Electronics – 96032

POLITECNICO DI MILANO





Loop Compensation

Alessandro Spinelli

Phone: (02 2399) 4001

alessandro.spinelli@polimi.it spinelli.faculty.polimi.it

Disclaimer

Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes

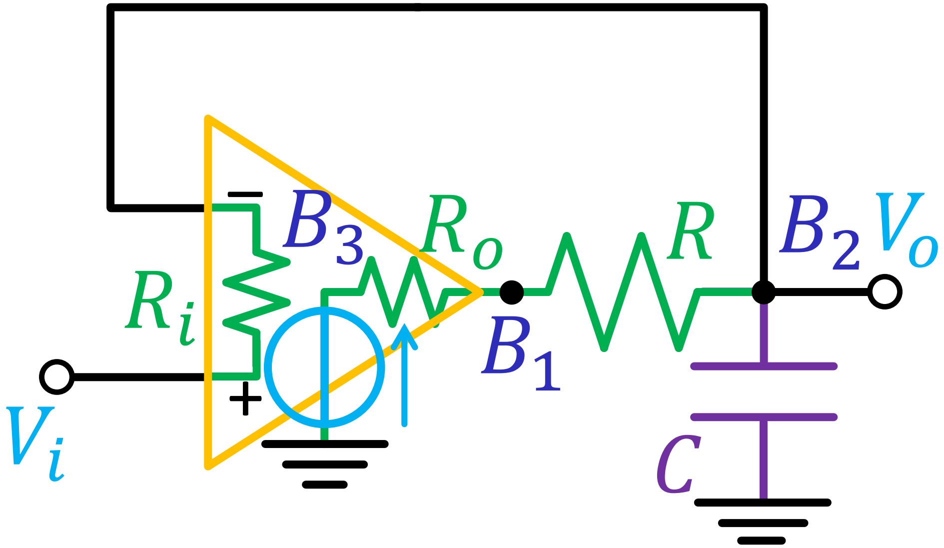
Outline

- Impedance reconstruction
- Compensation
- Appendix: poles and zeros of a network

Breaking the loop

- In a block scheme, it makes no difference where you choose to break the loop
- In a real circuit, you must terminate the loop end with the impedance that existed before you broke the loop
- In reality, things are even more complex because you must be careful not to modify the DC operating point

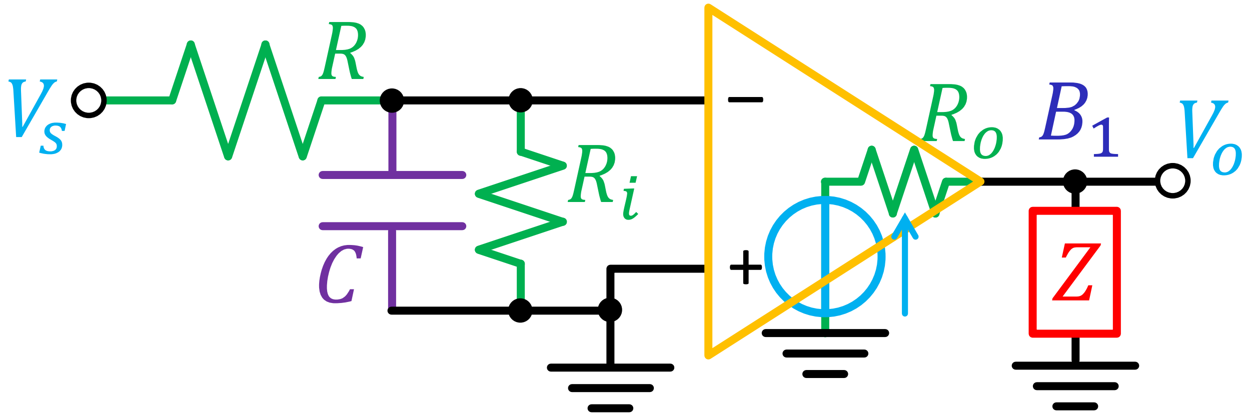
Example: buffer stage



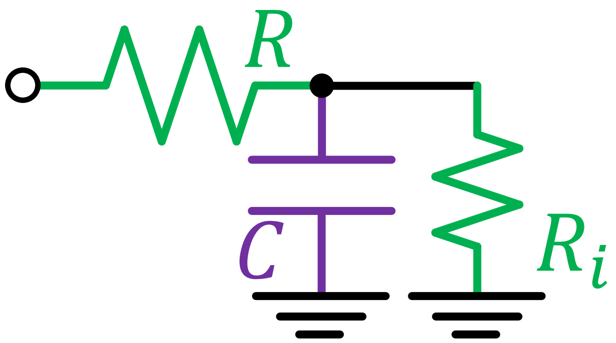
- $R = 900\ \Omega, C = 20$ nF
- $R_i = 1\ \text{M}\Omega, R_o = 10\ \Omega$
- $GBWP = 1$ MHz, $A_0 = 100$ dB

1. Evaluate the loop gain using B_1, B_2 and B_3 as breakpoints
2. Compensate if needed (neglect R_i for simplicity)

Loop breaking – case B_1



$Z =$

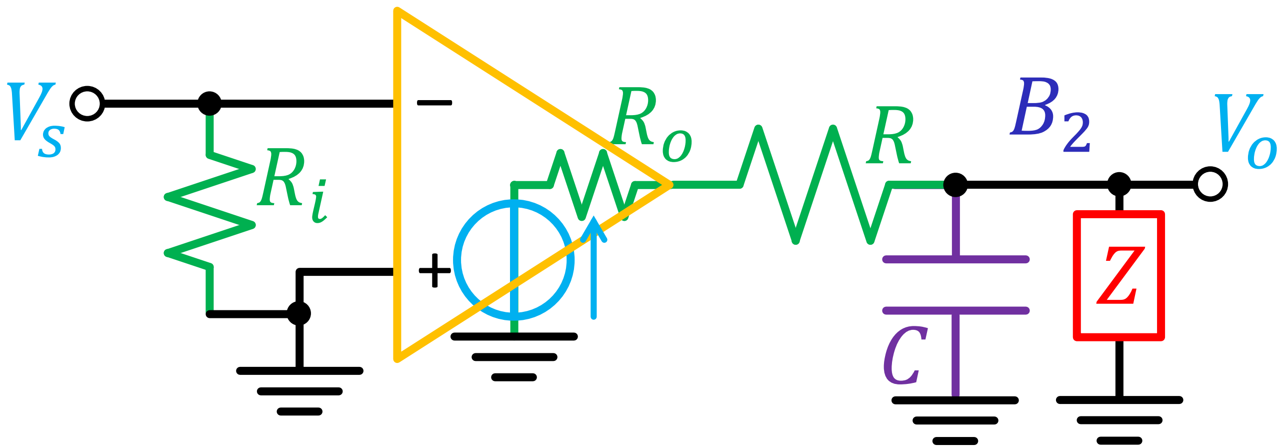


$$= R + \frac{1}{sC} \parallel R_i = (R + R_i)\frac{1 + sC(R \parallel R_i)}{1 + sCR_i}$$

Calculations – case B_1

$$G_{loop}^{1} = -\frac{\frac{1}{sC} \parallel R_i}{R + \frac{1}{sC} \parallel R_i} A(s) \frac{Z}{Z + R_o} = -A(s) \frac{\frac{1}{sC} \parallel R_i}{R + R_o + \frac{1}{sC} \parallel R_i}$$

Loop breaking – case B_2

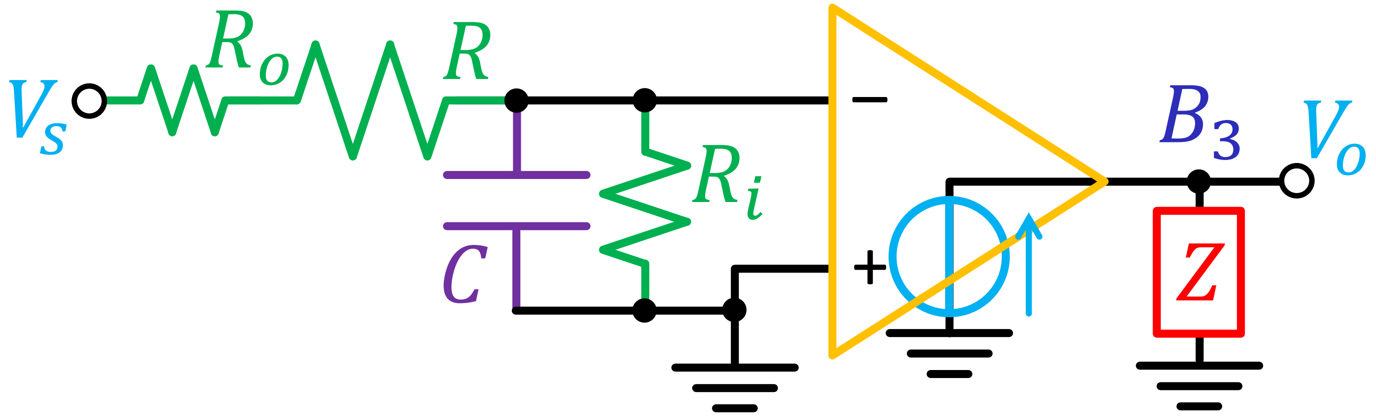


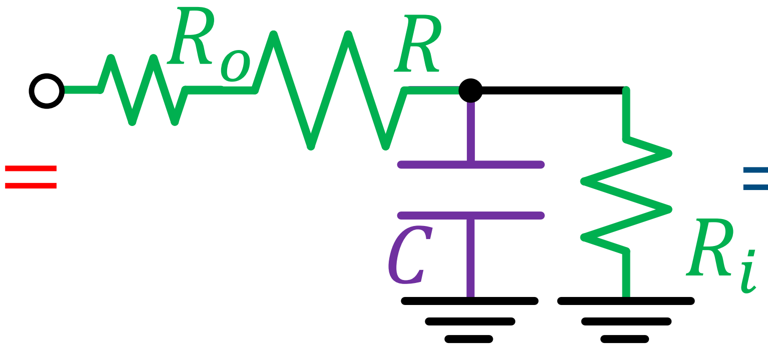
$$Z = R_i$$

Calculations – case B_2

$$G_{loop}^{2} = -A(s)\frac{\frac{1}{sC} \parallel Z}{R + R_o + \frac{1}{sC} \parallel Z} = -A(s)\frac{\frac{1}{sC} \parallel R_i}{R + R_o + \frac{1}{sC} \parallel R_i}$$

Loop breaking – case B_3



$Z =$ 

$= R + R_o + \frac{1}{sC} \parallel R_i$

(but it is not needed for the calculations!)

Calculations – case B_3

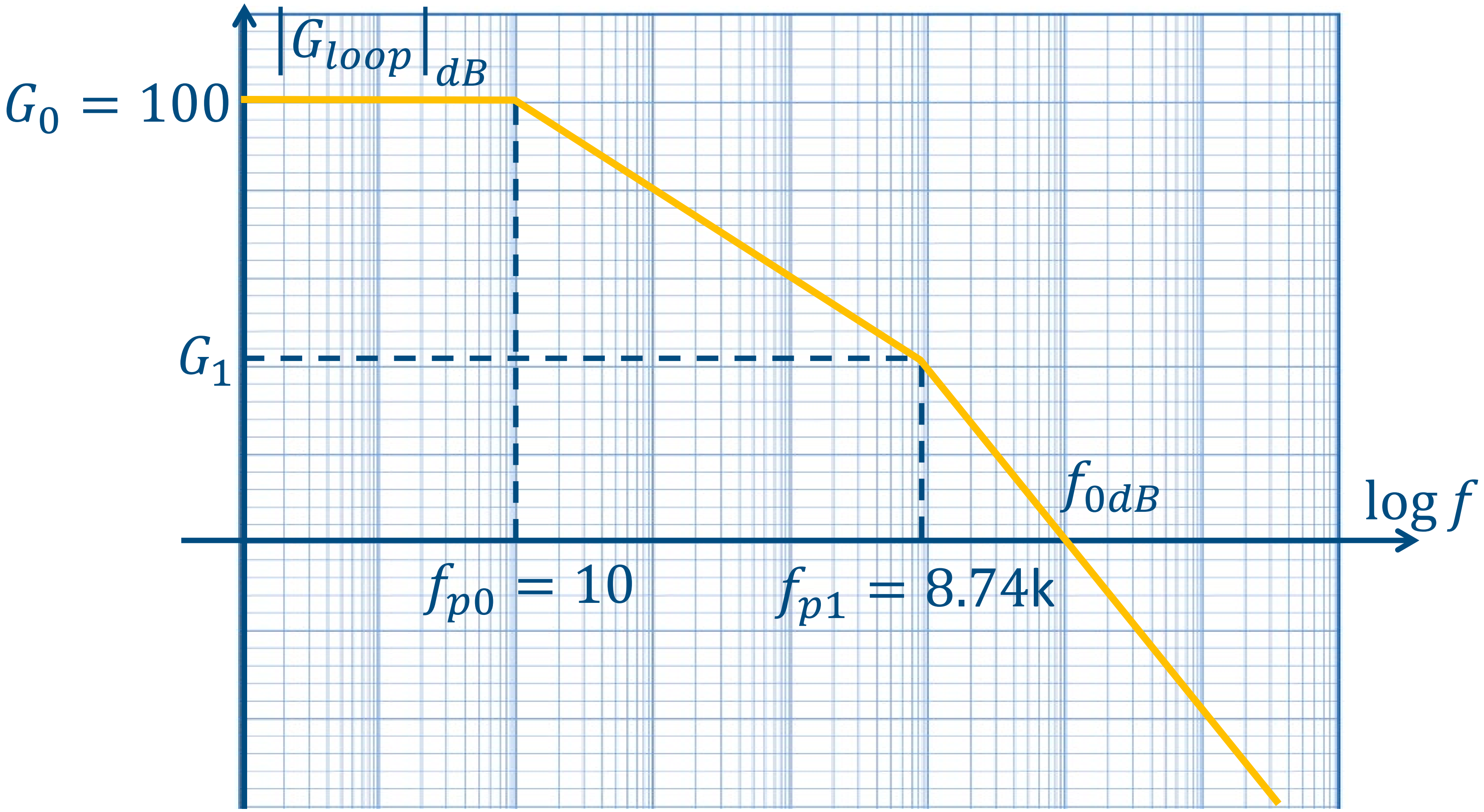
$$G_{loop}^{3} = -A(s)\frac{\frac{1}{sC} \parallel R_i}{R + R_o + \frac{1}{sC} \parallel R_i}$$

$$= -\frac{A_0}{1+s\tau}\frac{R_i}{R+R_o+R_i}\frac{1}{1+sC\left(R_i \parallel (R+R_o)\right)}$$

$$\approx -\frac{A_0}{1+s\tau}\frac{1}{1+sC(R+R_o)}$$

10 Hz 8.74 kHz

Bode plot



Phase margin

$$G_0 f_{p0} = G_1 f_{p1} \Rightarrow G_1 = G_0 \frac{f_{p0}}{f_{p1}} = 114 = 41\ \mathrm{dB}$$

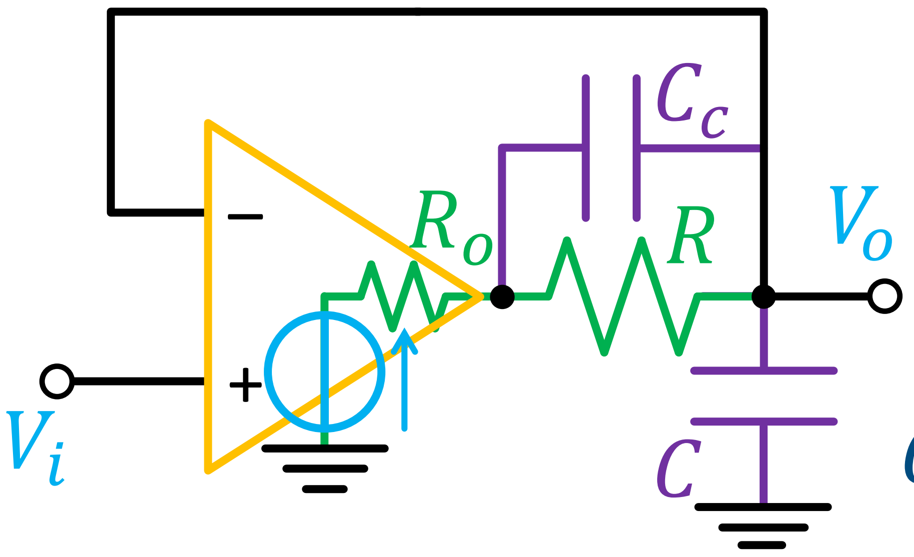
$$G_1 f_{p1}^2 = f_{0dB}^2 \Rightarrow f_{0dB} = f_{p1}\sqrt{G_1} = 93.2\ \mathrm{kHz}$$

$$\phi_m = 180 - 90 - \arctan\left(\frac{f_{0dB}}{f_{p1}}\right) = 5°$$

Outline

- Impedance reconstruction
- Compensation
- Appendix: poles and zeros of a network

Compensation (negl. R_i)



$$G_{loop} = -A(s)\frac{\frac{1}{sC}}{R_o + R \parallel \frac{1}{sC_c} + \frac{1}{sC}}$$

Effect of C_c

$$G_{loop} = -\frac{A_0}{1+s\tau}\frac{1+sC_cR}{1+s(CR+CR_o+C_cR)+s^2CC_cRR_o}$$

1. A zero is introduced in G_{loop}
2. A second pole also arises
3. The poles time constants are entangled

Sizing of C_c – LF/HF approximation

- LF: C_c = open circuit. The pole is

$$f_{p1} = \frac{1}{2\pi C(R + R_o)} = 8.74 \text{ kHz},$$

obviously not affected by C_c. So f_{0dB} is still 93.2 kHz. To achieve a phase margin of 45° we then need $f_z = f_{0dB}$, i.e.

$$\frac{1}{2\pi C_c R} = 93.2 \times 10^3 \Rightarrow C_c = 1.88 \text{ nF}$$

- HF: C = short circuit. The pole is now

$$f_{p2} = \frac{1}{2\pi C_c(R \parallel R_o)} = 8.47 \text{ MHz}$$

Sizing of C_c – Analytical approximation

- Given a 2nd order polynomial with well-separate real roots (i.e., $\tau_1 \gg \tau_2$), we can write
  $$(1+s\tau_1)(1+s\tau_2) = 1 + s(\tau_1+\tau_2) + s^2\tau_1\tau_2 \approx 1 + s\tau_1 + s^2\tau_1\tau_2$$
- Approximate values of the roots are

$$as^2 + bs + 1 = 0 \Rightarrow \begin{array}{l} s_{LF} \approx -\dfrac{1}{b} \\ s_{HF} \approx -\dfrac{b}{a} \end{array}$$

- The error is of the order of the ratio of the roots

Analytical approximation – caveat

- Consider the previous cases with $\tau_1 \gg \tau_2$:

$$s^2\tau_2\tau_1 + s\tau_1 + 1 = 0 \Rightarrow \begin{array}{l} \omega_{LF} \approx \dfrac{1}{\tau_1} \\ \omega_{HF} \approx \dfrac{1}{\tau_2} \end{array} \qquad s^2\tau_2\tau_1 + s\tau_2 + 1 = 0 \Rightarrow \begin{array}{l} \omega_{LF} \approx \dfrac{1}{\tau_2} \\ \omega_{HF} \approx \dfrac{1}{\tau_1} \end{array}$$

- In the second case we have $\omega_{LF} \gg \omega_{HF}$, which is impossible $\Rightarrow$ the solution is wrong and actual poles are complex!

- Always check the significance of your results!

Pole position

- Pole frequencies become:
  - $f_{p1} = \frac{1}{2\pi(C(R+R_o)+C_c R)}$
  - $f_{p2} = \frac{C(R+R_o)+C_c R}{2\pi C C_c R R_0}$
- Values are similar to the previous ones, if $C_c \ll C$. As an example, with the previous value $C_c = 1.88$ nF we get

$$f_{p1} = 8.0\ \text{kHz};\ f_{p2} = 9.36\ \text{MHz}$$

Sizing of C_c – 2

$$\begin{cases} G_0 f_{p0} = G_1 f_{p1} \\ G_1 f_{p1}^2 = f_z^2 \end{cases} \Rightarrow f_z^2 = G_0 f_{p0} f_{p1}$$

$$\left(\frac{1}{C_c R}\right)^2 = \frac{A_0}{\tau(CR + CR_0 + C_c R)}$$

- If $C_c \ll C$ we can neglect the last term, obtaining $C_c = \sqrt{\tau(CR + CR_0)/A_0}/R = 1.88$ nF (same expression as before)
- The correct solution gives $C_c = 1.98$ nF

Phase margin > 45°

- If – say – $\phi_m = 60°$ is required, we have

$$\arctan\left(\frac{f_{0dB}}{f_z}\right) = 60° \Rightarrow f_{0dB} = 1.73 f_z$$

$$\begin{cases} G_1 f_{p1}^2 = G_z f_z^2 \\ G_z f_z = f_{0dB} \end{cases} \Rightarrow f_z^2 = \frac{G_0 f_{p0} f_{p1}}{1.73}$$

- Result is $C_c \approx 2.5$ nF

Homework

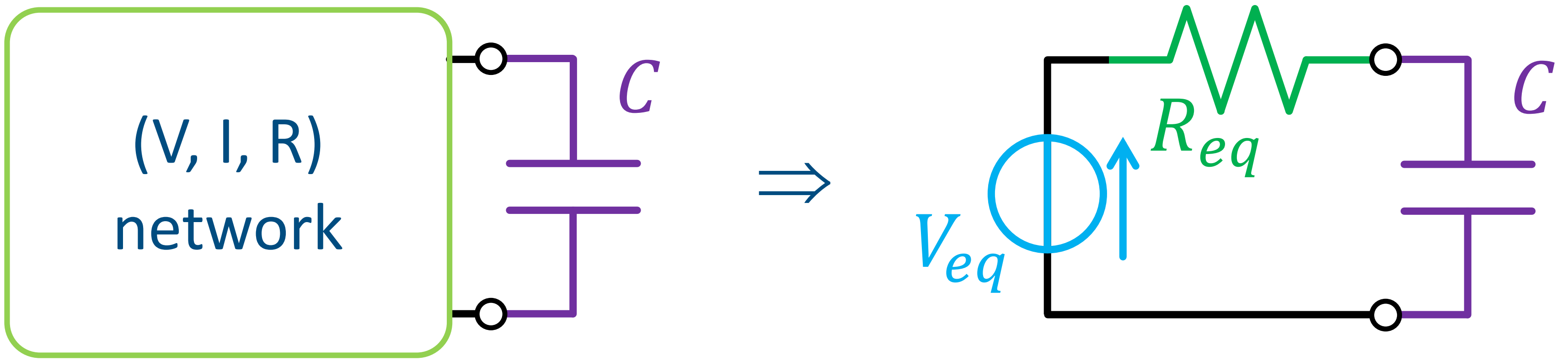
1. Compute the loop gain for the buffer stage (slide #5) using a current source
2. Compensate the buffer stage using a resistor rather than a capacitor

Outline

- Impedance reconstruction
- Compensation
- Appendix: poles and zeros of a network

Network pole

- We consider a generic network with only one reactive element (say, a capacitor)
- Thevenin theorem can be applied at the capacitor port:



- The time constant of the pole is CR_{eq}, where R_{eq} is the equivalent resistance seen at the capacitor port

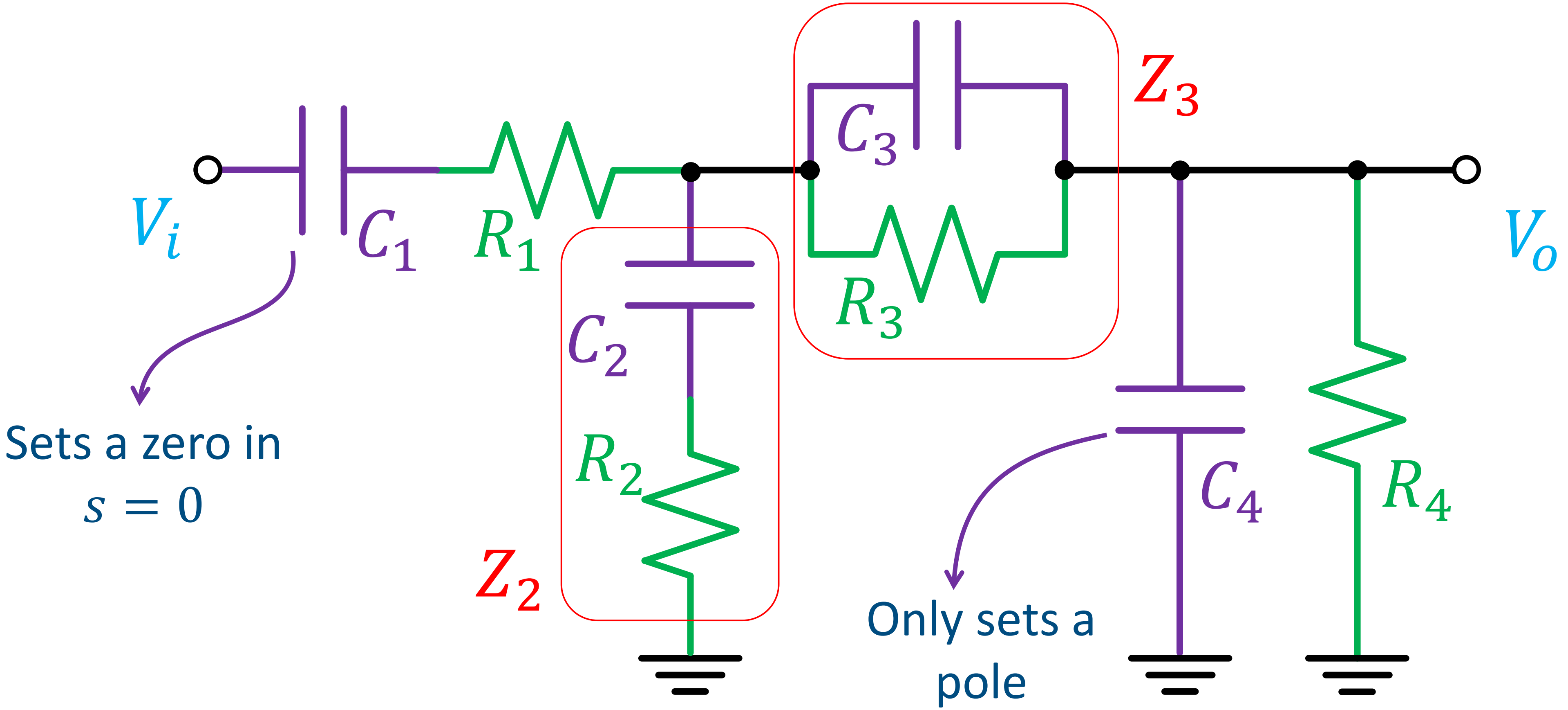
Presence of a zero

- A capacitor always adds a pole, but does not always add a zero to the transfer function
- To tell when this is the case, consider these two limits:

$$\lim_{s\to\infty} \frac{A}{1 + s\tau_p} = 0 \qquad\qquad \lim_{s\to\infty} A\frac{1 + s\tau_z}{1 + s\tau_p} \neq 0$$

- In practice: replace the capacitor with a short-circuit
  - If the output is null, there is only a pole
  - If the output is non-zero, there is a pole and a zero

Position of the zero: example



Evaluation

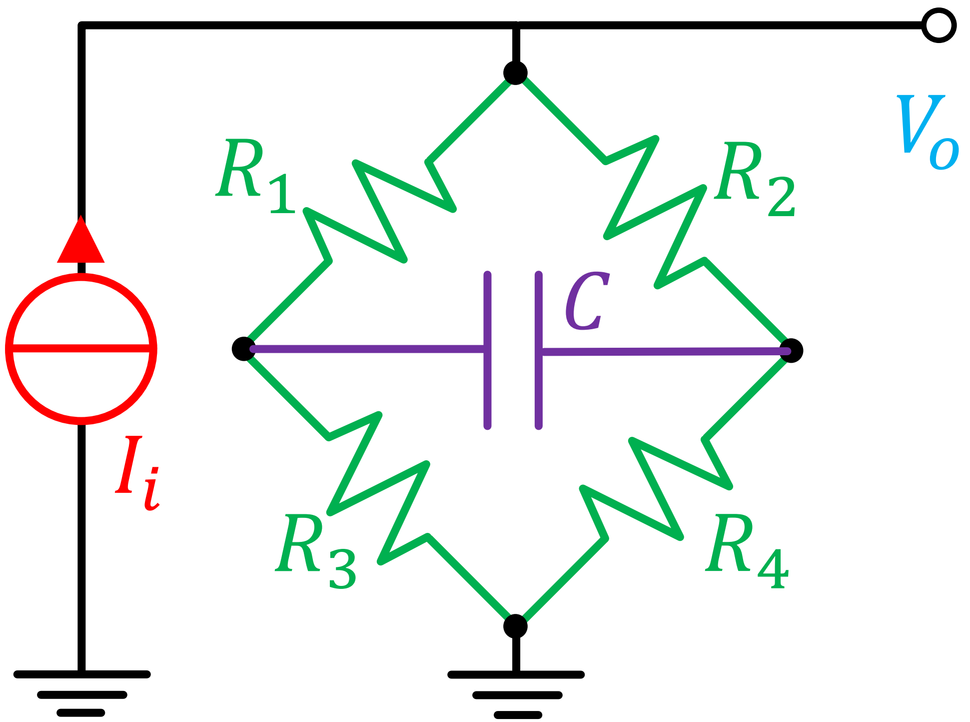
- If $Z_2(s) = 0$, the output is nulled and a zero arises

$$Z_2 = R_2 + \frac{1}{sC_2} = \frac{1 + sC_2R_2}{sC_2} = 0 \Rightarrow s = -\frac{1}{C_2R_2}$$

- If $Z_3(s) = \infty$, the output is nulled and a zero arises

$$Z_3 = R_3 \parallel \frac{1}{sC_3} = \frac{R_3}{1 + sC_3R_3} = \infty \Rightarrow s = -\frac{1}{C_3R_3}$$

However… not everything is straightforward



- The pole time constant is
  $\tau_p = C\big((R_1 + R_2) \parallel (R_3 + R_4)\big)$
- $V_o(\infty) \neq 0 \Rightarrow C$ also adds a zero
- What is the time constant of the zero?

A possible approach

- The TF can be written as

$$H(s) = H(0)\frac{1+s\tau_z}{1+s\tau_p} \Rightarrow H(\infty) = H(0)\frac{\tau_z}{\tau_p} \Rightarrow \tau_z = \tau_p\frac{H(\infty)}{H(0)}$$

- In our case

$$H(0) = (R_1 + R_3) \parallel (R_2 + R_4) \qquad H(\infty) = R_1 \parallel R_2 + R_3 \parallel R_4$$

$$\tau_z = C\big((R_1 + R_2) \parallel (R_3 + R_4)\big)\frac{R_1 \parallel R_2 + R_3 \parallel R_4}{(R_1 + R_3) \parallel (R_2 + R_4)} =$$

$$C(R_1 \parallel R_3 + R_2 \parallel R_4)$$

Comment

- The last method works fine when the numerical value is needed (or in a Bode plot), but can return lenghty symbolic expressions
- A different technique, called the double null injection, can be used but is outside the scope of this class
- In these cases, it might be better to directly solve the circuit (other theorems are available)