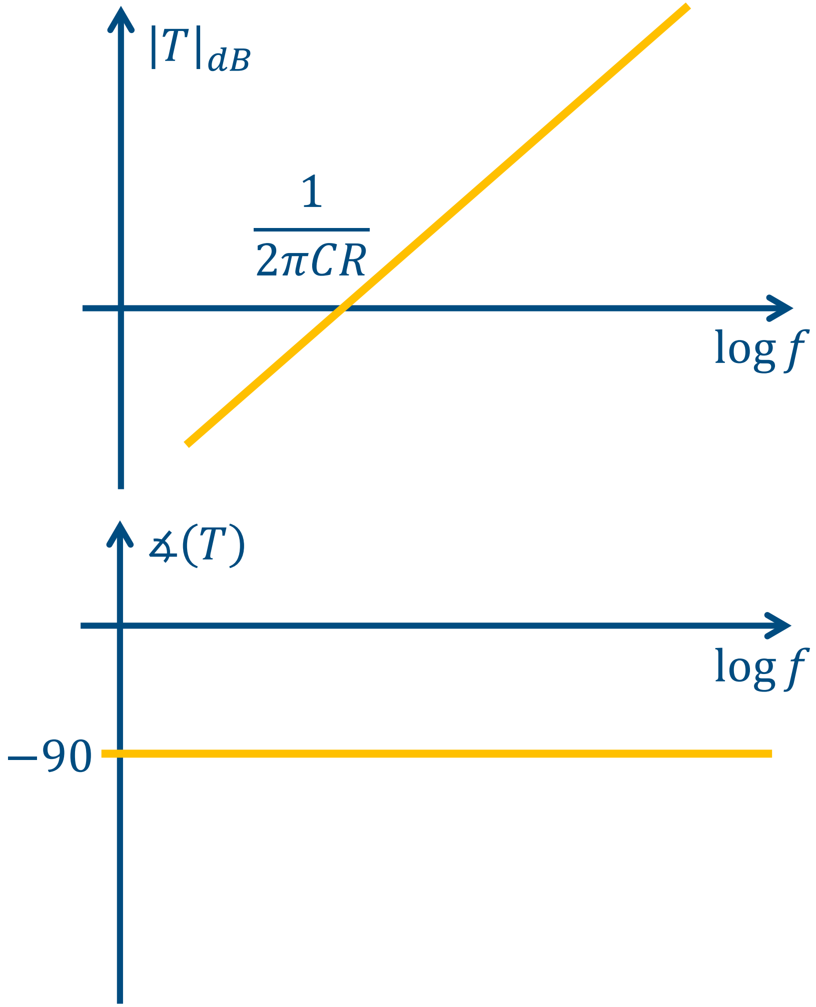
- Integrator stage
- Differentiator stage
- Phase shifter

Ideal differentiator

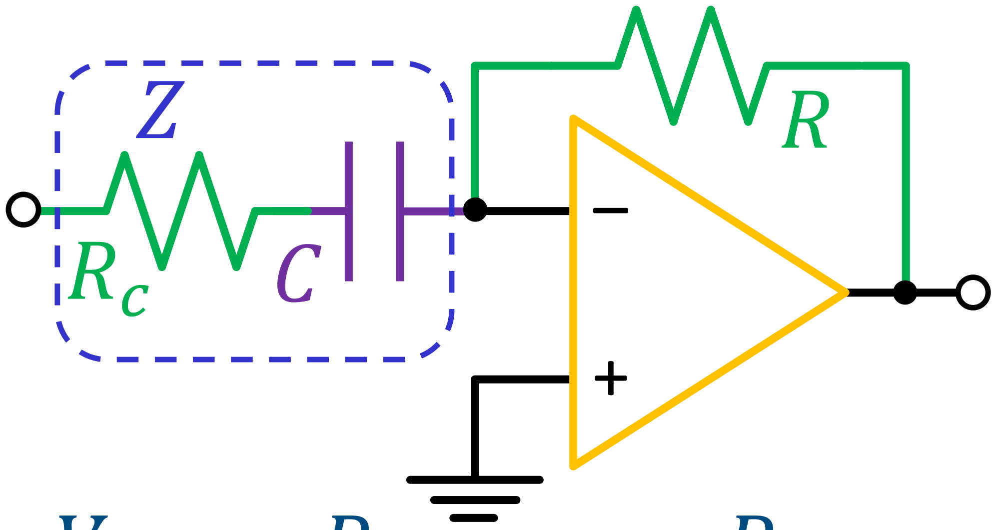


$$\frac{V_o}{V_i} = T(s) = -sCR$$

$$T(j\omega) = -j\omega CR$$

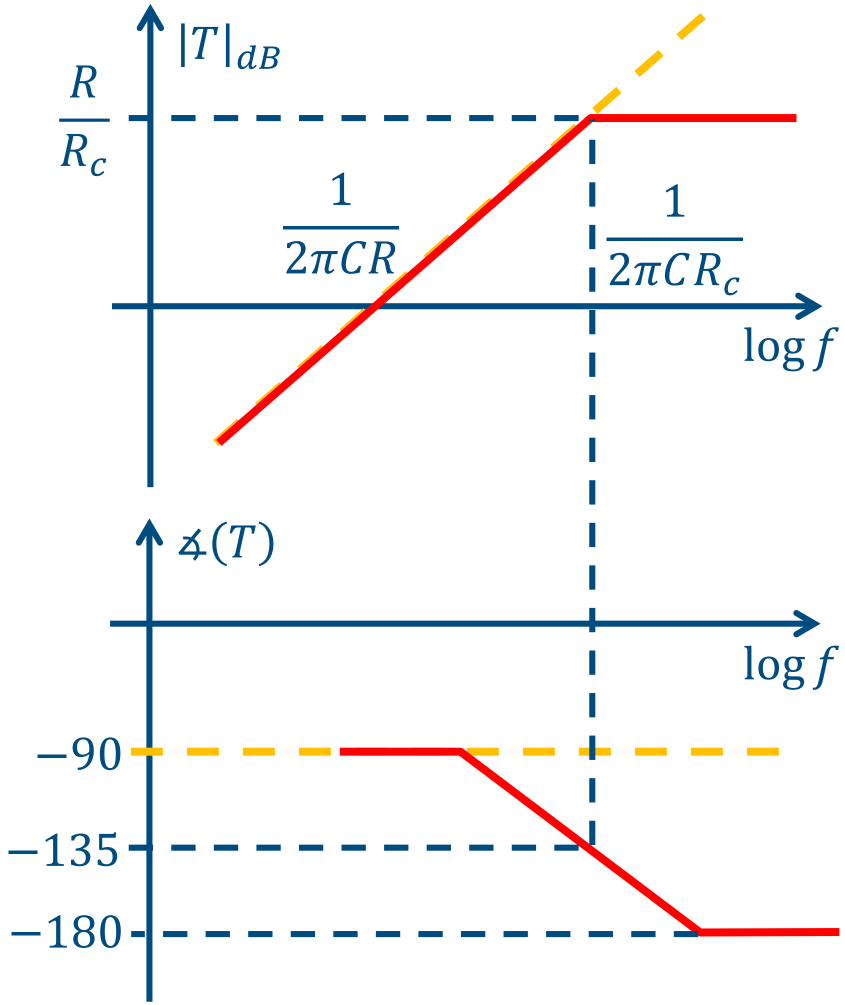


Practical differentiator



$$\frac{V_o}{V_i} = -\frac{R}{Z} = -\frac{R}{R_c + 1/sC}$$

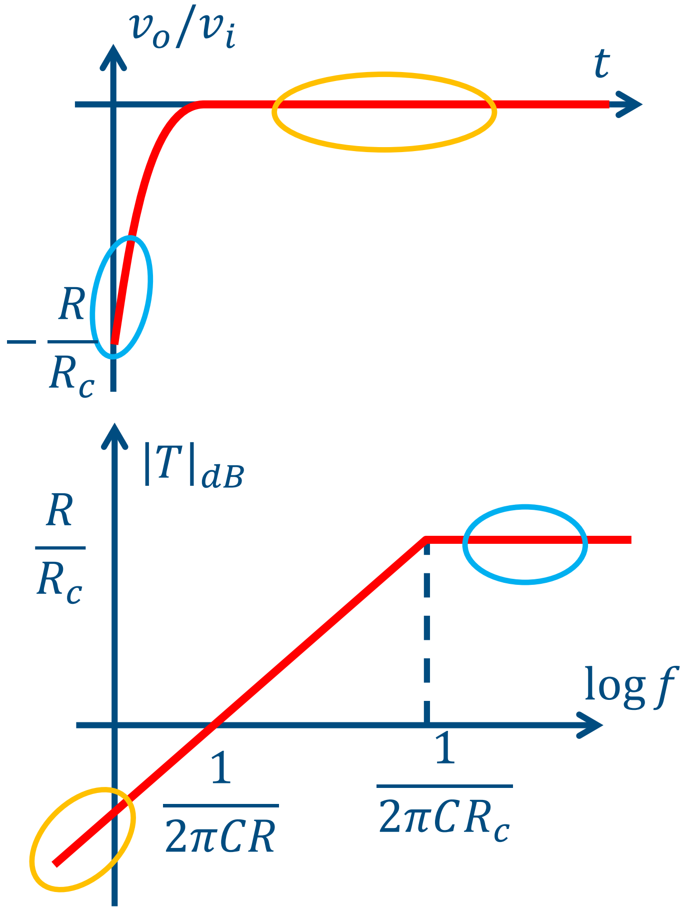
$$= -\frac{sCR}{1 + sCR_c}$$



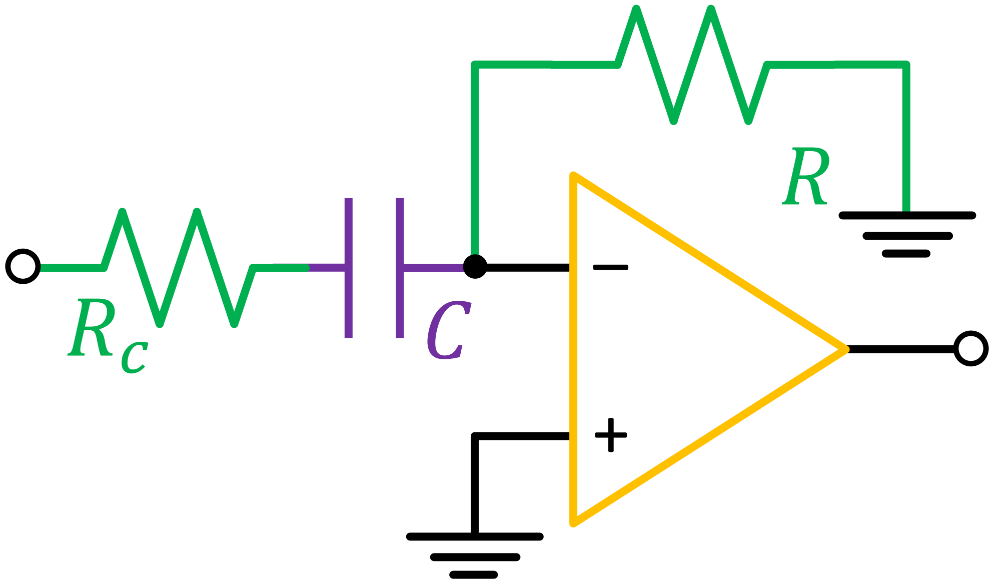
Step response

$$\frac{v_o(t)}{V_i} = -\frac{R}{R_c} e^{-\frac{t}{CR_c}}$$

$$\approx \begin{cases} -\dfrac{R}{R_c} & t \ll CR_c \\ \approx 0 & t \gg CR_c \end{cases}$$



Open-loop gain



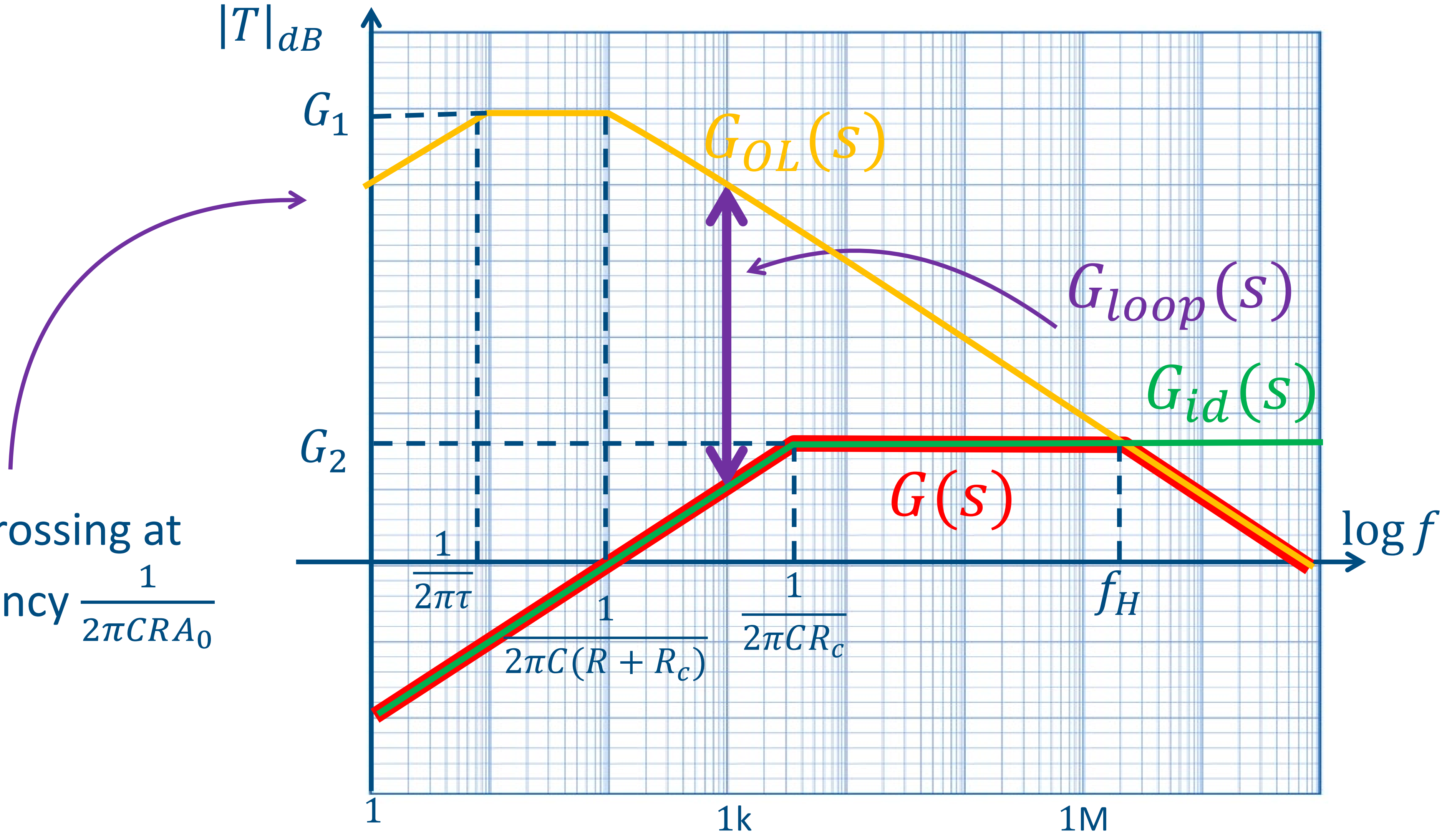
$C = 100$ nF
$R = 16$ kΩ
$R_c = 450\ \Omega$
$\tau = 22.5$ ms
$A_0 = 10^7$

$$G_{OL} = -A(s)\frac{R}{R+Z} = -\frac{A_0}{1+s\tau}\frac{sCR}{1+sC(R+R_c)}$$

$$G_{id} = -\frac{sCR}{1+sCR_c}$$

Closed-loop gain

0 dB crossing at frequency $\frac{1}{2\pi CRA_0}$



Calculations

- Poles:

$$\frac{1}{2\pi C R_c} = 3.54\text{ kHz};\quad \frac{1}{2\pi C(R+R_c)} = 97\text{Hz};\quad \frac{1}{2\pi\tau} \approx 7\text{Hz}$$

- Gains:

$$G_2 = \frac{R}{R_c} = 35.5 = 31\text{ dB}$$

$$2\pi\tau G_1 = 2\pi C R A_0 \Rightarrow G_1 = \frac{A_0 C R}{\tau} = 7.04 \cdot 10^5 = 117\text{ dB}$$

- BW:

$$\frac{1}{2\pi C(R+R_c)} G_1 = f_H G_2 \Rightarrow f_H = 97\frac{G_1}{G_2} = 1.9\text{ MHz}$$

Calculations – alternative

- Beyond the 2nd pole, G_{OL} can be approximated as (we drop the sign)

$$G_{OL} \approx \frac{A_0}{s\tau} \frac{R}{R + R_c}$$

- Then:

$$G_{OL}(s) = G_{id} = \frac{R}{R_c} \Rightarrow s = \frac{A_0}{\tau} \frac{R_c}{R + R_c} \Rightarrow f_H = \frac{A_0}{2\pi\tau} \frac{R_c}{R + R_c}$$

$$= 1.9 \text{ MHz}$$

Outline

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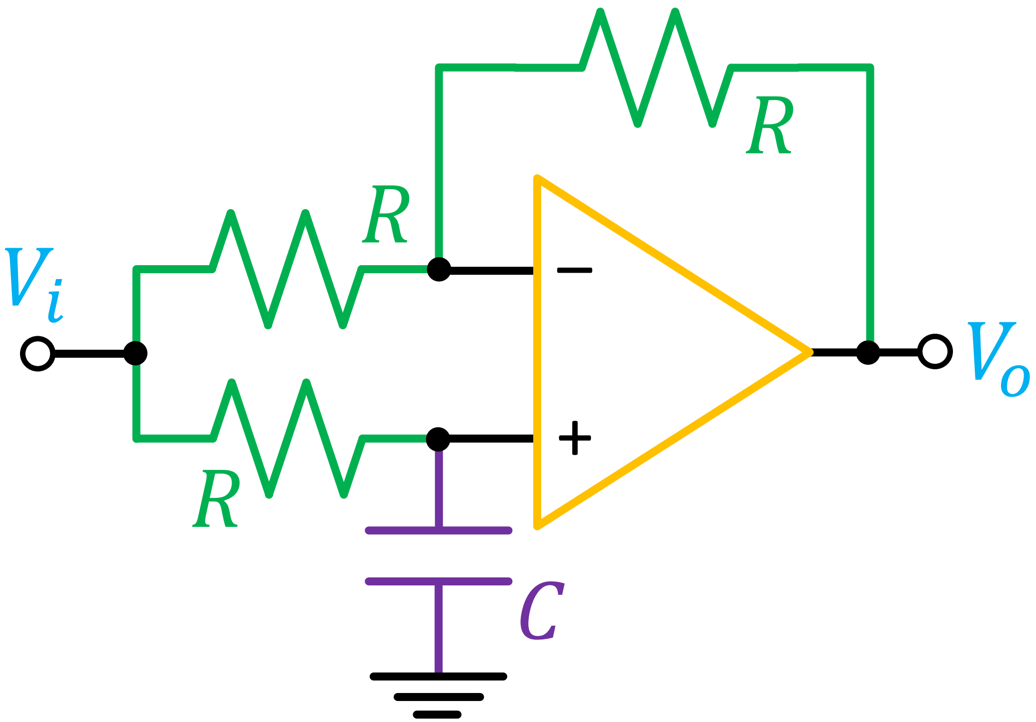
Phase shifter



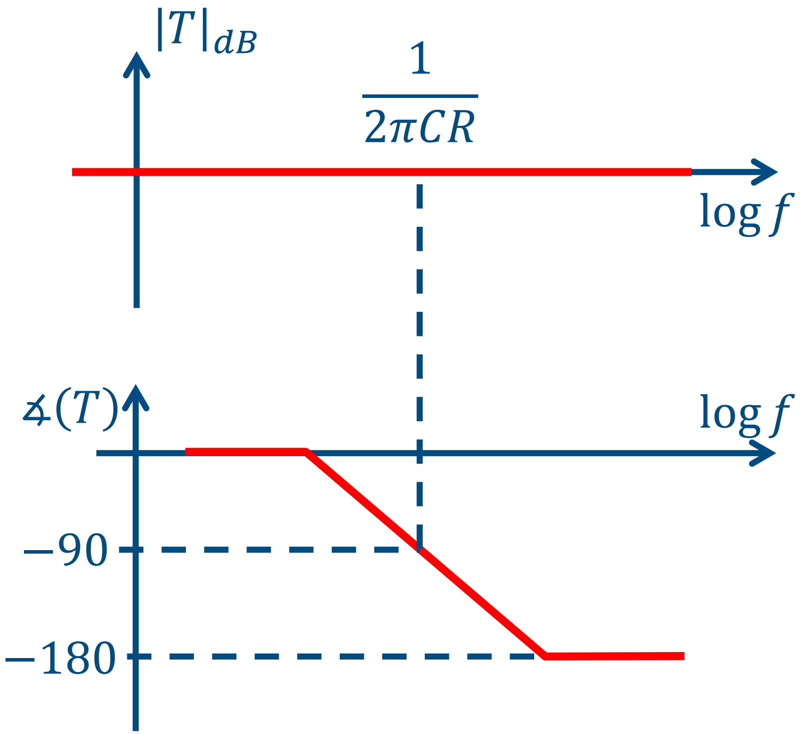
$C = 10$ nF
$R = 2$ kΩ
$\tau = 16$ ms
$A_0 = 10^6$

$$V_0 = -V_i + 2\frac{V_i}{1+sCR} = V_i\frac{1-sCR}{1+sCR} \Rightarrow G_{id} = \frac{1-sCR}{1+sCR}$$

Ideal gain

$$|G_{id}| = \left|\frac{1 - j\omega CR}{1 + j\omega CR}\right| = 1$$

$$\measuredangle(G_{id}) = \arctan(-\omega CR) - \arctan(\omega CR) = -2\arctan(\omega CR)$$



Step response

$$V_o(s) = \frac{1 - sCR}{1 + sCR}\frac{V_i}{s} = V_i\left(\frac{1}{s} - \frac{2CR}{1 + sCR}\right)$$

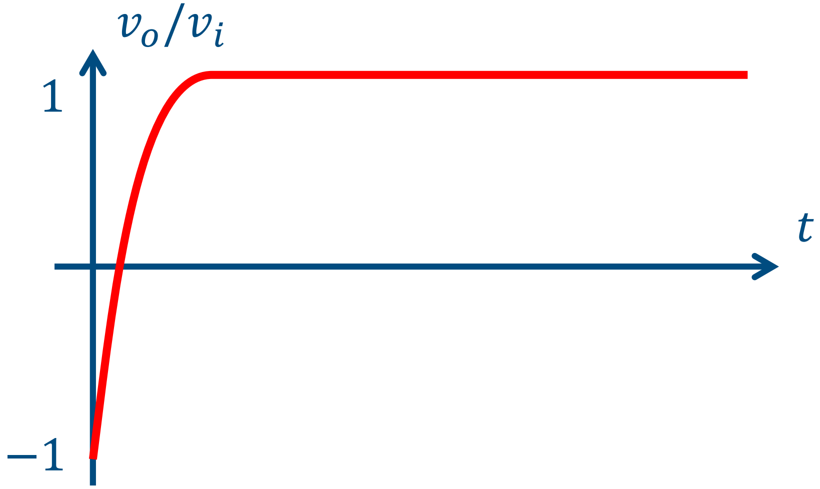
$$v_o(t) = V_i\left(1 - 2e^{-t/CR}\right)$$

Or, alternatively:

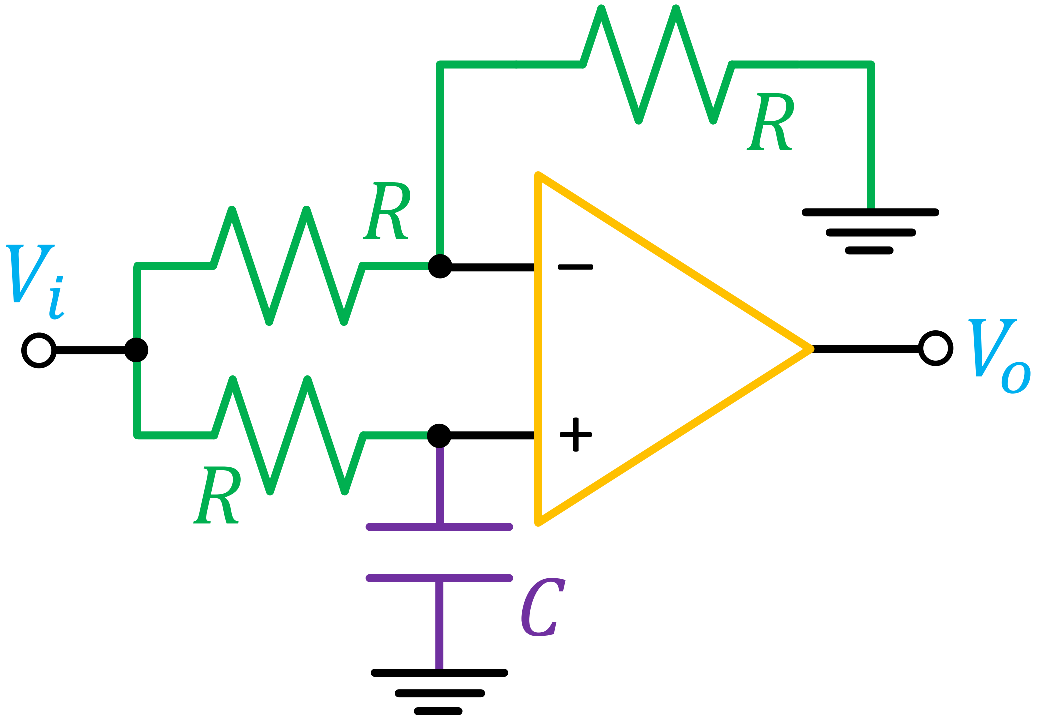
$$v_o(0) = \lim_{s\to\infty} sV_o(s) = -V_i$$

$$v_o(\infty) = \lim_{s\to 0} sV_o(s) = V_i$$

Pole time constant = $CR = 20\ \mu s$



Open-loop gain



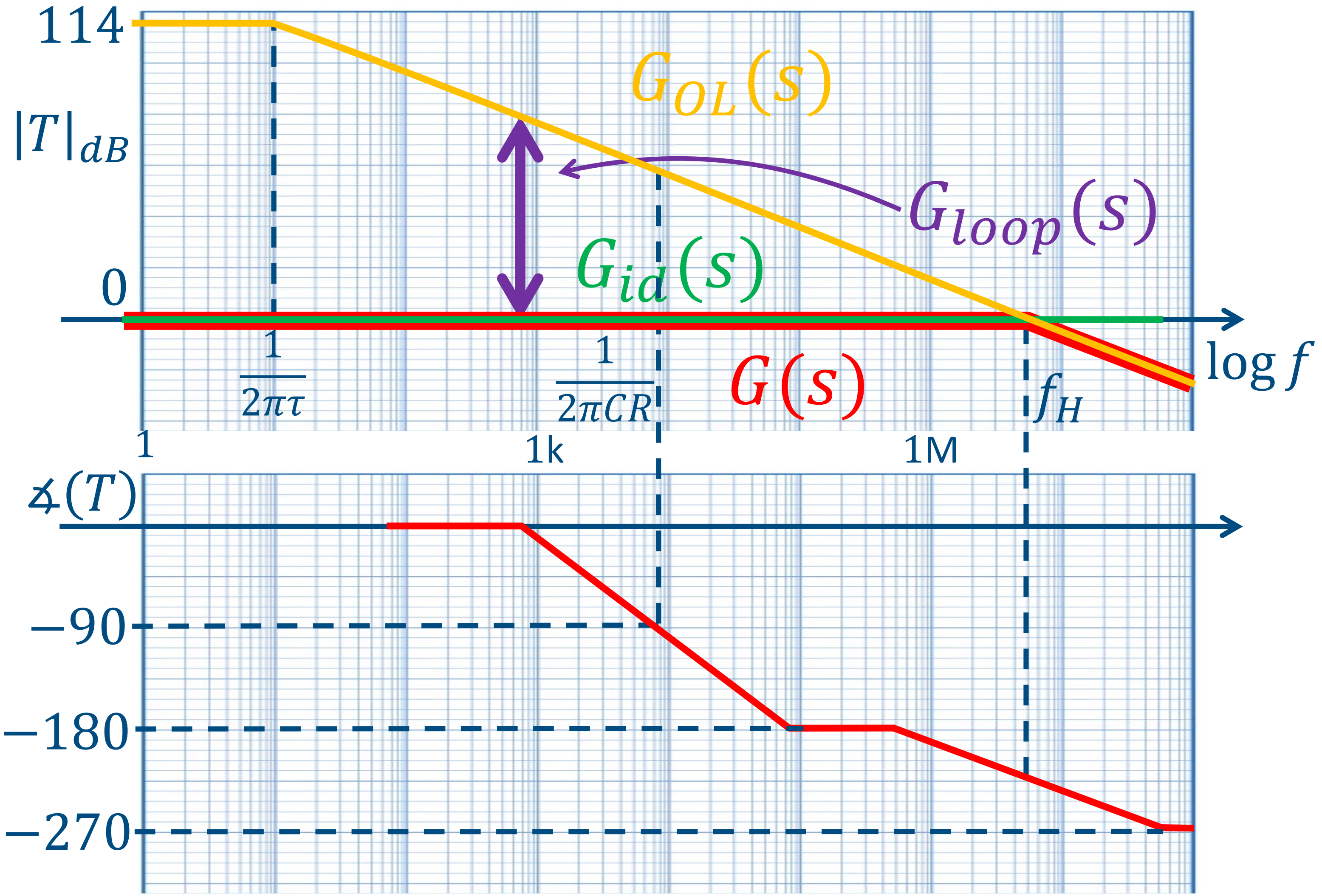
$$\frac{V_0}{V_i} = A(s)(V^+ - V^-)$$

$$= A(s)\left(\frac{1}{1+sCR} - \frac{1}{2}\right)$$

$$= \frac{A(s)}{2}\frac{1-sCR}{1+sCR} = G_{OL}$$

$$G_{loop} = -\frac{G_{OL}}{G_{id}} = -\frac{1}{2}A(s)$$

Closed-loop gain



Homework

1. Plot the Bode diagram of the closed-loop gain for the scheme in slide #11
2. In the scheme of slide #15, remove resistor R_c and place a capacitor C_c in parallel to R. Compute the gain
3. Compute (approximately) the step response of the phase-shifter circuit accounting for the real gain