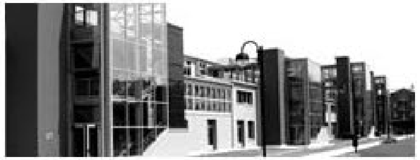
Electronics – 96032

POLITECNICO DI MILANO





I/O Impedances Calculation

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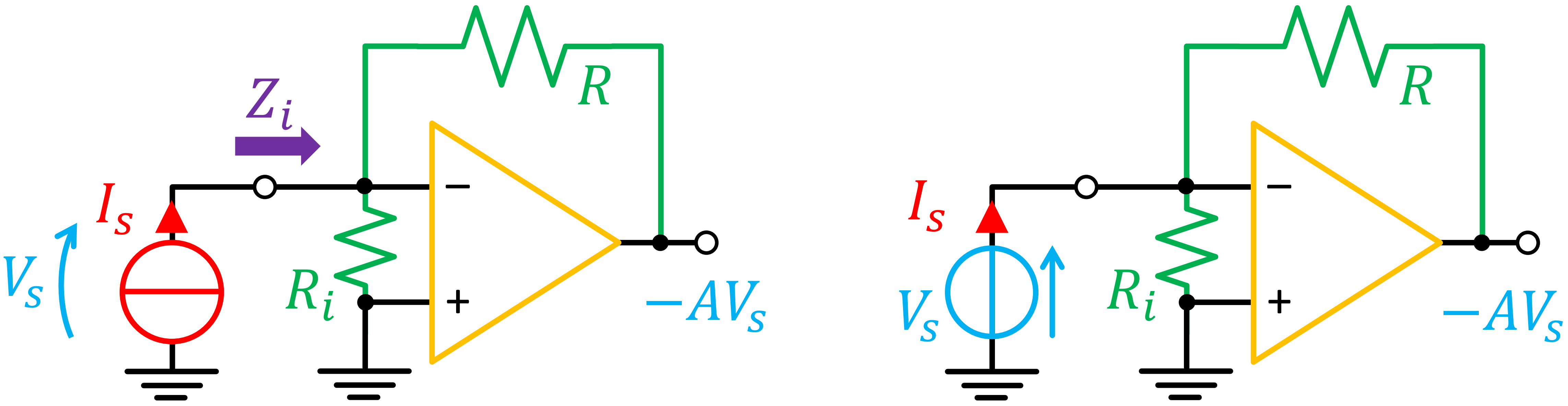
Disclaimer

Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes

Outline

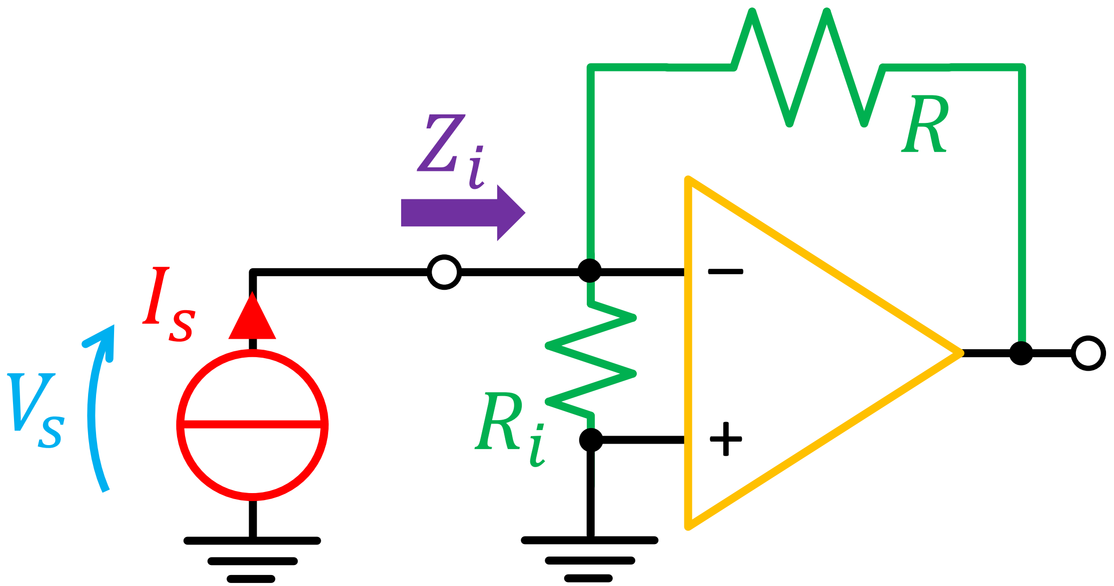
- Choice of a suitable test source
- Ex: Amplifier
  - Input impedance (I input)
  - Output impedance
  - Input impedance (NI input)
- Differential amplifiers
- Appendix: impedance removal

Direct calculation (no feedback theory)



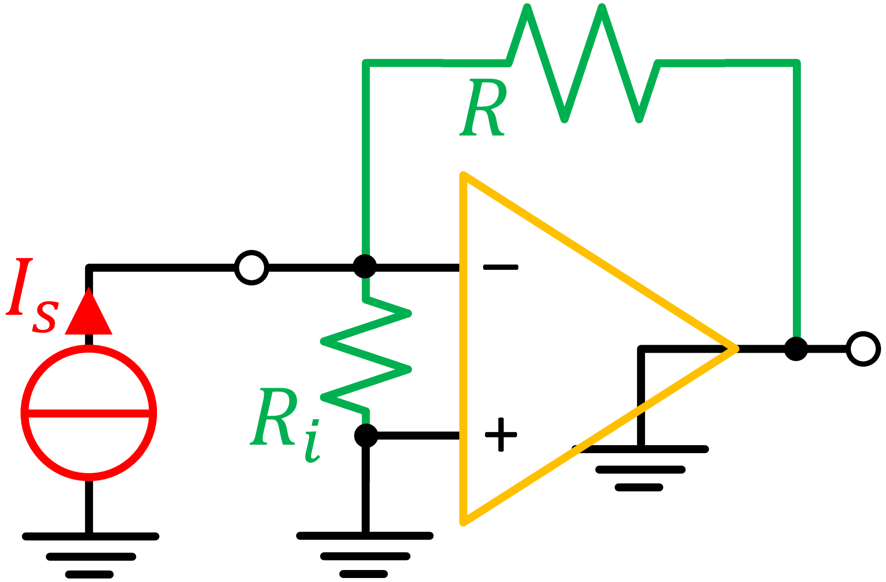
$$I_s = \frac{V_s}{R_i} + \frac{V_s(1+A)}{R} \Rightarrow Z_{in} = \frac{1}{\frac{1}{R_i} + \frac{1+A}{R}} = \frac{RR_i}{R + R_i + AR_i}$$

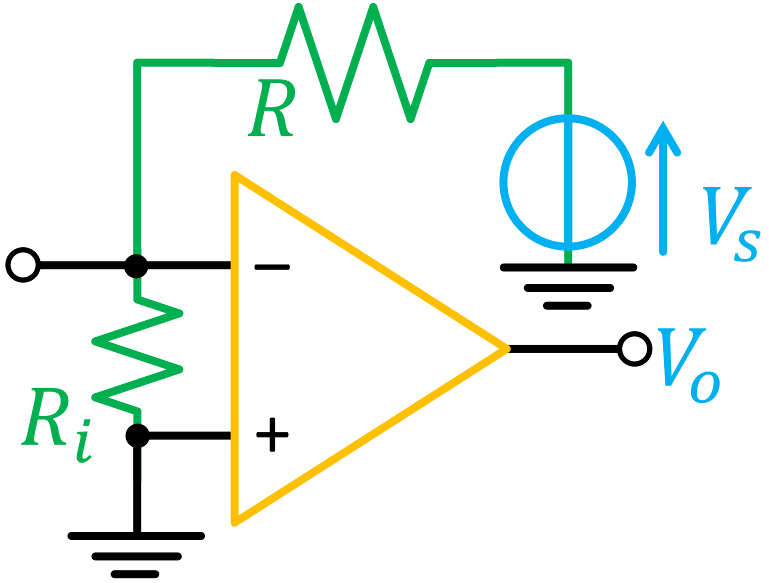
Ideal case (current source)



$$V_s = V^- = 0 \Rightarrow Z_{id} = \frac{V_s}{I_s} = 0 \Rightarrow Z_{in} = \frac{Z_{OL}}{1 - G_{loop}}$$

Z_{OL} and G_{loop}





$$Z_{OL} = R_i \parallel R$$

$$G_{loop} = -A(s)\frac{R_i}{R + R_i}$$

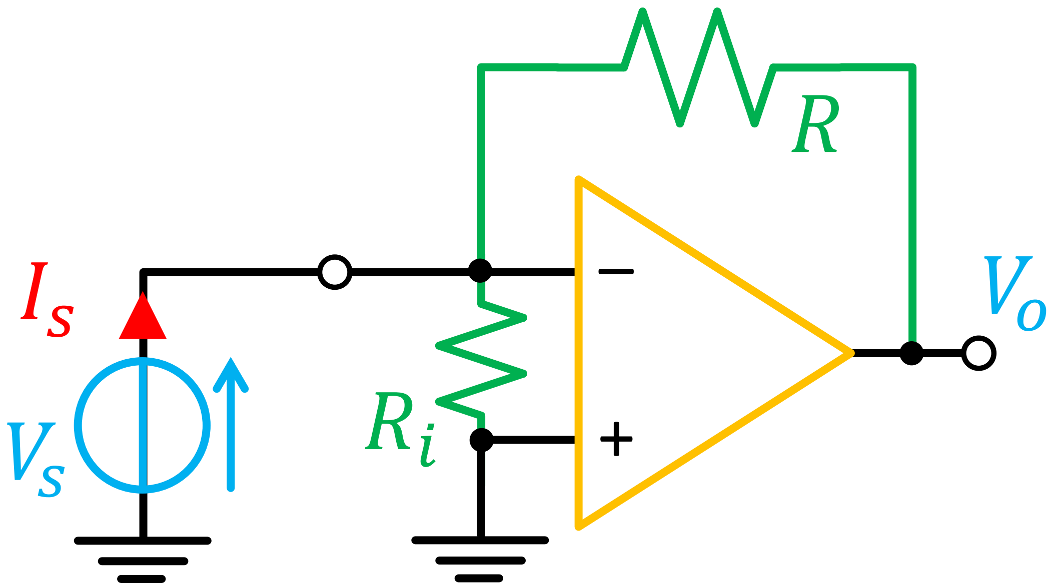
Final result

$$Z_{in} = \frac{Z_{OL}}{1 - G_{loop}} = \frac{\dfrac{RR_i}{R + R_i}}{1 + \dfrac{AR_i}{R + R_i}} = \frac{RR_i}{R + R_i + AR_i}$$

Side note: another way to look at the circuit is to consider R_i in parallel to the rest. The remaining circuit has $Z_{OL} = R$ and $G_{loop} = -A$, so that

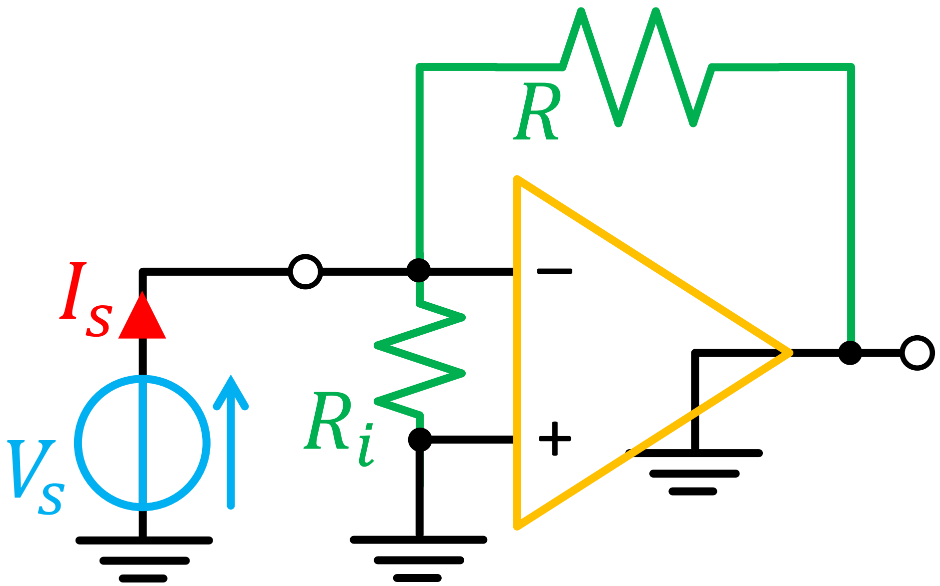
$$Z_{in} = R_i \parallel \frac{R}{1 + A} = \frac{RR_i}{R + R_i + AR_i}$$

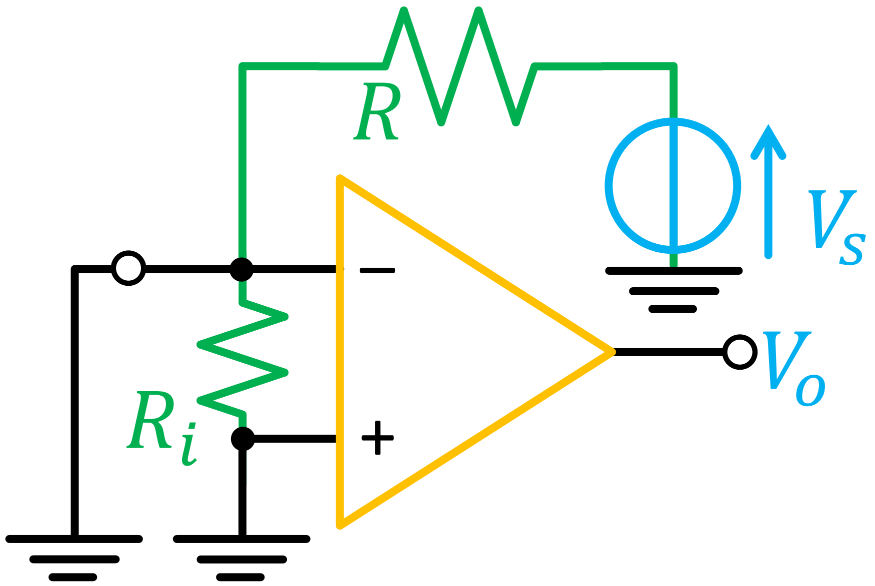
Ideal case (voltage source)



$$V_o = -\infty \Rightarrow I_s = \infty \Rightarrow Z_{id} = \frac{V_s}{I_s} = 0 \Rightarrow Z_{in} = \frac{Z_{OL}}{1 - G_{loop}}$$

Z_{OL} and G_{loop}





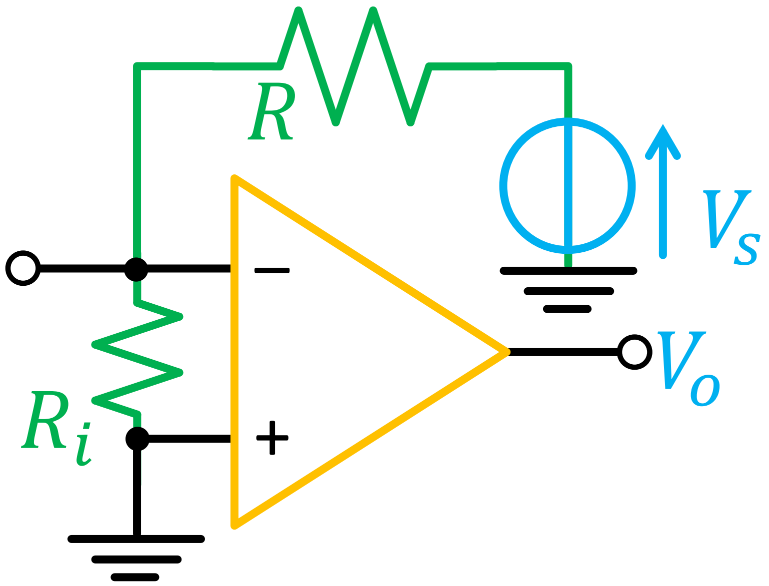
$$Z_{OL} = R_i \parallel R$$

$$G_{loop} = 0\ (?)$$

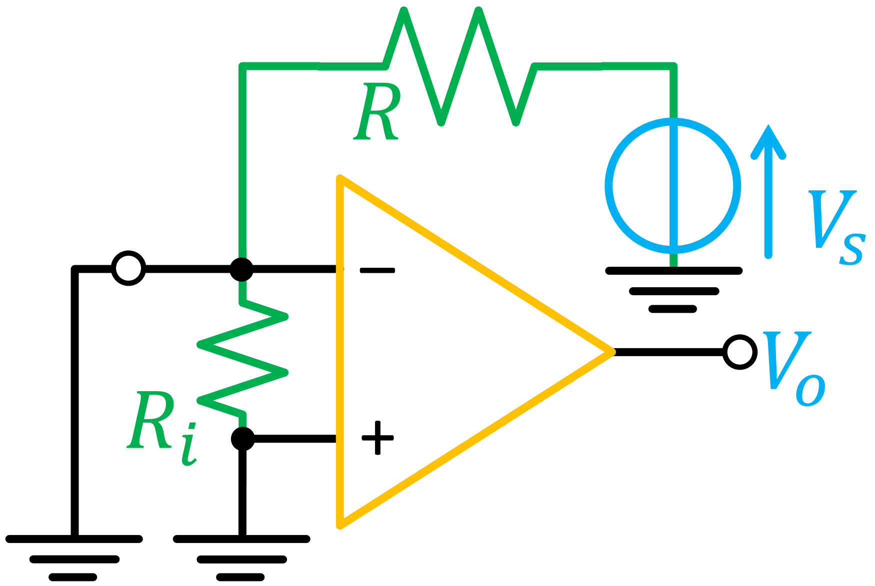
Comments

- Feedback circuit theory works if the loop is not broken, i.e., if $G_{loop} \neq 0$
- If you use the «wrong» source, you break the loop and end up with ∞ in voltages and/or currents, i.e., with inconsistencies in the equations in the ideal case (e.g., biasing a virtual ground or forcing currents through an infinite impedance)
- Practically
  - Low Z (e.g., virtual ground, inv. input of OAs and OA output) ⇒ current source
  - High Z (e.g., non inv. input of OAs) ⇒ voltage source

Blackman's formula



$G_{loop}^{oc} \neq 0$



$G_{loop}^{sc} = 0$

$$Z = Z_{OL}\frac{1 - G_{loop}^{sc}}{1 - G_{loop}^{oc}}$$

Outline

- Choice of a suitable test source
- Ex: Amplifier
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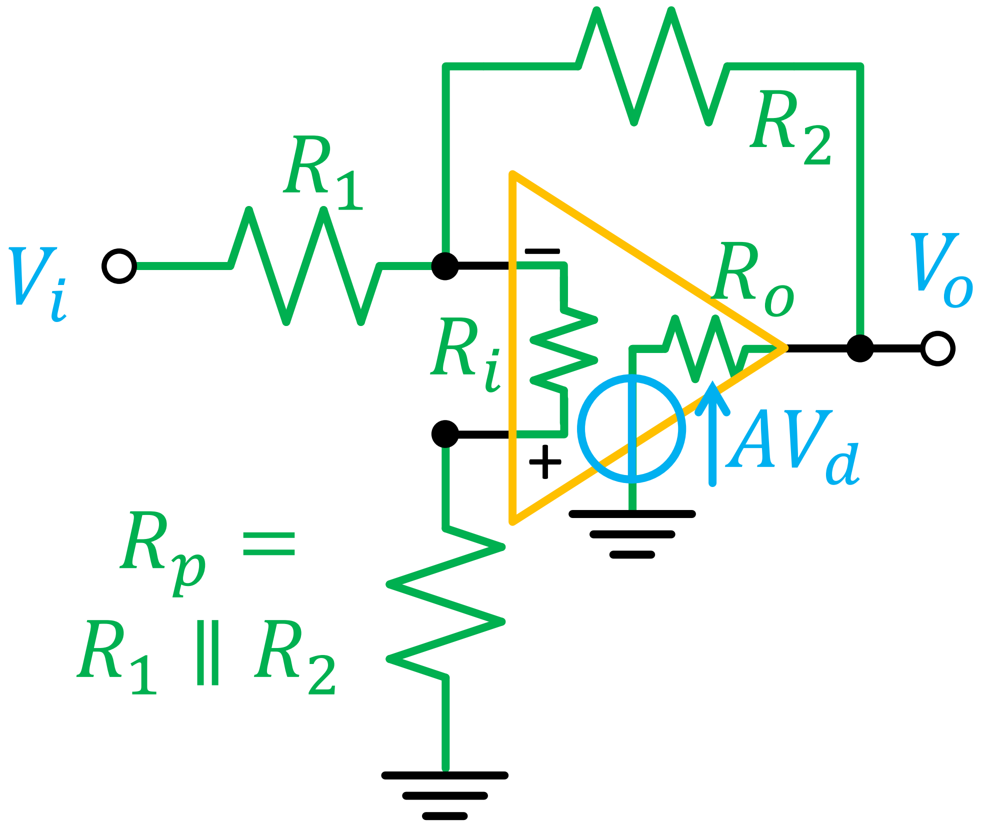
- Difference amplifiers
- Appendix: impedance removal

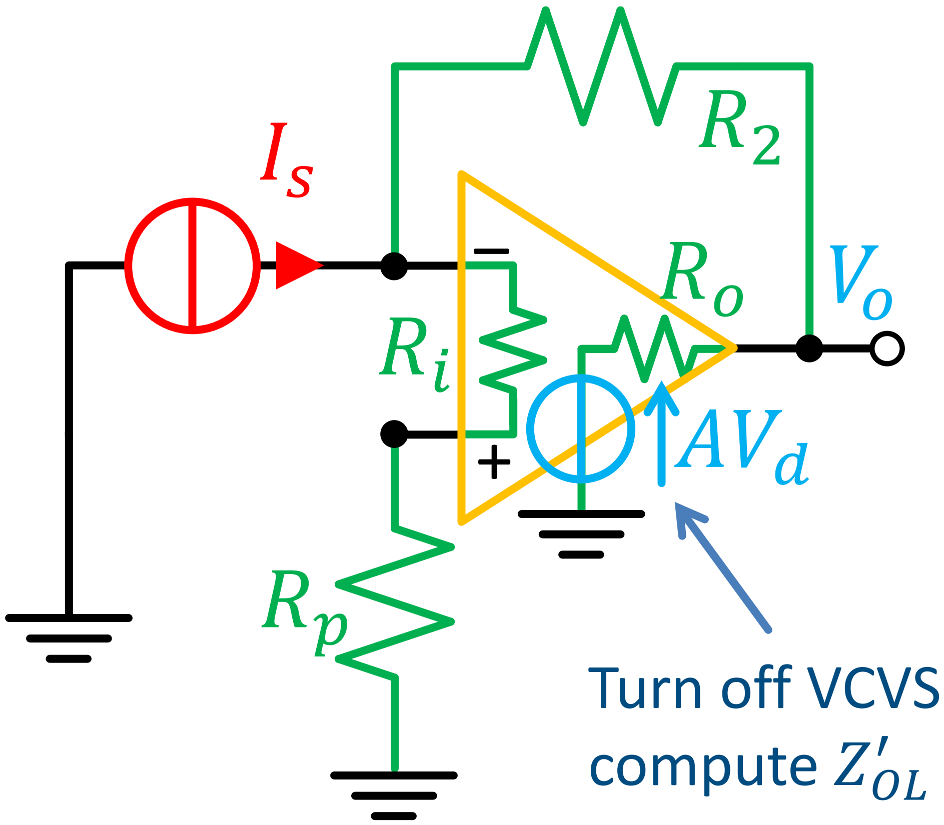
Inverting amplifier



- $R_1 = 10\ \text{k}\Omega$
- $R_2 = 100\ \text{k}\Omega$
- $R_i = 2\ \text{M}\Omega$
- $R_o = 75\ \Omega$
- $A_0 = 106\ \text{dB}$
- $GBWP = 1\ \text{MHz}$

1. Compute input and output impedances

Z_{in}



Turn off VCVS to compute Z'_{OL}

$$Z_{in} = R_1 + Z'$$

$$Z'_{id} = 0 \text{ (virtual ground)}$$

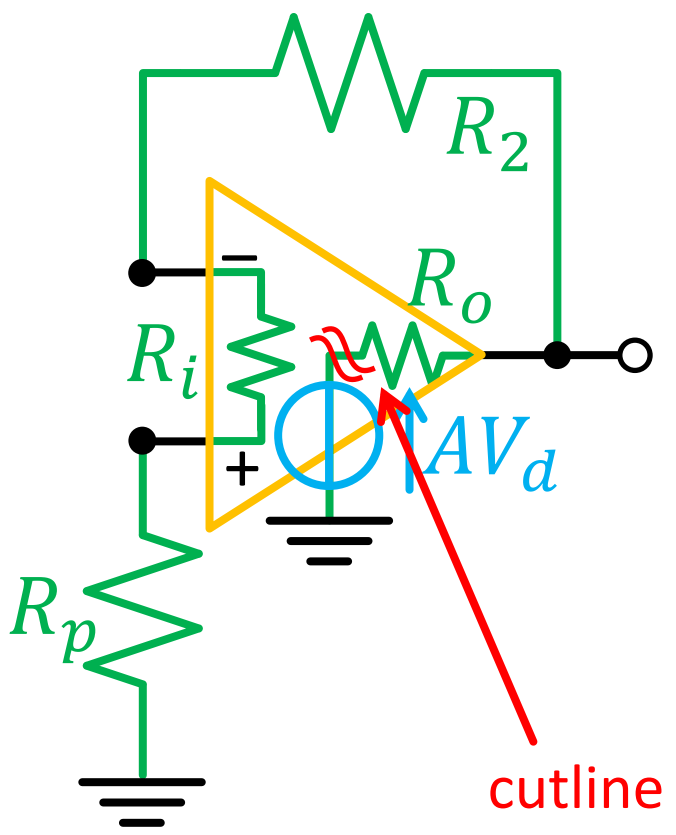
⇙ ⇘

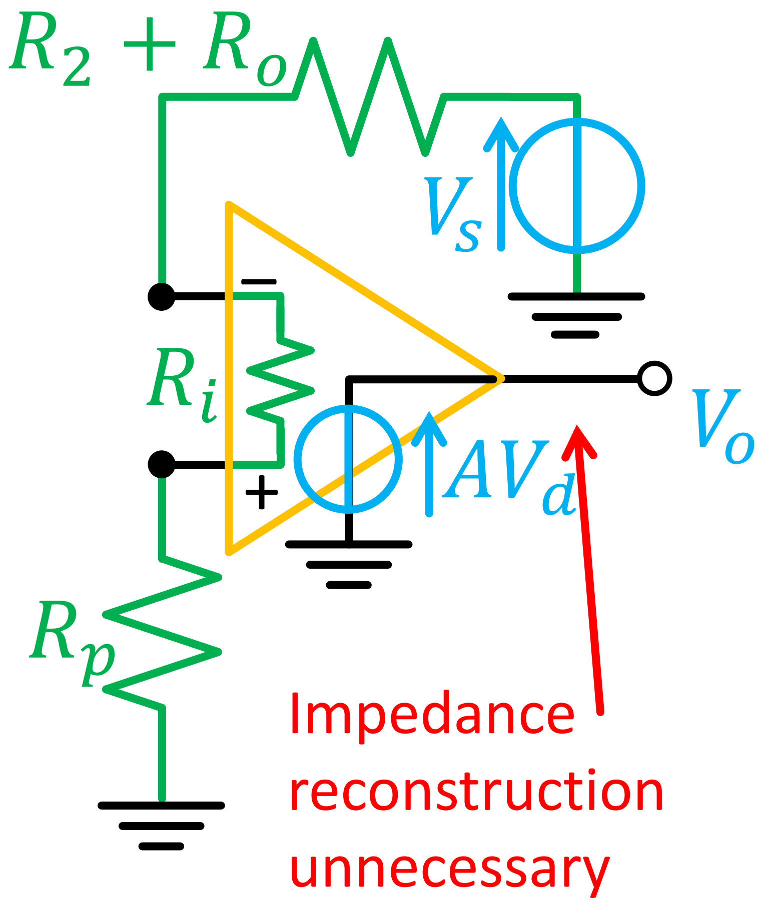
$$Z' = \frac{Z'_{OL}}{1 - G'_{loop}}$$

A current source must be used as test signal

$$Z'_{OL} = \left(R_i + R_p\right) \parallel \left(R_2 + R_0\right) = 95.2\ \text{k}\Omega$$

G'_{loop}


R2
Ri
Ro
+
AVd
Rp
cutline


R2 + Ro
Vs
Ri
+
AVd
Vo
Rp
Impedance
reconstruction
unnecessary

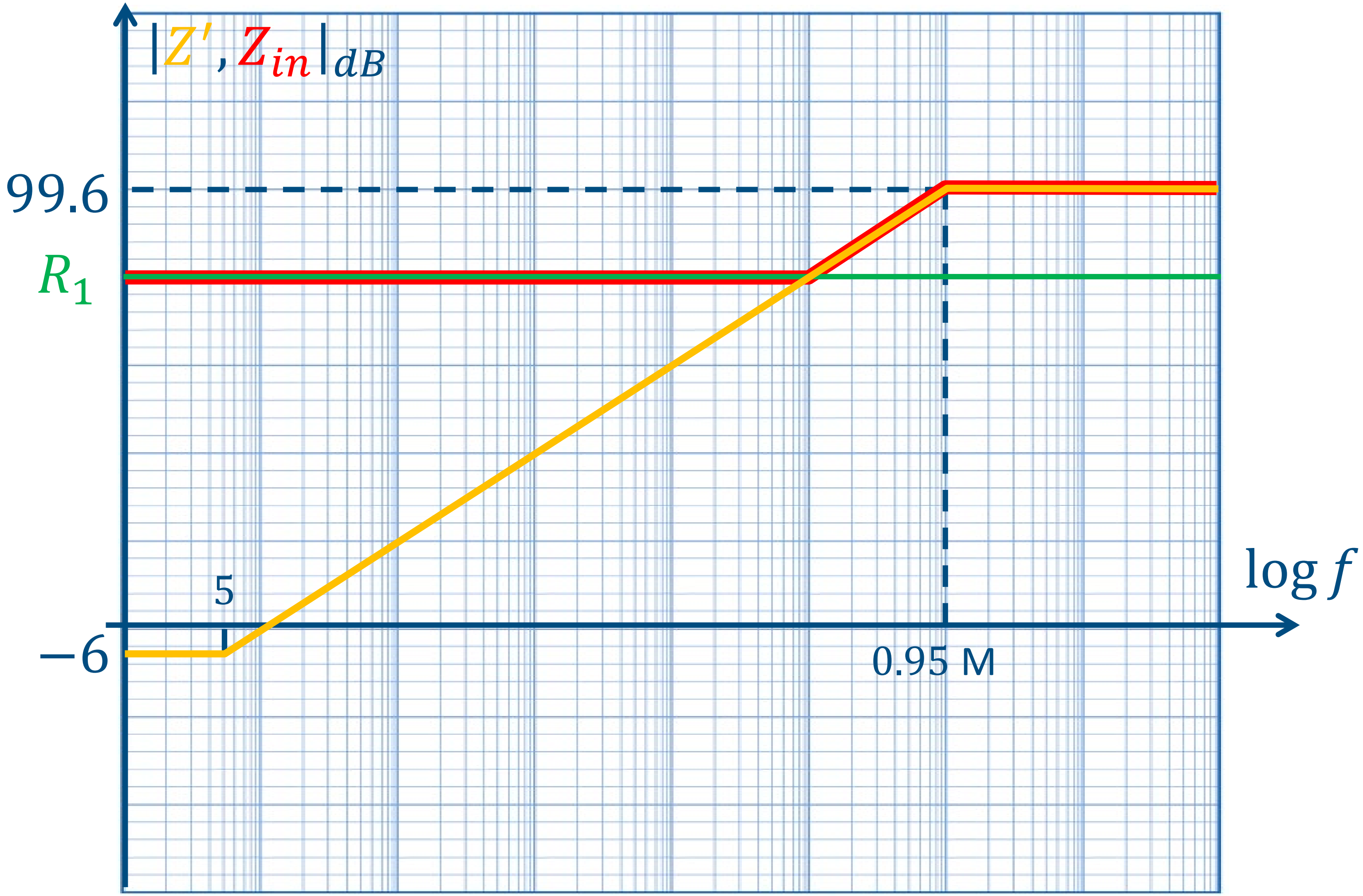
Result

$$G'_{loop} = -A(s)\frac{R_i}{R_i + R_p + R_2 + R_o} = -0.95\,A(s)$$

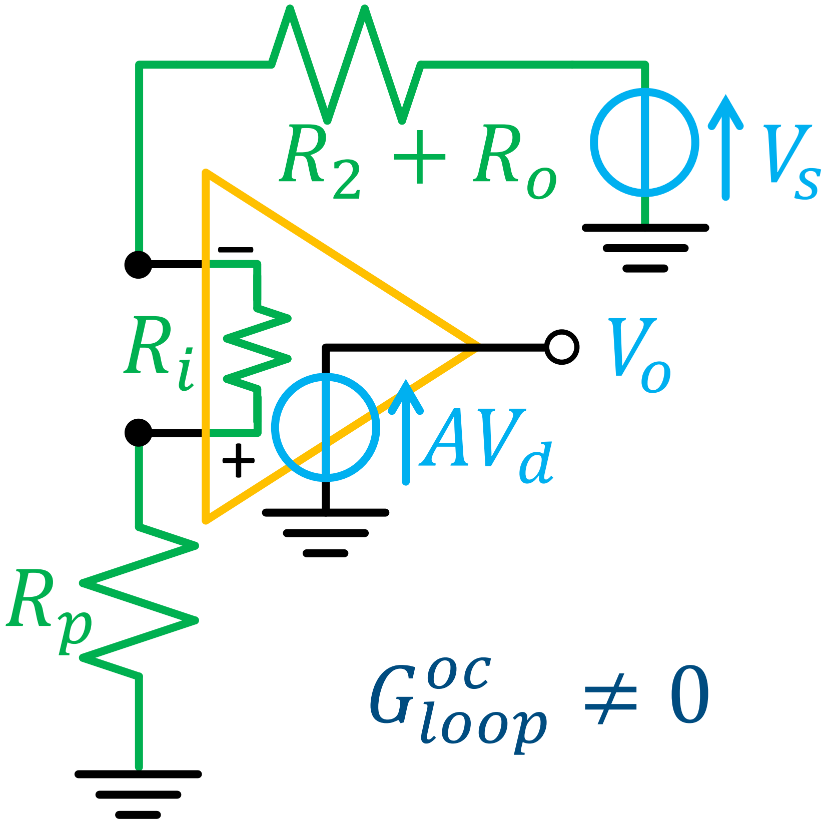
- Under DC condition, $A(s) = A_0 = 2 \times 10^5 \Rightarrow \left|G'_{loop}\right| = 1.9 \times 10^5 \Rightarrow Z' = 0.5\ \Omega \Rightarrow Z_{in} = R_1 + Z' = R_1 = 10\ \text{k}\Omega$
- At higher frequencies

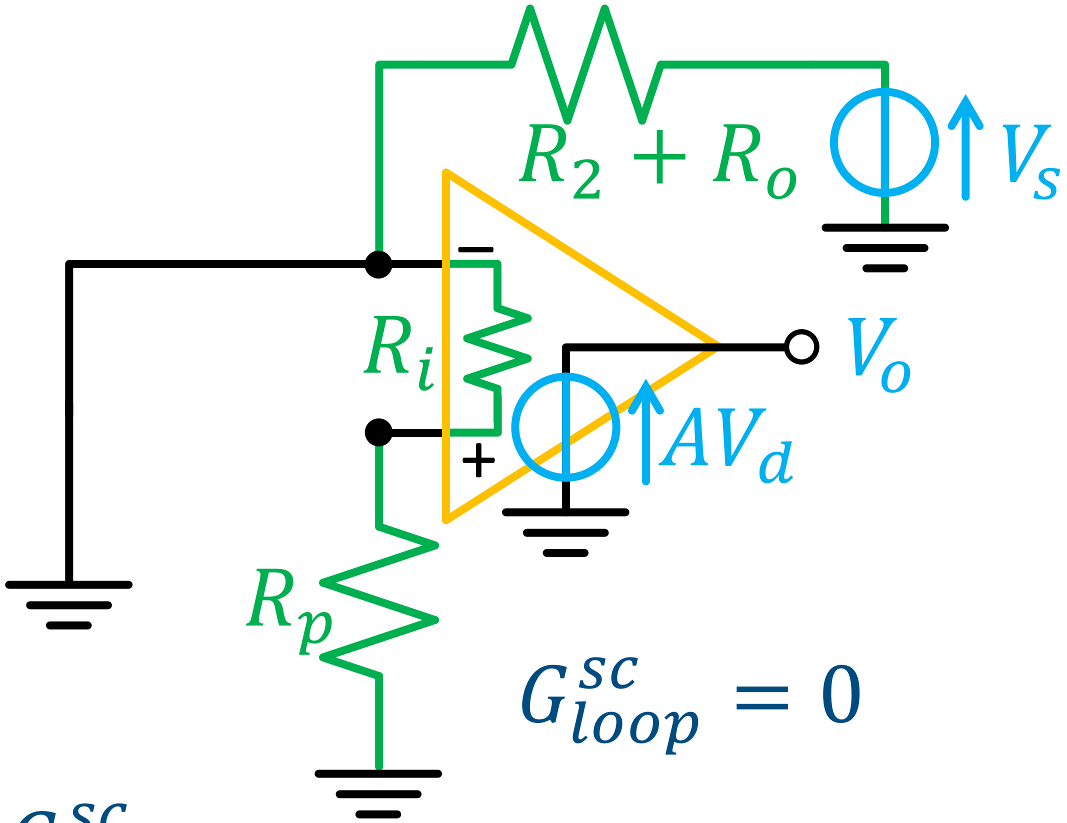
$$Z' = \frac{Z_{OL}}{1 + \dfrac{1.9 \times 10^5}{1 + s\tau}} \approx Z_{OL}\frac{1 + s\tau}{1.9 \times 10^5 + s\tau}$$

Bode plot



Blackman's formula





$$Z = Z_{OL}\frac{1 - G_{loop}^{sc}}{1 - G_{loop}^{oc}}$$

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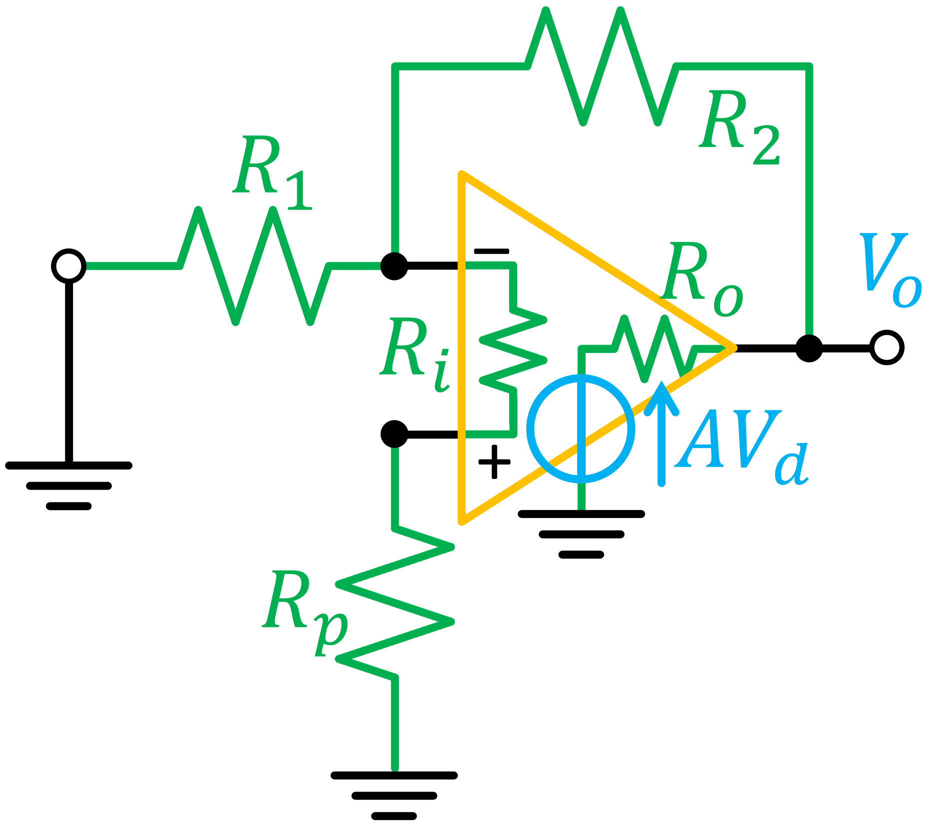
Z_{out}



$$Z_{id} = 0$$

⇙ ⇘

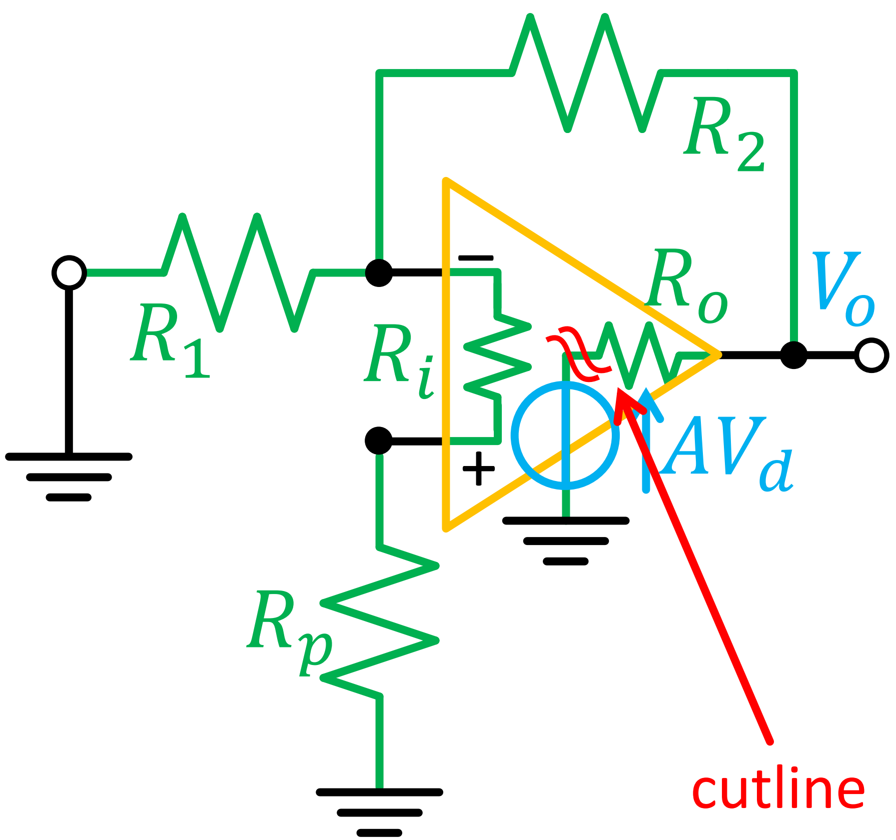
$$Z_{out} = \frac{Z_{OL}}{1 - G_{loop}}$$

A current source must be used as test signal

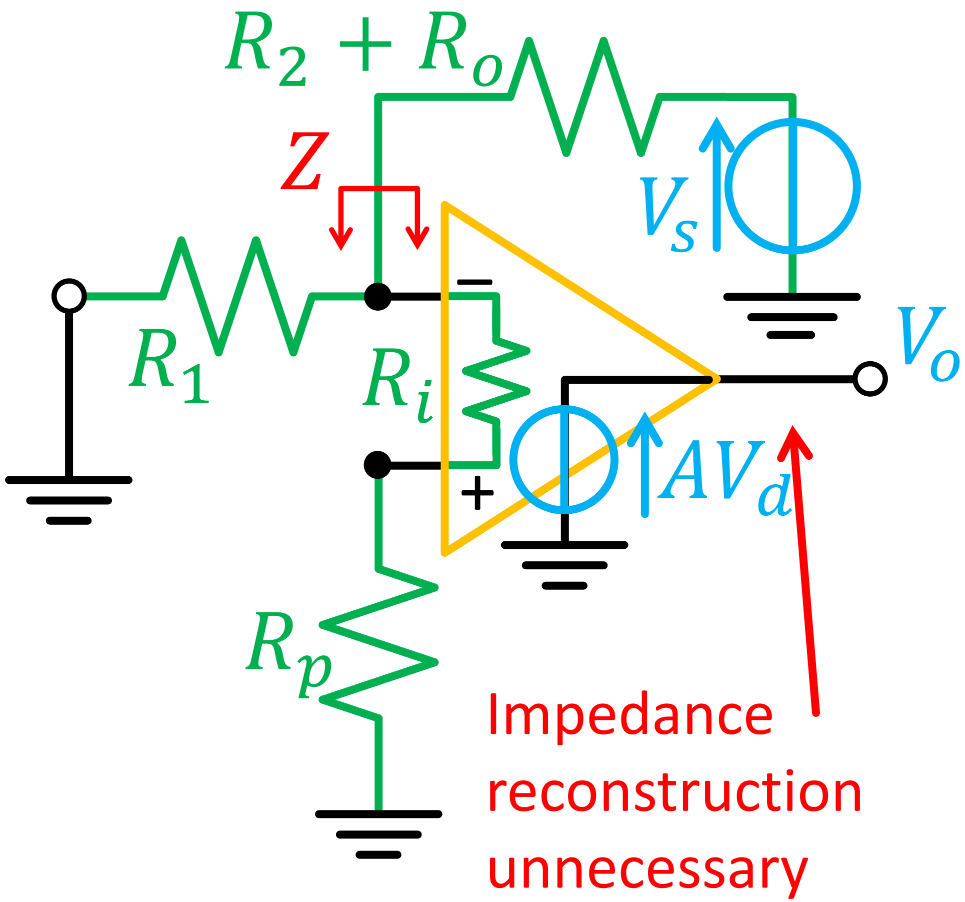
$$Z_{OL} = \left(\left(R_i + R_p\right) \parallel R_1 + R_2\right) \parallel R_0$$

$$\approx R_o = 75\ \Omega$$

G_{loop}



R_2
R_1
R_i
R_o
V_o
AV_d
R_p
−
+
cutline



$R_2 + R_o$
Z
V_s
R_1
R_i
V_o
AV_d
R_p
−
+
Impedance
reconstruction
unnecessary

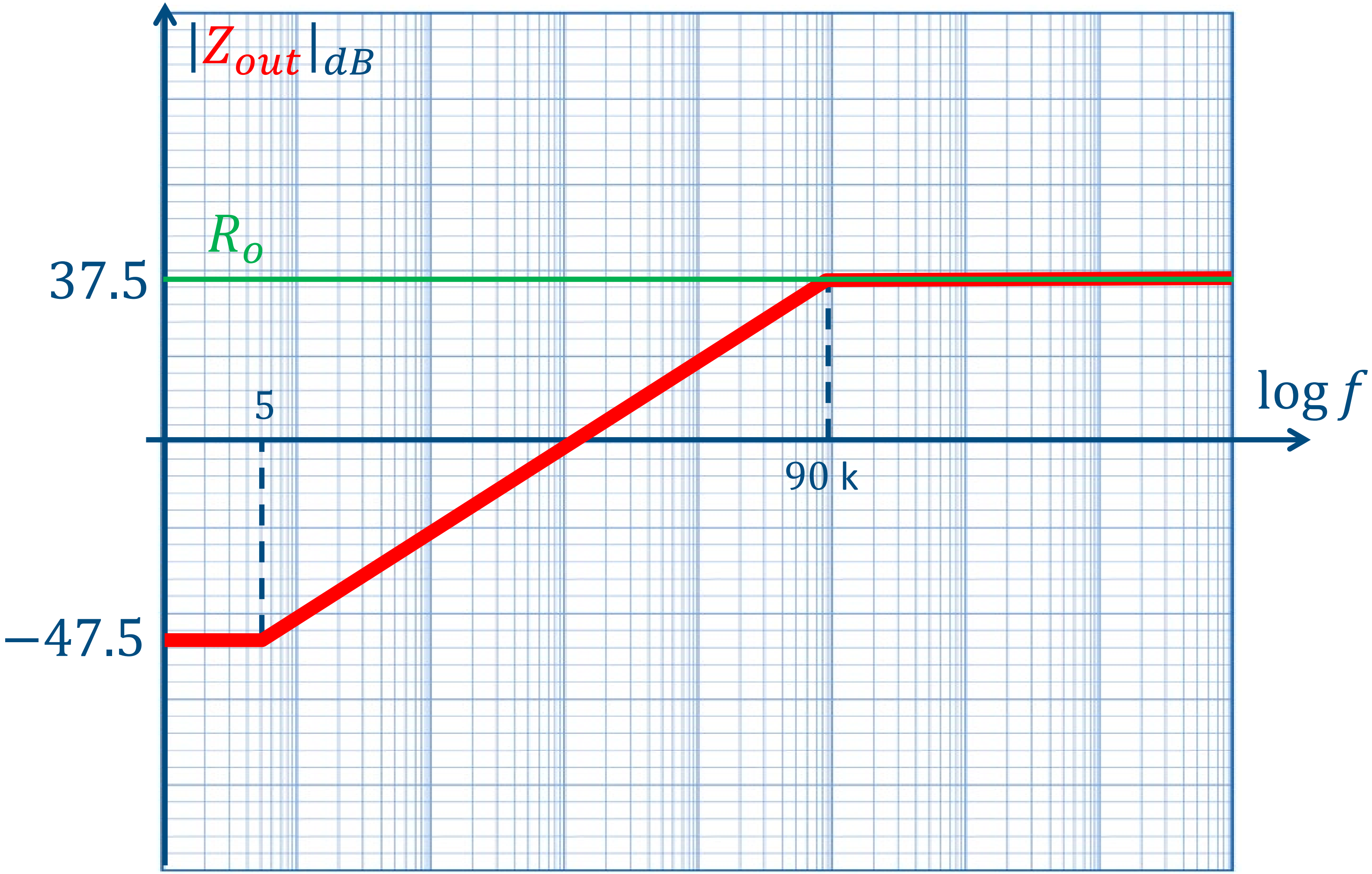
Result

$$G_{loop} = -A(s)\frac{Z}{Z + R_2 + R_o}\frac{R_i}{R_i + R_p} = -0.09\,A(s)$$

- Under DC condition, $A(s) = A_0 = 2 \times 10^5 \Rightarrow G'_{loop} = 1.8 \times 10^4$ and $Z_{out} = 4.2\ \text{m}\Omega$
- At high frequencies

$$Z_{out} = \frac{Z_{OL}}{1 + \dfrac{1.8 \times 10^4}{1 + s\tau}} \approx Z_{OL}\frac{1 + s\tau}{1.8 \times 10^4 + s\tau}$$

Bode plot



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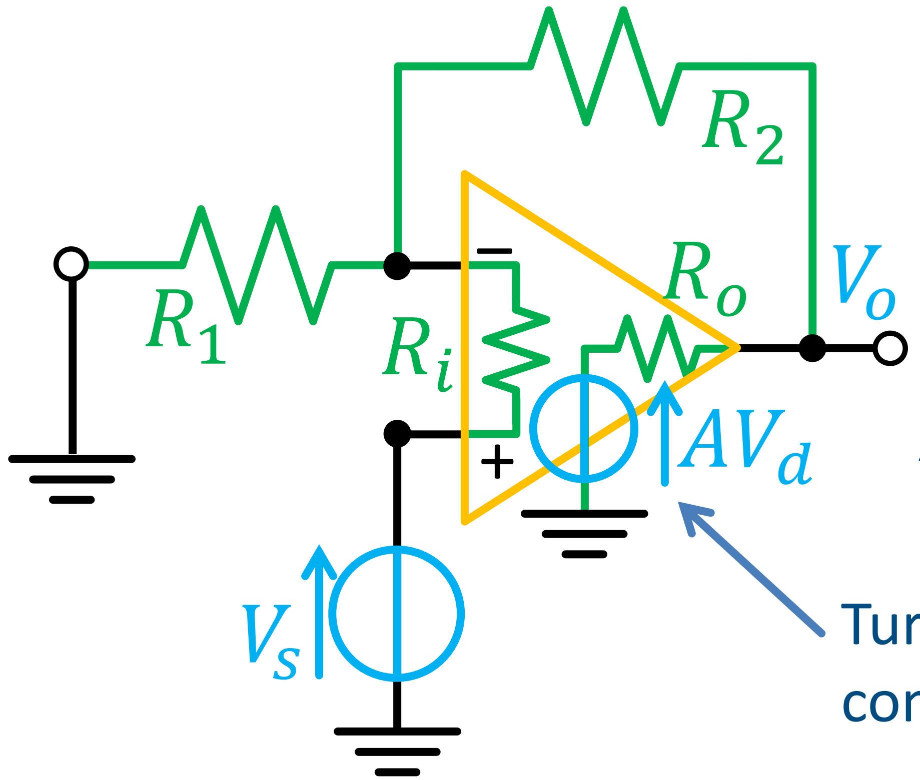
Z_{in}



Turn off VCVS to compute Z'_{OL}

$$Z_{in} = R_p + Z'$$

$$Z'_{id} = \infty \; (V_d = 0)$$

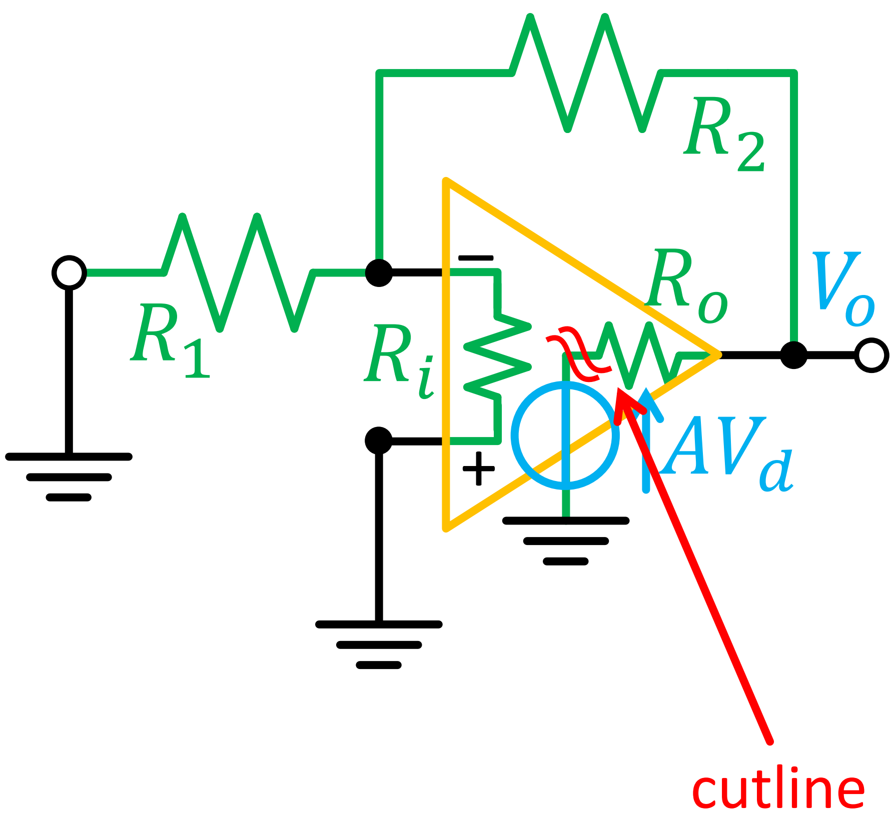
⇙ ⇘

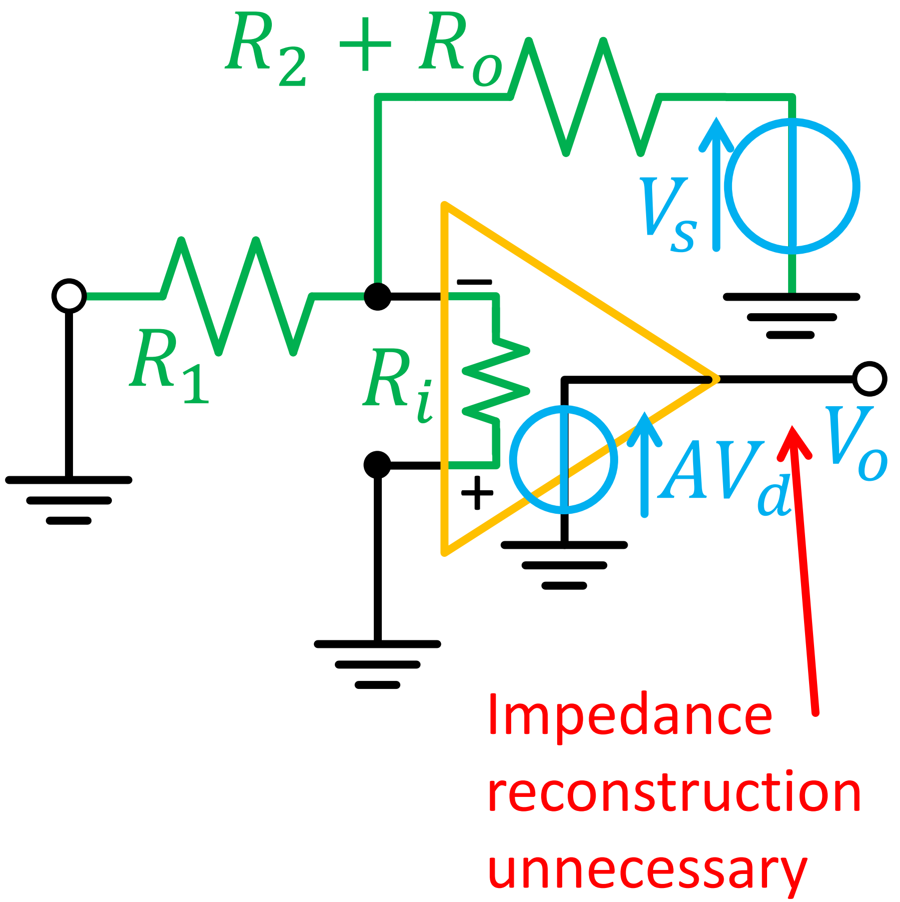
$$Z' = Z'_{OL}(1 - G'_{loop})$$

A voltage source must be used as test signal

$$Z'_{OL} = R_i + R_1 \parallel (R_2 + R_o) \approx R_i = 2\ \text{M}\Omega$$

G'_{loop}


R_2
R_1
R_i
R_o
V_o
AV_d
cutline


$R_2 + R_o$
V_s
R_1
R_i
AV_d
V_o
Impedance reconstruction unnecessary

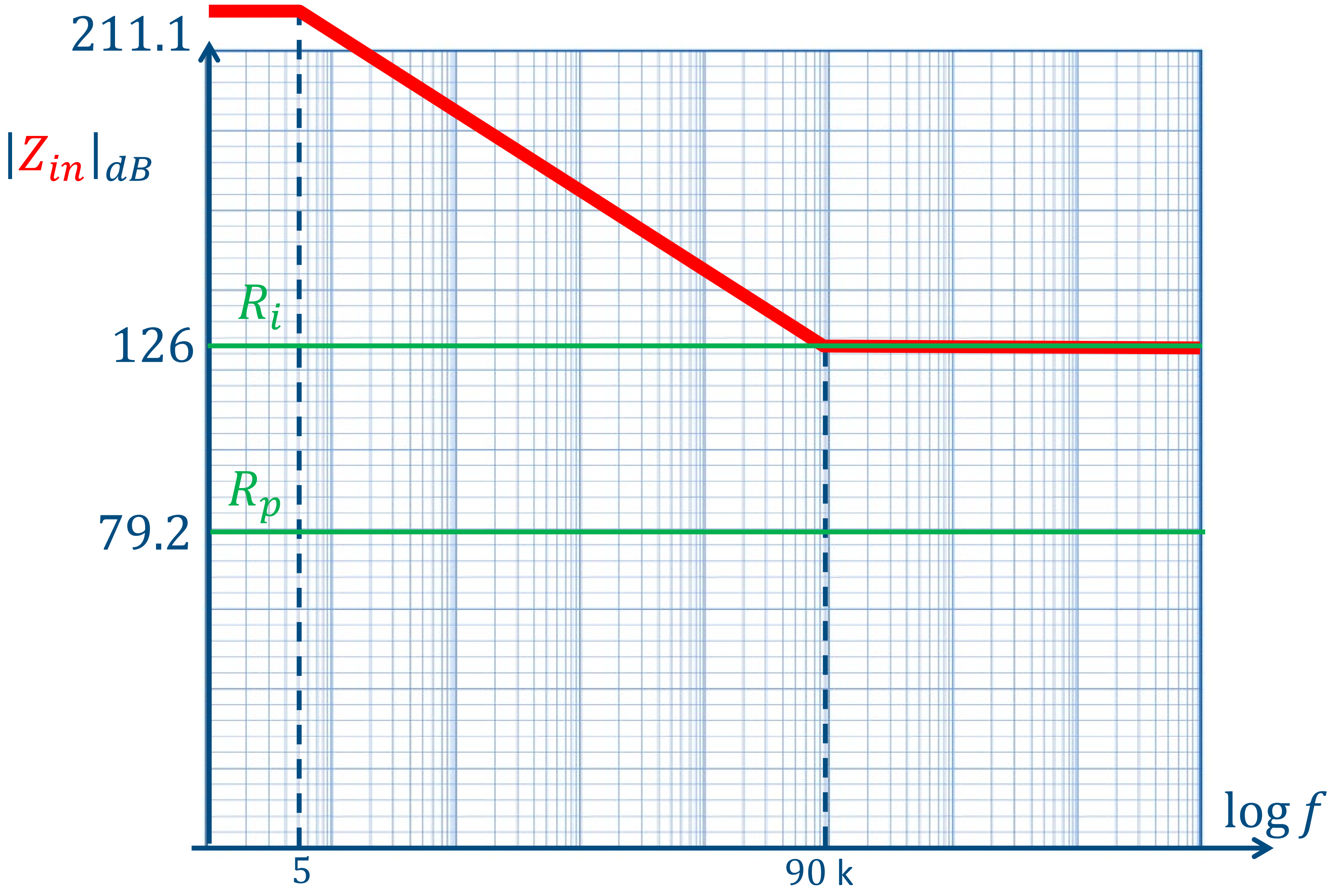
Result

$$G'_{loop} = -A(s)\frac{R_1 \parallel R_i}{R_1 \parallel R_i + R_2 + R_o} = -0.09\,A(s)$$

- At low frequencies, $A(s) = A_0 = 2 \times 10^5 \Rightarrow G'_{loop} = 1.8 \times 10^4$ and $Z' = 3.6 \times 10^{10}\ \Omega =$ 36 GΩ $\Rightarrow Z_{in} = R_p + Z' = Z'$
- At high frequencies

$$Z_{in} \approx Z_{OL}\left(1 + \frac{1.8 \times 10^4}{1 + s\tau}\right) \approx Z_{OL}\frac{1.8 \times 10^4 + s\tau}{1 + s\tau}$$

Bode plot



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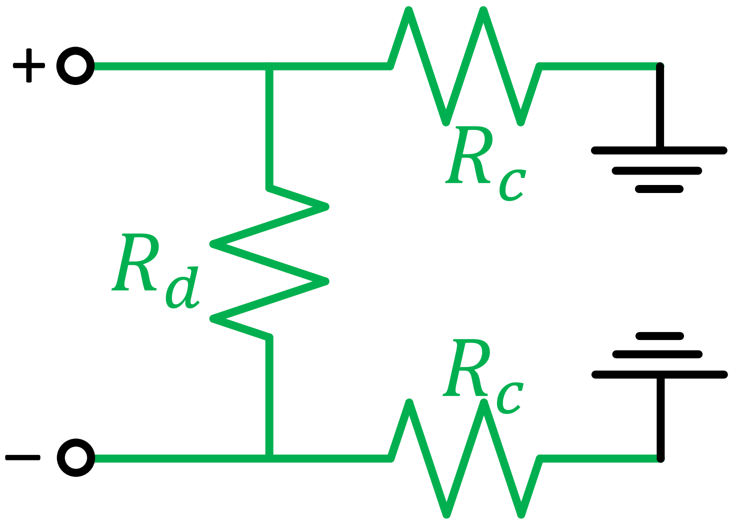
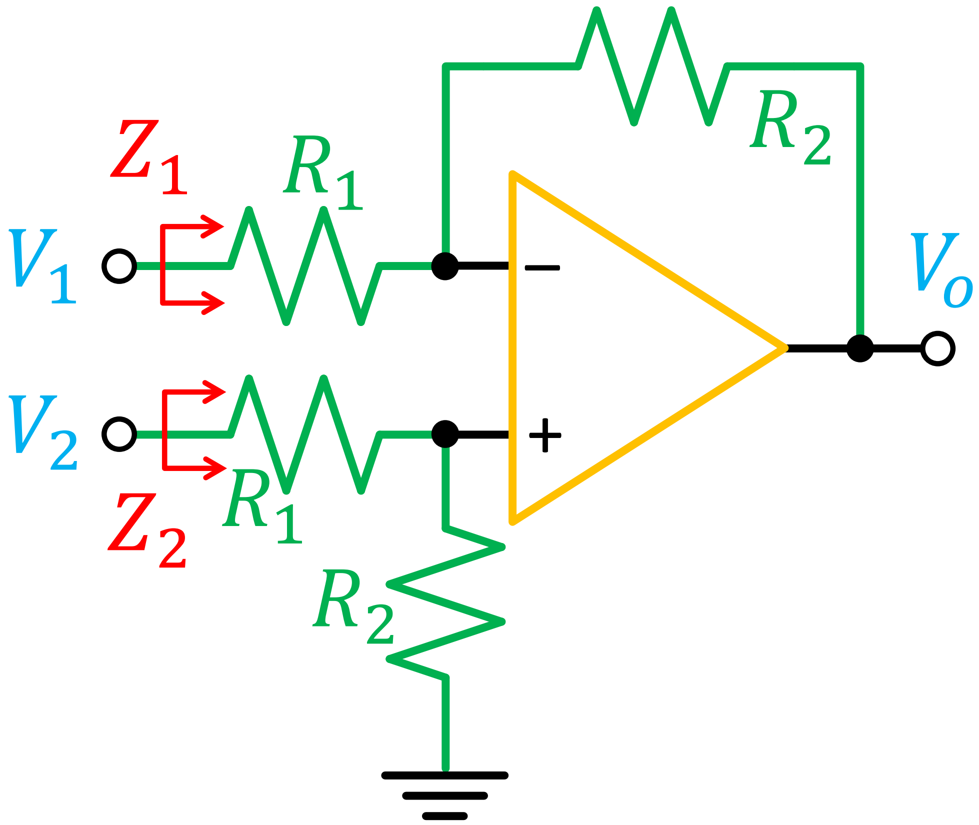
Differential amplifiers

- In the general case it is not possible to provide a purely passive equivalent of the amplifier inputs
- If the amplifier is symmetric (or antisymmetric), the input can usually be represented by a pi network

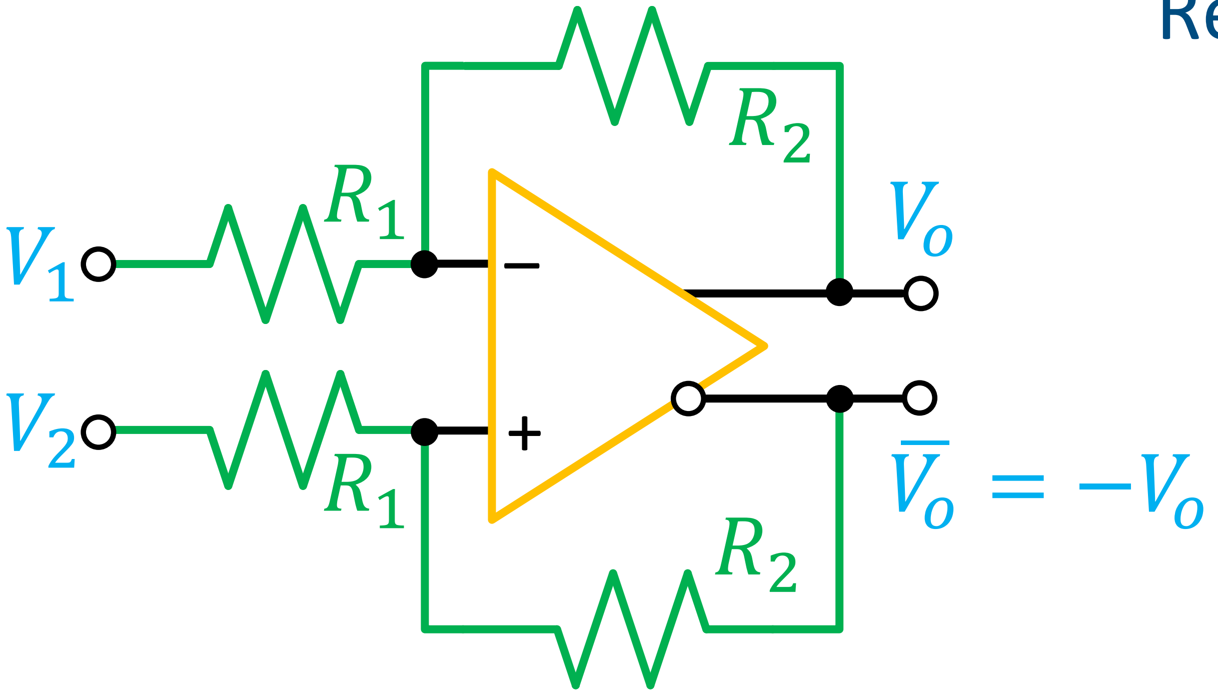


Subtractor



- The circuit is non-symmetric
- $Z_2 = R_1 + R_2$
- Z_1 cannot be broken down into a common- and differential-mode resistance
- Sometimes a differential input resistance is defined, related to a floating differential input voltage

Fully-differential amplifier (FDA)



Results are

$$V_o = \frac{R_2}{2R_1}(V_2 - V_1)$$

$$R_c = R_1 + R_2$$

$$R_d = \frac{2R_1}{R_2}(R_1 + R_2)$$

Homework

1. Find the relation between the differential impedance defined as in slide #31 and the resistances of the pi network
2. Derive the results for gain and impedances of the FDA

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Impedance removal

- The calculation of Z_{in} (NI input) was conducted after removal of R_p, placed in series (slide #25)
- However, R_p does not affect $Z_{id} = \infty$, and can be left in the loop
- Calculations for the two cases are shown in the following (hint: results do not change)
- An analogous case is that of impedances in parallel to a node where $Z_{id} = 0$. They can be either removed or left

R_p outside the loop

$$Z'_{OL} = R_i + R_1 \parallel R_2 \qquad G'_{loop} = -A(s)\frac{R_1 \parallel R_i}{R_1 \parallel R_i + R_2}$$

$$Z_{in} = R_p + (R_i + R_1 \parallel R_2)\left(1 + A(s)\frac{R_1 \parallel R_i}{R_1 \parallel R_i + R_2}\right) =$$

$$R_p + R_i + R_1 \parallel R_2 +$$

$$A(s)\frac{R_1 R_i}{R_1 R_i + R_2(R_1 + R_i)}\,\frac{R_i(R_1 + R_2) + R_1 R_2}{R_1 + R_2}$$

$$Z_{in} = R_p + R_i + R_1 \parallel R_2 + A(s)\frac{R_1 R_i}{R_1 + R_2}$$

R_p inside the loop

$$Z_{OL} = R_p + R_i + R_1 \parallel R_2$$

$$G_{loop} = -A(s)\frac{R_1 \parallel (R_i + R_p)}{R_1 \parallel (R_i + R_p) + R_2}\frac{R_i}{R_i + R_p} = -A(s)\frac{R_1 R_i}{R_1(R_i + R_p) + R_2(R_1 + R_i + R_p)}$$

$$Z_{in} = (R_p + R_i + R_1 \parallel R_2)\left(1 + A(s)\frac{R_1 R_i}{R_1(R_i + R_p) + R_2(R_1 + R_i + R_p)}\right) =$$

$$R_p + R_i + R_1 \parallel R_2 +$$

$$A(s)\frac{R_1 R_i}{\cancel{R_1(R_i + R_p) + R_2(R_1 + R_i + R_p)}}\frac{\cancel{(R_p + R_i)(R_1 + R_2)} + R_1 R_2}{R_1 + R_2}$$

$$Z_{in} = R_p + R_i + R_1 \parallel R_2 + A(s)\frac{R_1 R_i}{R_1 + R_2}$$