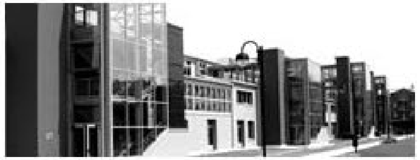
Electronics – 96032

POLITECNICO DI MILANO





Multiple Feedback Loops

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Disclaimer

Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes

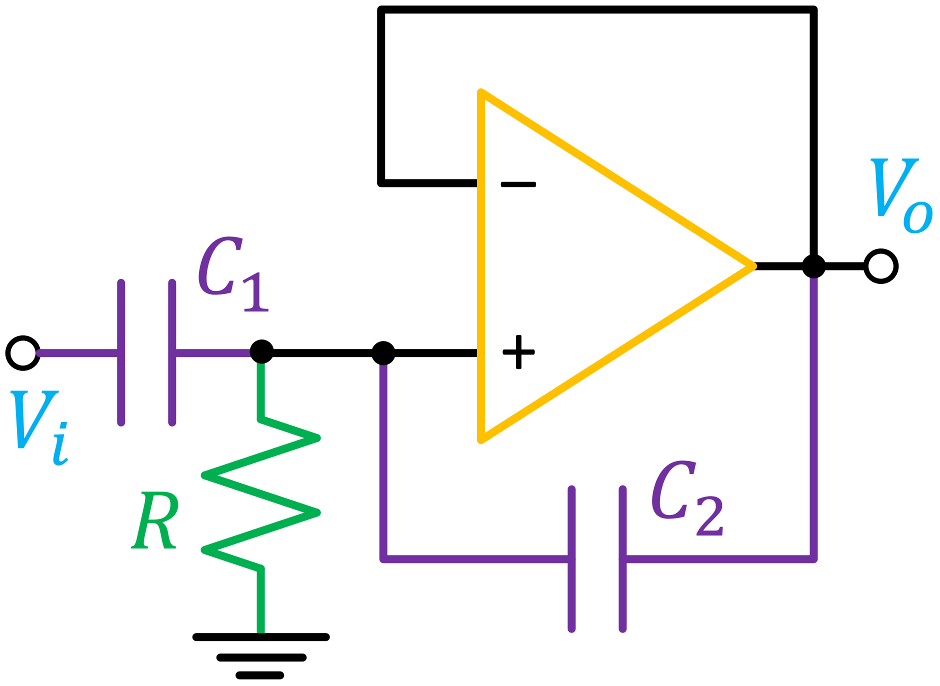
Multiple feedback loops

- Stability is easily computed in two cases:
  - There is a node which is common to all loops ⇒ break all loops at that position
  - There is a «global» loop which contains «local» ones ⇒ check local loops and replace them with their closed-loop transfer functions; check then the global loop
- For the general case, a sequential loop closure criterion (not discussed here) was developed by Bode

Outline

- HP amplifier
- LP filter
- Current source
- Appendix: relation between G_{id}, G_{OL}, G_{loop} in a multiple-feedback loop

HP Amplifier



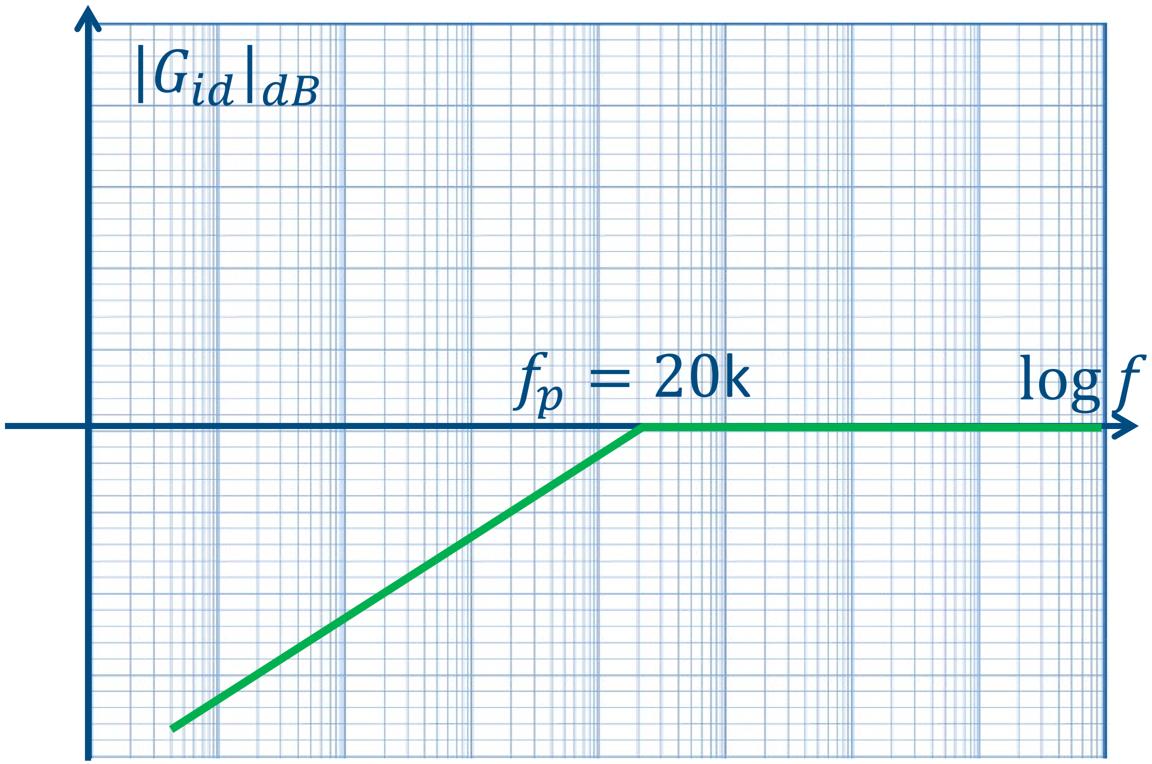
- $R = 8\ \text{k}\Omega$
- $C_1 = 1\ \text{nF}$, $C_2 = 10\ \text{nF}$
- $GBWP = 10\ \text{MHz}$
- $A_0 = 120\ \text{dB}$

1. Find the TF of the stage
2. Check the stability
3. Compute the closed-loop gain

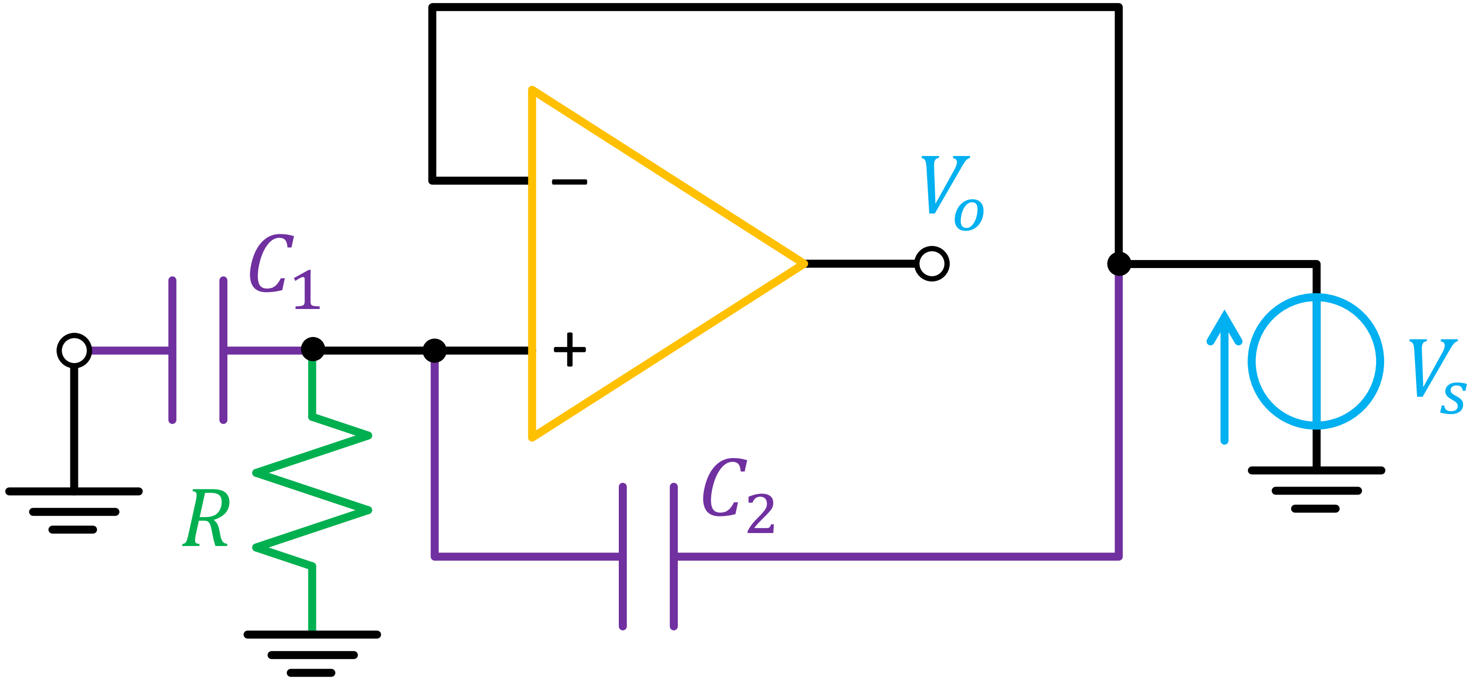
Gain

$$V_o = V^+ \Rightarrow G_{id} = \frac{sC_1R}{1 + sC_1R}$$

$$f_p = \frac{1}{2\pi C_1 R} \approx 20\ \text{kHz}$$



Loop gain



Result

$$\frac{V^+}{V_s} = \frac{R \parallel 1/sC_1}{R \parallel 1/sC_1 + 1/sC_2} = \frac{sC_2R}{1 + s(C_1 + C_2)R}$$

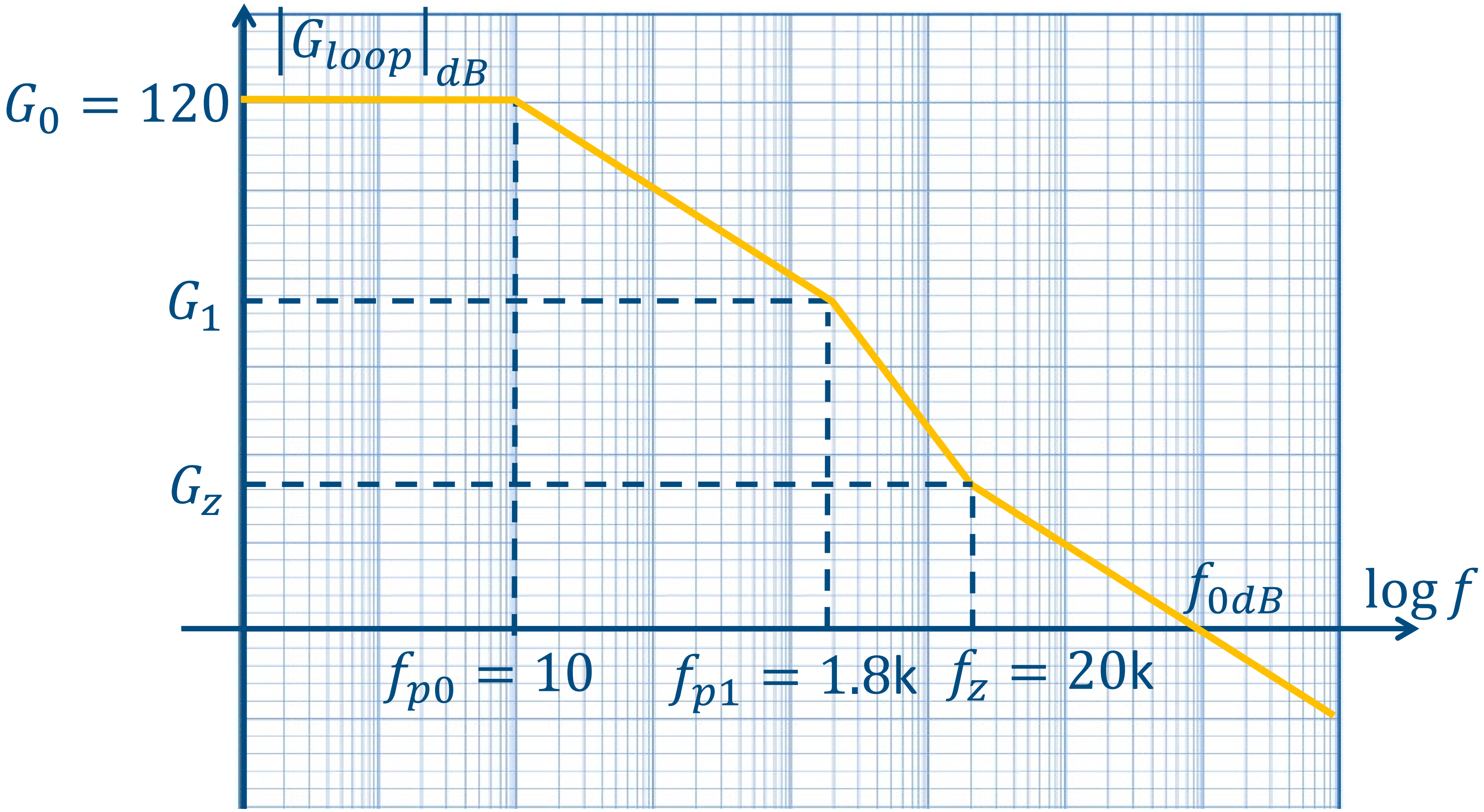
$$G_{loop} = -A(s)\frac{1 + sC_1R}{1 + s(C_1 + C_2)R}$$

Poles and zeros:

$$f_{p0} = \frac{GBWP}{A_0} = 10\ \text{Hz} \qquad f_{p1} = \frac{1}{2\pi(C_1 + C_2)R} \approx 1.8\ \text{kHz}$$

$$f_z = \frac{1}{2\pi C_1 R} \approx 20\ \text{kHz}$$

Bode plot



Calculations

$$G_0 f_{p0} = G_1 f_{p1} \Rightarrow G_1 = 5530 \approx 75 \text{ dB}$$

$$G_1 f_{p1}^2 = G_z f_z^2 \Rightarrow G_z = 45 \approx 33 \text{ dB}$$

$$G_z f_z = f_{0dB} \approx 900 \text{ kHz}$$

The circuit is obviously stable. The phase margin is

$$\phi_m = \arctan\left(\frac{f_{0dB}}{f_z}\right) = 89°$$

Comment

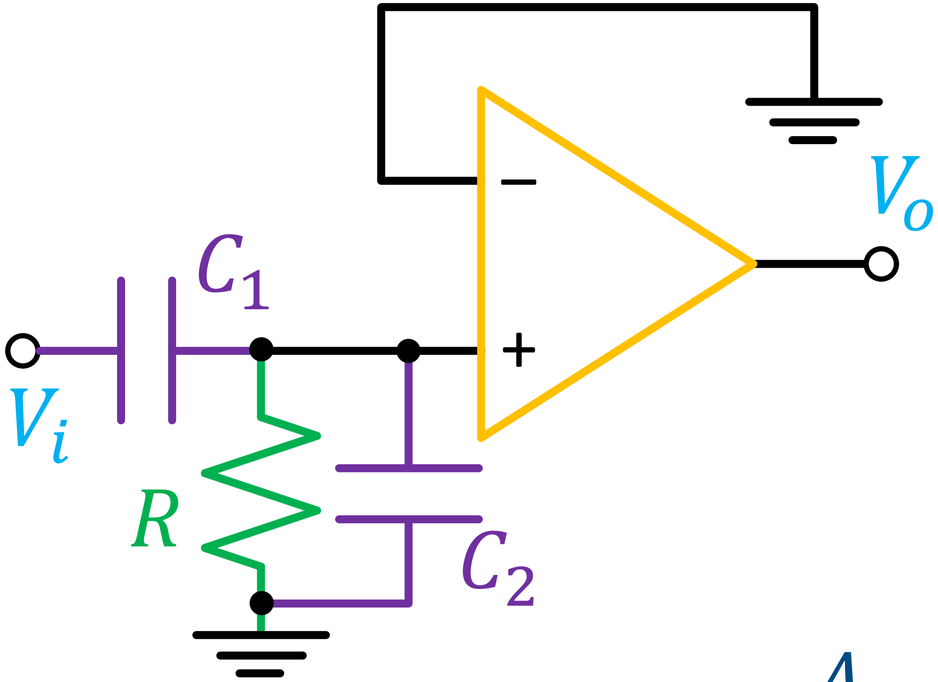
At HF, beyond the pole and zero introduced by the capacitors, we have

$$G_{loop} = -A(s)\frac{C_1}{C_1 + C_2}$$

The zero-dB crossing takes place at

$$f_{0dB} = A_0\frac{C_1}{C_1 + C_2}f_{p0} = GBWP\frac{C_1}{C_1 + C_2} \approx 909\ \text{kHz}$$

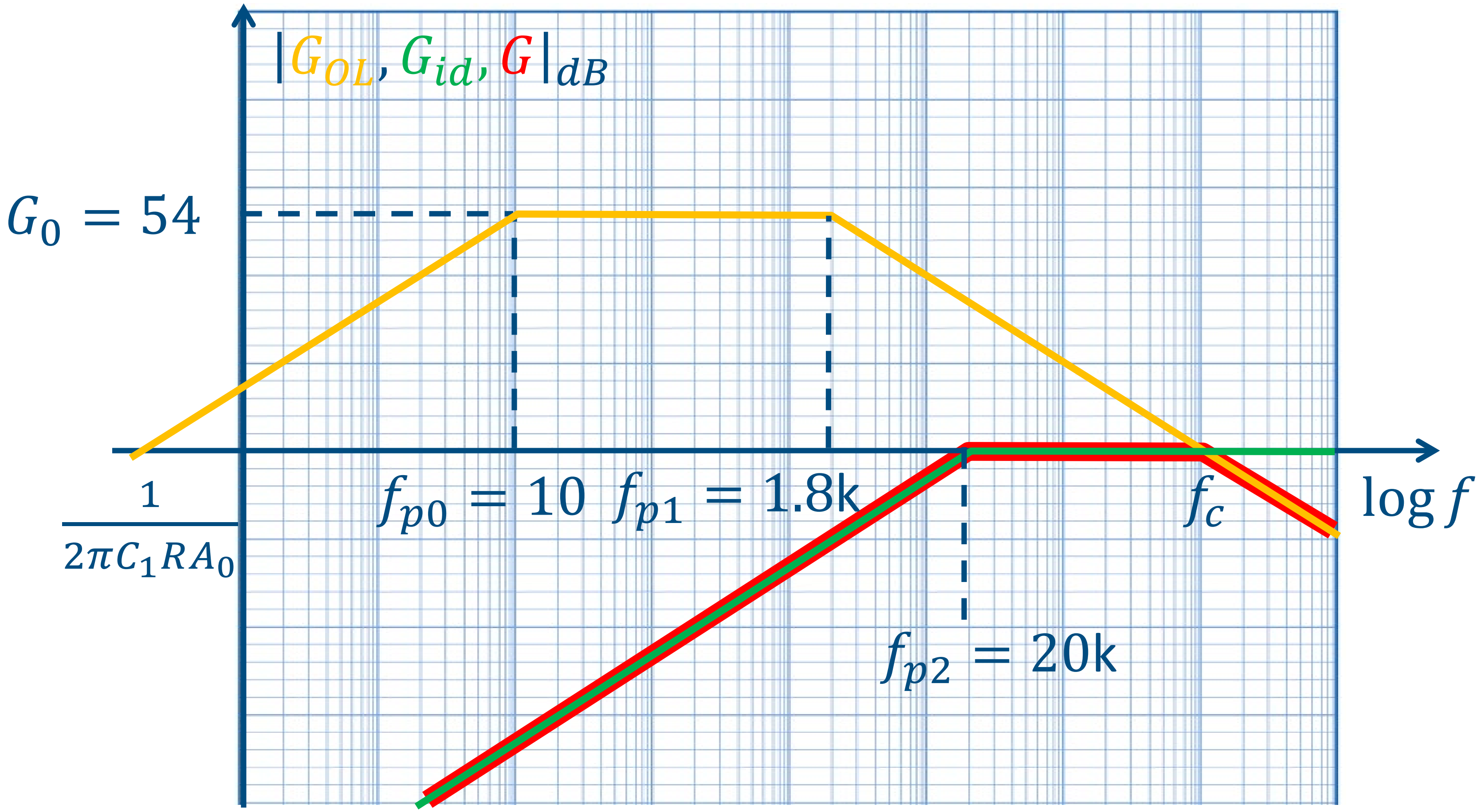
Open-loop gain



$$G_{OL} = A(s)\frac{R \parallel \frac{1}{sC_2}}{R \parallel \frac{1}{sC_2} + \frac{1}{sC_1}} =$$

$$\frac{A_0}{1+s\tau}\frac{sC_1R}{1+s(C_1+C_2)R} = -G_{loop}G_{id}$$

Bode plot



$|G_{OL}, G_{id}, G|_{dB}$
$G_0 = 54$
$\frac{1}{2\pi C_1 R A_0}$
$f_{p0} = 10$
$f_{p1} = 1.8\text{k}$
$f_{p2} = 20\text{k}$
f_c
$\log f$

Calculations

At low frequencies, $G_{OL} \approx sC_1RA_0$

- 0 dB frequency is $f_0 = 1/(2\pi C_1RA_0\ ) \approx 0.02$ Hz

MF gain:

$$\frac{G_0}{f_{p0}} = \frac{1}{f_0} = 2\pi C_1RA_0 \Rightarrow G_0 = 503 = 54\ \text{dB}$$

0 dB frequency:

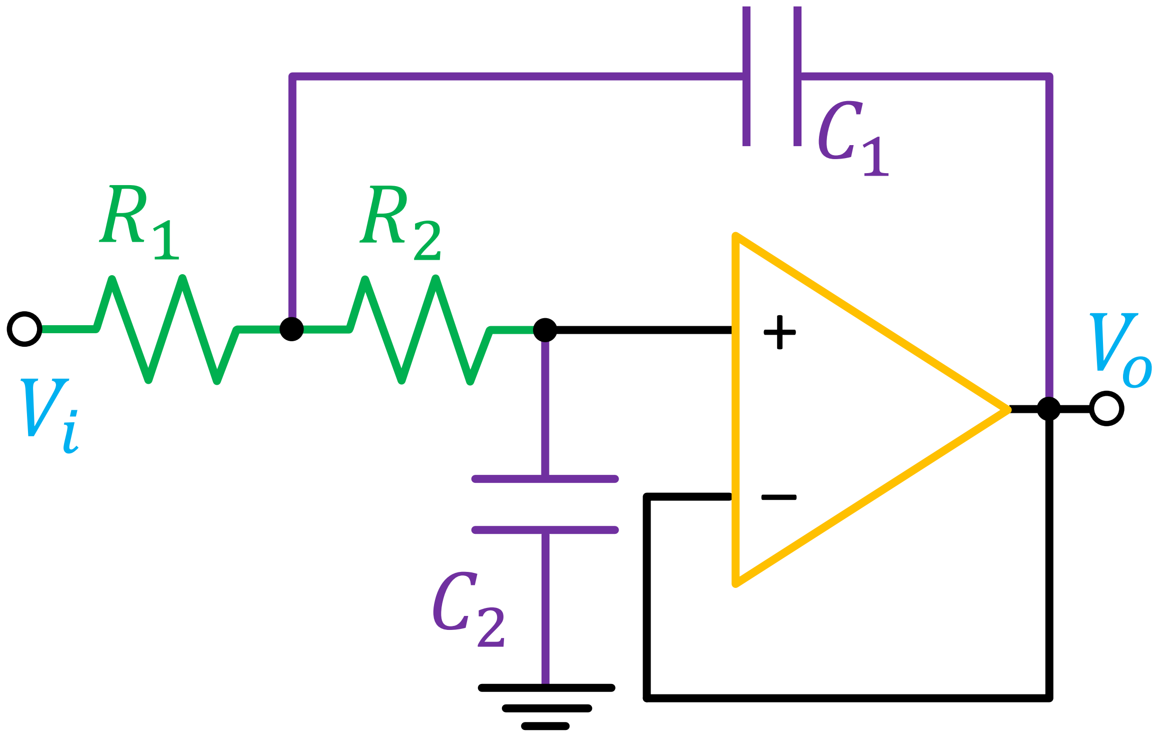
$$G_0 f_{p1} = f_c = 905\ \text{kHz} = f_{0dB}$$

(differences are due to rounding)

Outline

- HP amplifier
- LP filter
- Current source
- Appendix: relation between G_{id}, G_{OL}, G_{loop} in a multiple-feedback loop

Sallen-Key LP filter



- $R_1 = R_2 = 1\ \mathrm{k\Omega}$
- $C_1 = C_2 = 1\ \mathrm{nF}$
- $GBWP = 10\ \mathrm{MHz}$
- $A_0 = 100\ \mathrm{dB}$

1. Find the TF of the stage
2. Check the stability
3. Compute the closed-loop gain

Transfer function

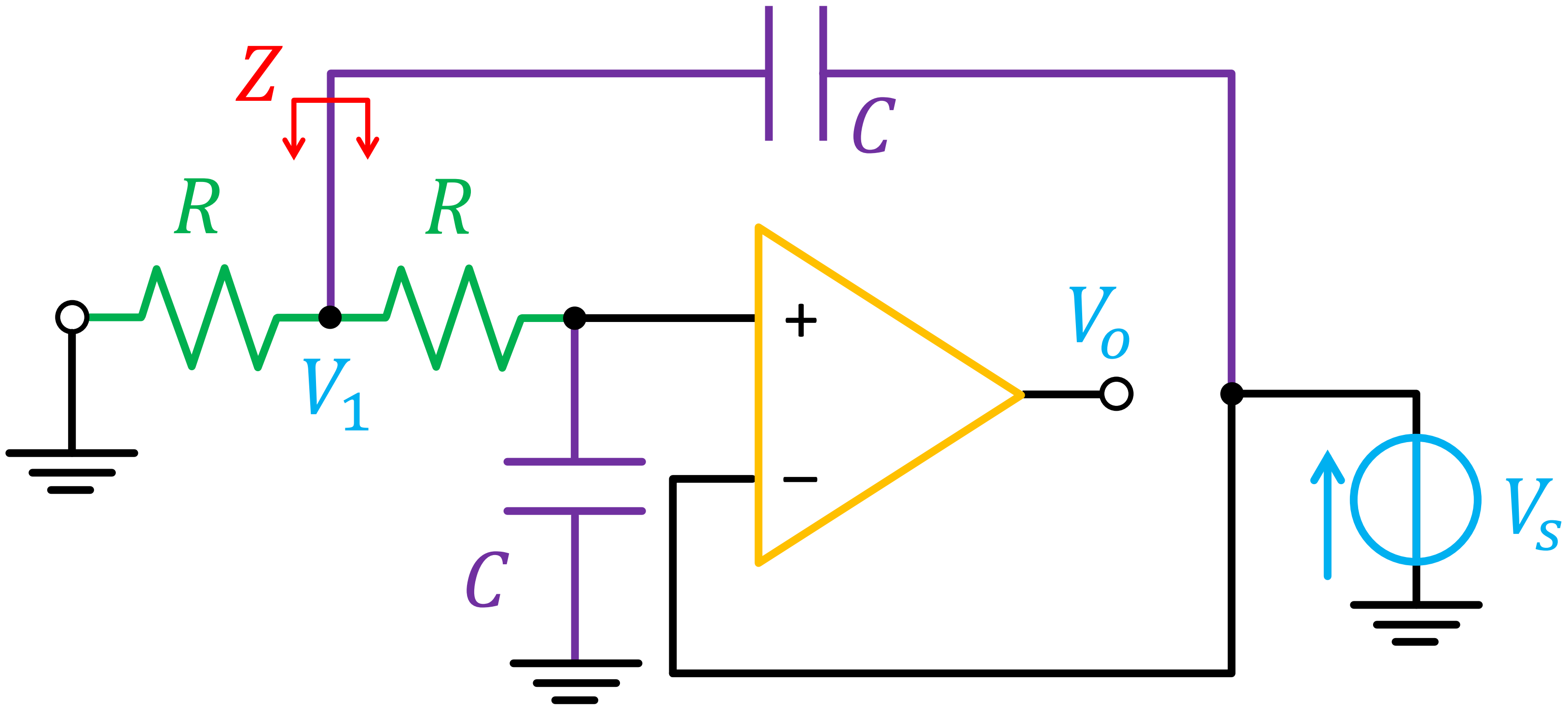
$$\begin{cases} sC_1(V_o - V_1) + \dfrac{V_i - V_1}{R_1} = sC_2 V_o \\ \dfrac{V_1}{1 + sC_2\, R_2} = V_o \end{cases}$$

$$R_1 = R_2 = R$$
$$C_1 = C_2 = C$$

$$\frac{V_o}{V_i} = \frac{1}{(sRC)^2 + 2sRC + 1} = \frac{1}{(1 + sRC)^2}$$

$$f_p = 159\ \text{kHz}$$

Loop gain



Result

$$Z = R \parallel \left(R + \frac{1}{sC} \right) = R \frac{1 + sCR}{1 + 2sCR}$$

$$V_1 = V_s \frac{Z}{Z + 1/sC} = V_s \frac{sCR(1 + sCR)}{(sCR)^2 + 3sCR + 1}$$

$$V^+ = \frac{V_1}{1 + sCR} \qquad V^- = V_s$$

$$G_{loop} = -A(s) \frac{(sCR)^2 + 2sCR + 1}{(sCR)^2 + 3sCR + 1}$$

Calculations

- Poles and zeros

$$f_{p0} = 100\ \text{Hz}, \quad f_{p1,2} = \frac{-3 \pm \sqrt{5}}{4\pi CR} \approx \begin{matrix} 60.8 \\ 416.7 \end{matrix}\ \text{kHz}$$

$$f_{z1,2} = \frac{1}{2\pi CR} = 159\ \text{kHz}$$

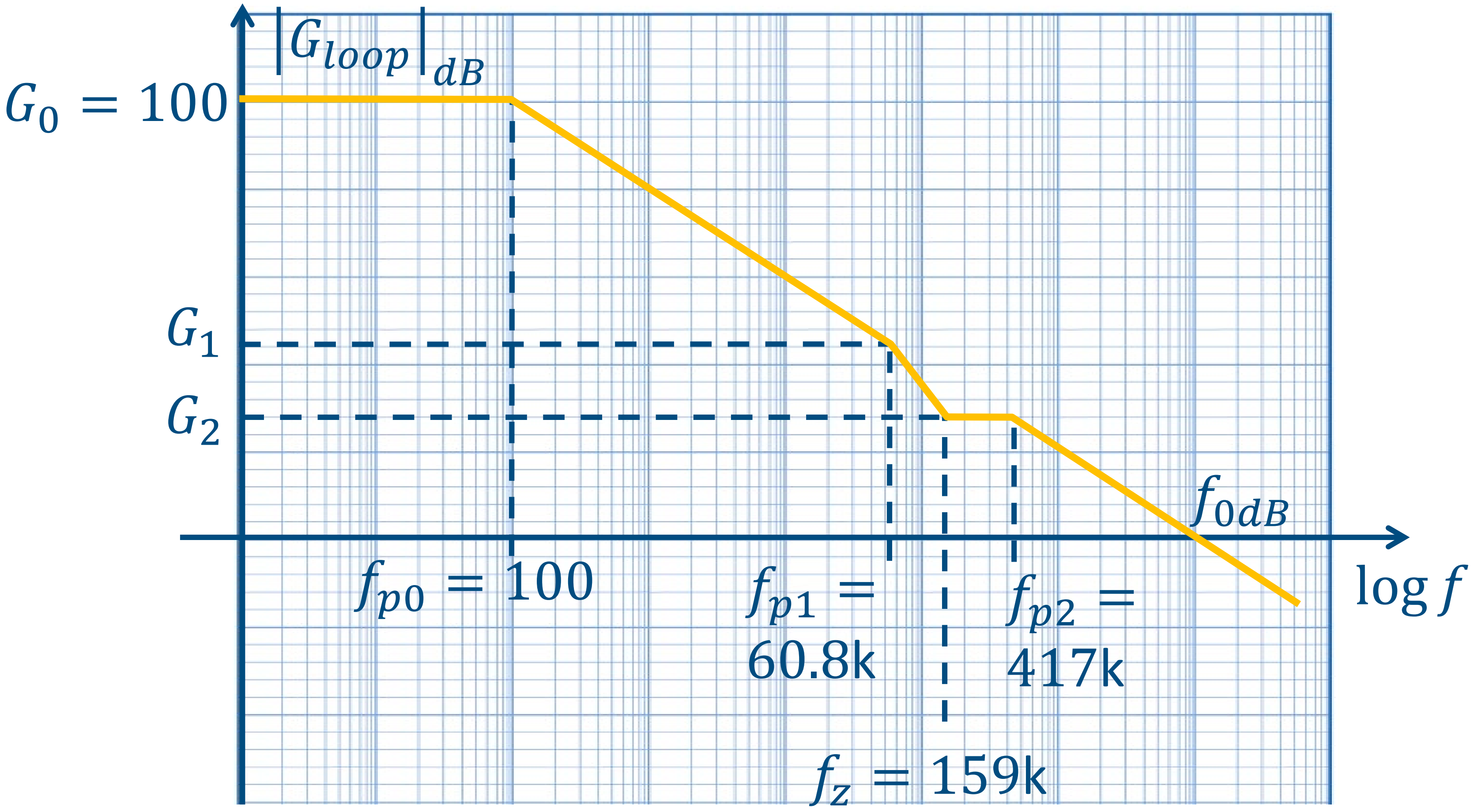
- Gains

$$G_0 f_{p0} = G_1 f_{p1} \Rightarrow G_1 = 164 \approx 44\ \text{dB}$$

$$G_1 f_{p1}^2 = G_2 f_z^2 \Rightarrow G_2 = 24 \approx 28\ \text{dB}$$

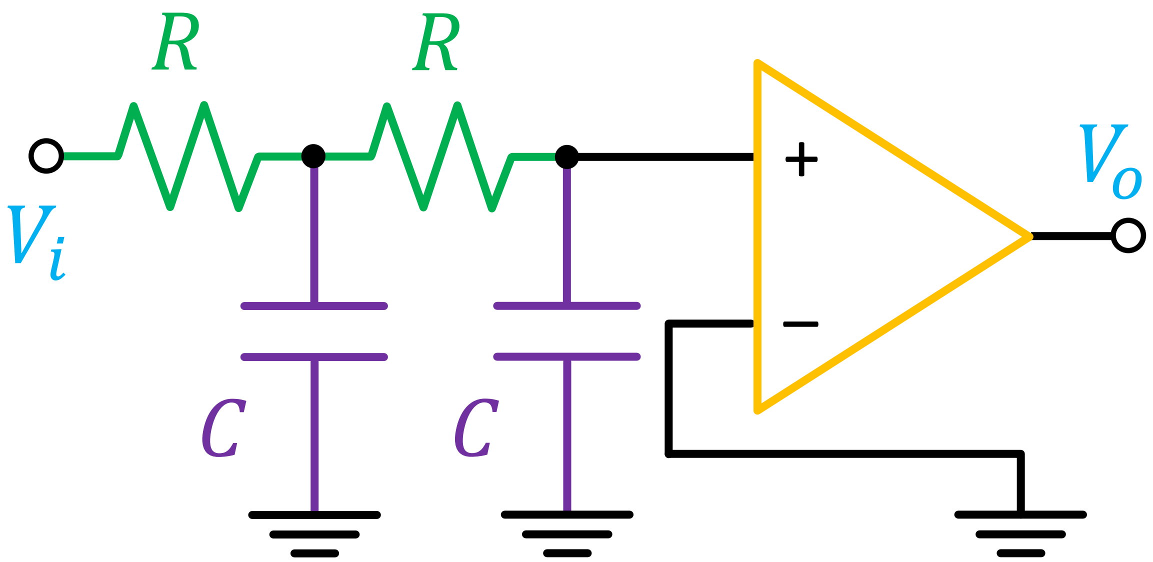
$$G_2 f_{p2} = f_{0dB} = 10\ \text{MHz}$$

Bode plot



$|G_{loop}|_{dB}$
$G_0 = 100$
G_1
G_2
f_{0dB}
$f_{p0} = 100$
$f_{p1} =$ 60.8k
$f_{p2} =$ 417k
$\log f$
$f_z = 159$k

Open-loop gain



$$G_{OL} = -G_{loop}G_{id} = A(s)\,\frac{1}{1+3sCR+(sCR)^2}$$

Gain calculations

$$G_0 f_{p0} = G_1 f_{p1} \Rightarrow G_1 = 164 \approx 44\text{ dB}$$

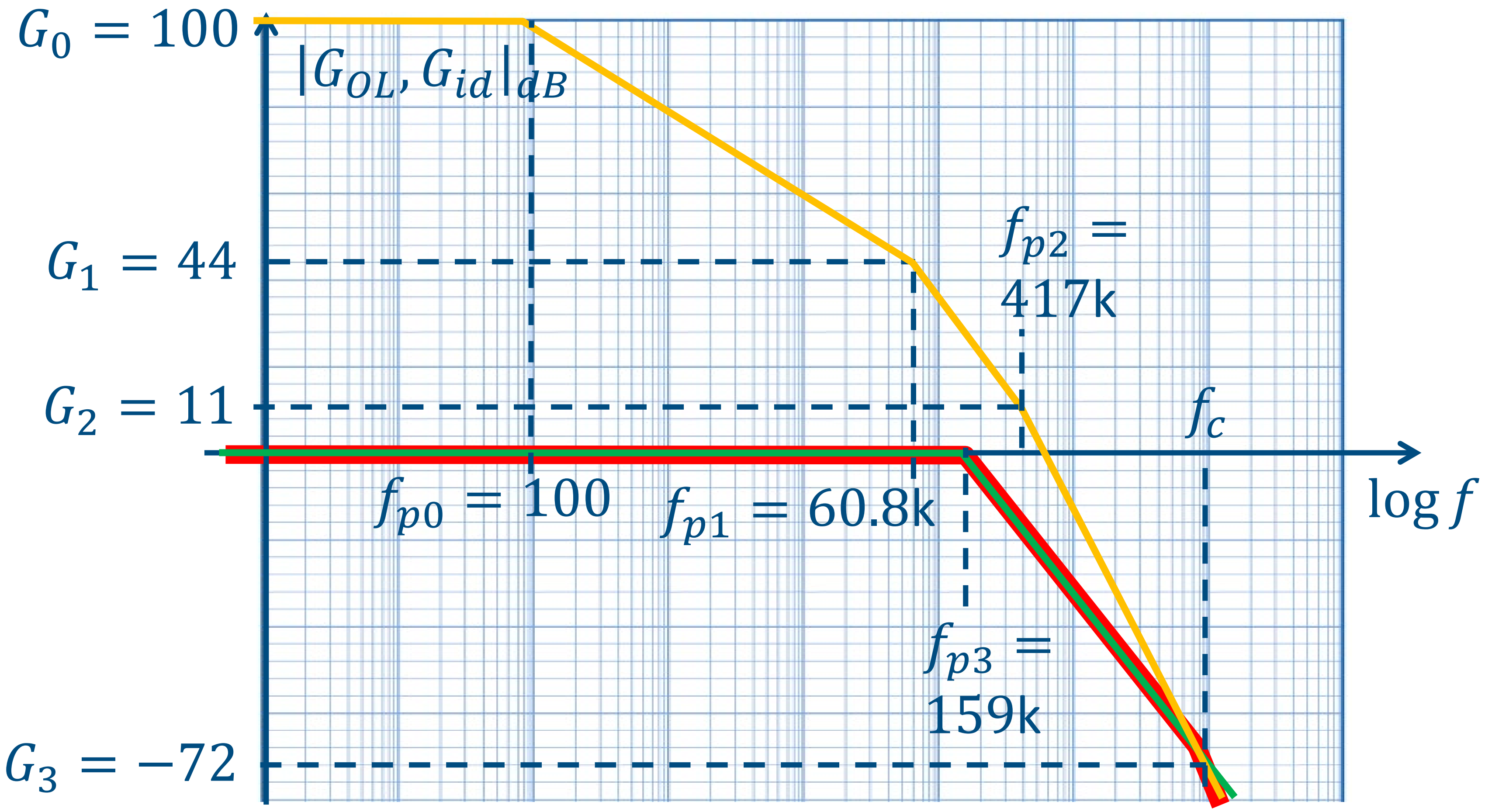
$$G_1 f_{p1}^2 = G_2 f_{p2}^2 \Rightarrow G_2 = 3.49 \approx 11\text{ dB}$$

$$f_{p3}^2 = G_3 f_c^2 \Rightarrow G_3 = 2.5 \times 10^{-4} \approx -72\text{ dB}$$

or equivalently

$$G_2 f_{p2}^3 = G_3 f_c^3$$

Bode plot



$G_0 = 100$
$|G_{OL}, G_{id}|_{dB}$
$G_1 = 44$
$G_2 = 11$
$G_3 = -72$
$f_{p2} =$ 417k
f_c
$f_{p0} = 100$
$f_{p1} = 60.8\text{k}$
log f
$f_{p3} =$ 159k

Outline

- HP amplifier
- LP filter
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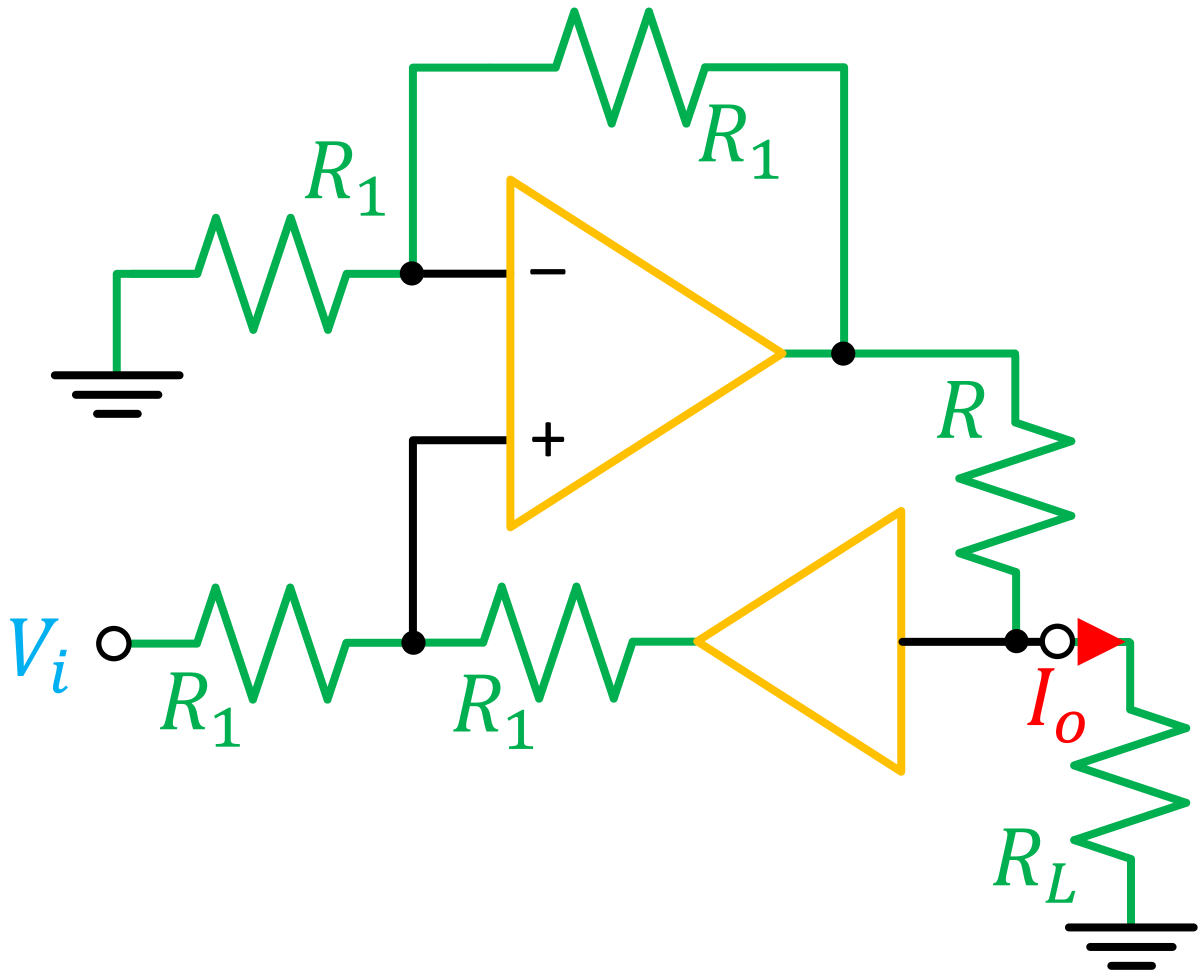
Current source



- $R_1 = 25\ \text{k}\Omega$, $R = 2.5\ \text{k}\Omega$
- $GBWP = 5$ MHz

1. Find the TF of the stage
2. Check the stability
3. Compute Z_{out}

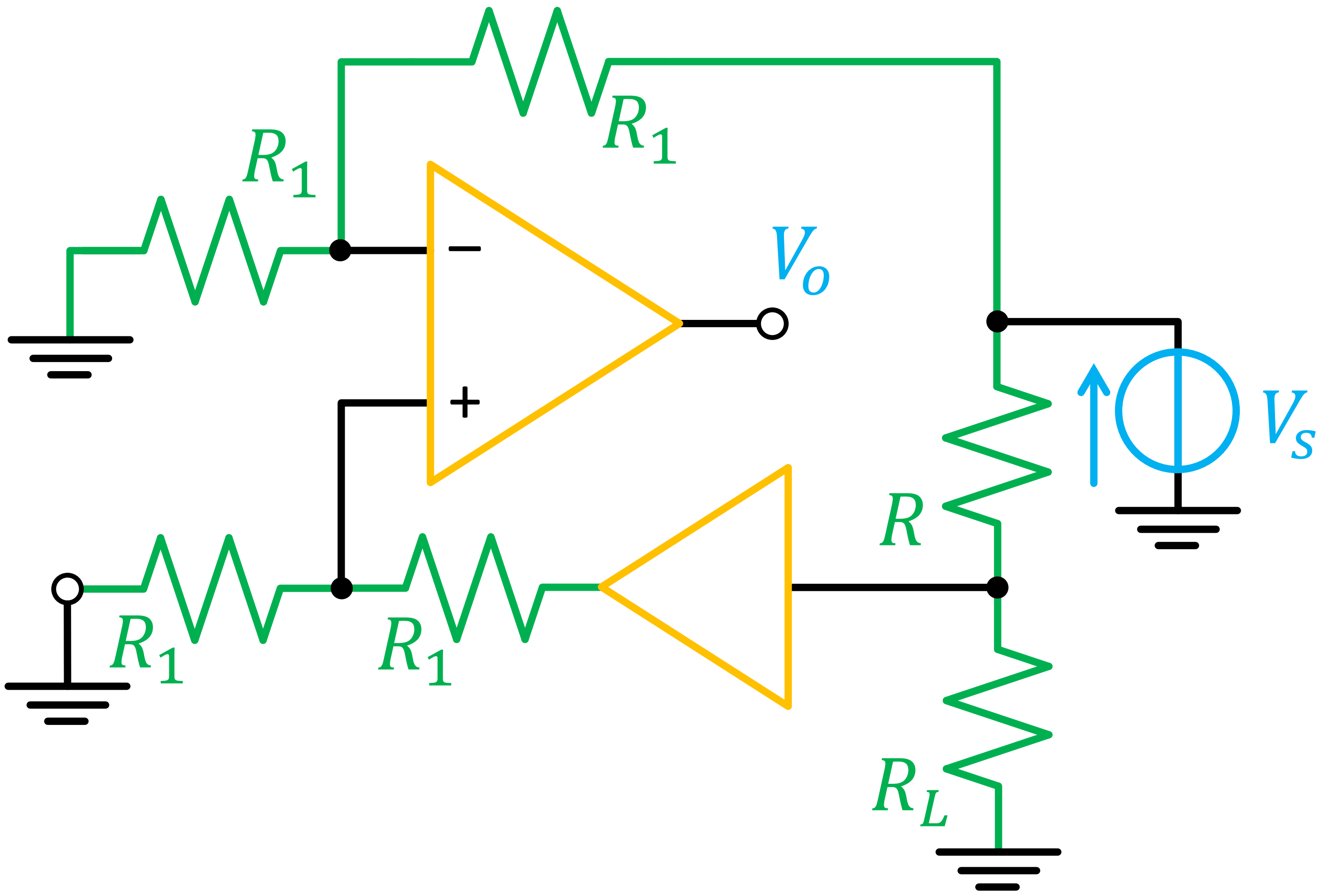
Gain

$$V^+ = \frac{V_i + V_o}{2} = V^- \Rightarrow V_o^{OA} = 2V^- = V_i + V_o$$

$$I_o = \frac{V_o^{OA} - V_o}{R} = \frac{V_i}{R}$$

The scheme is an ideal current source (note that the output current does not depend on the load)

Loop gain

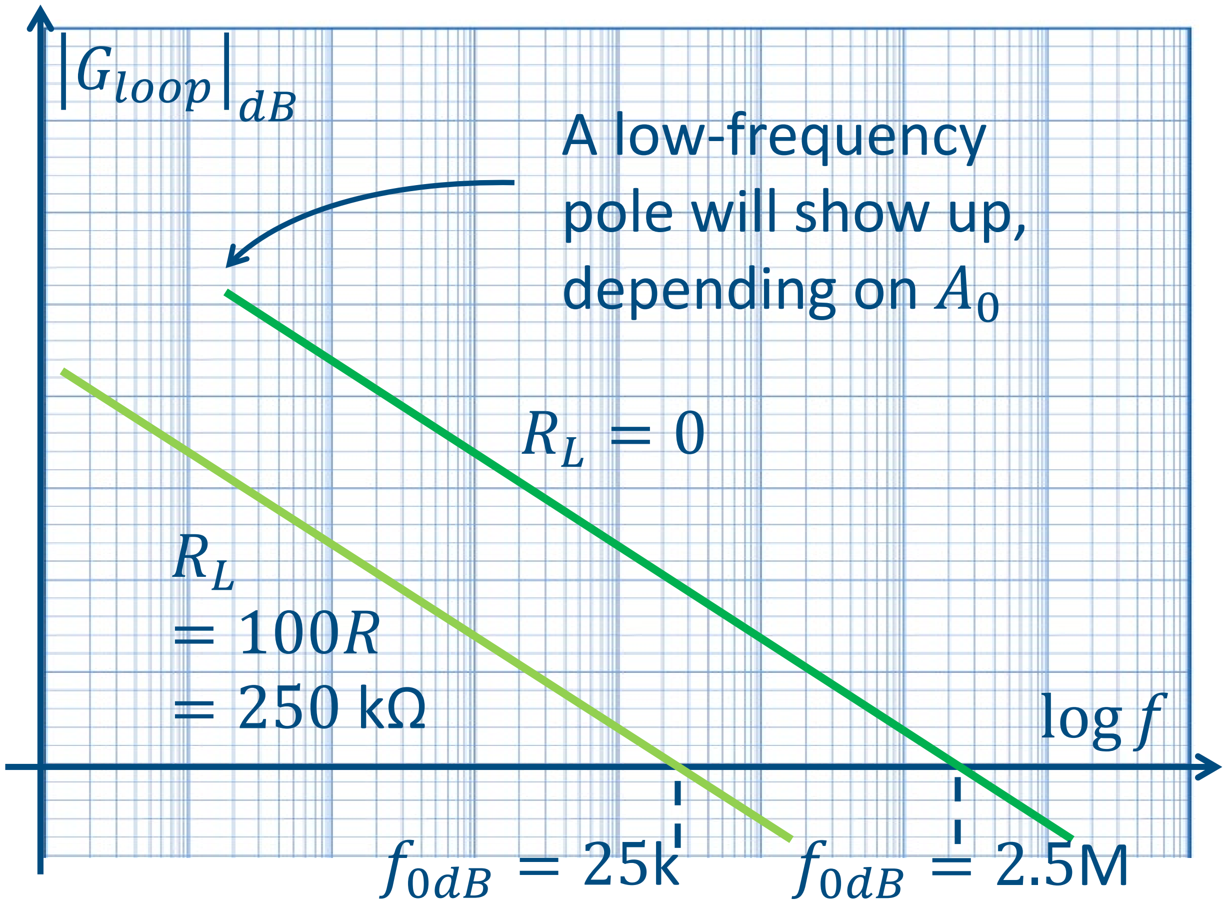


Result

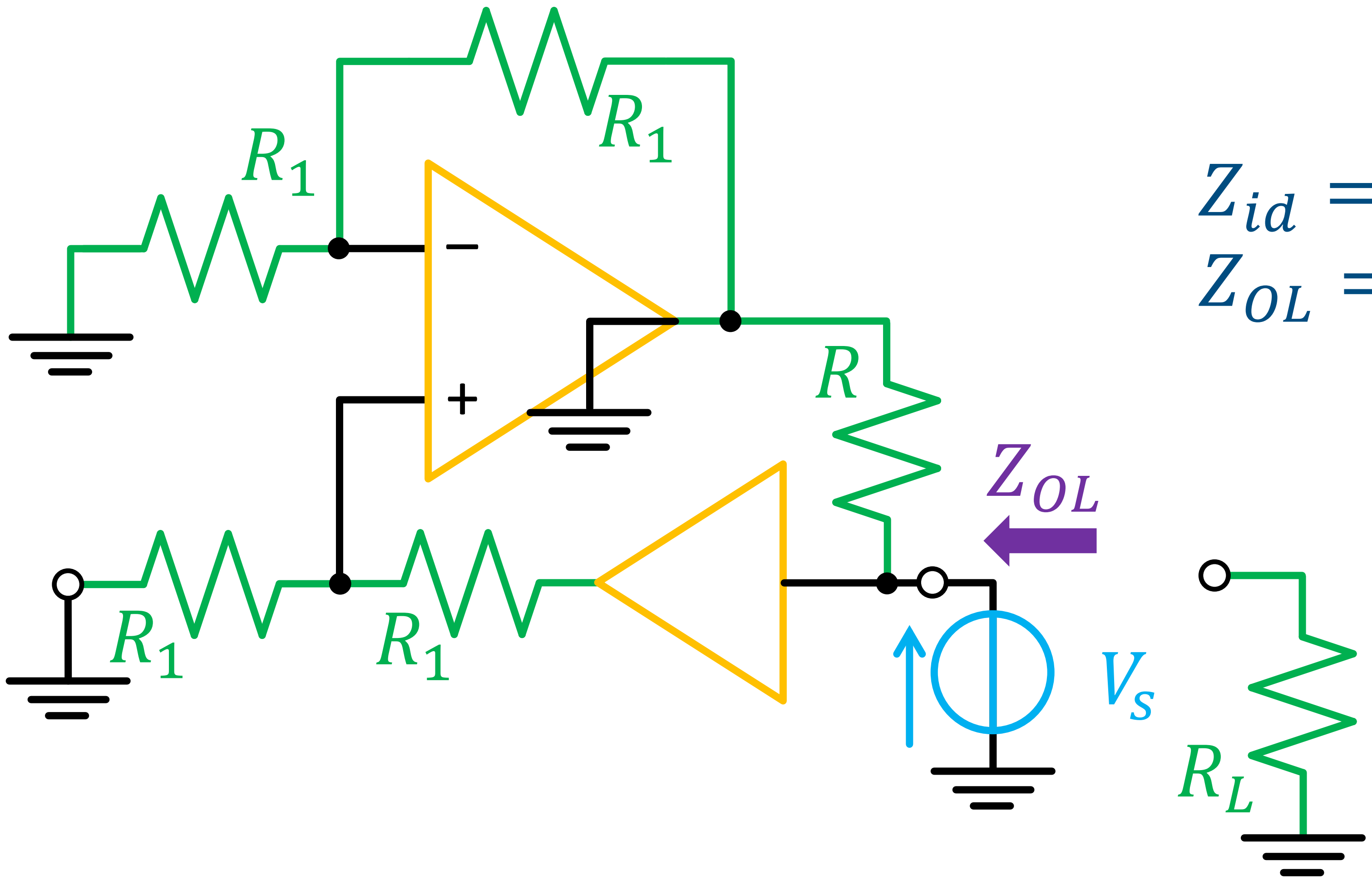
$$\frac{V^+}{V_s} = \frac{1}{2}\frac{R_L}{R + R_L} \qquad \frac{V^-}{V_s} = \frac{1}{2}$$

$$G_{loop} = -A(s)\frac{R}{2(R + R_L)}$$

Large values of R_L do not affect stability but reduce precision (related to $1/G_{loop}$)



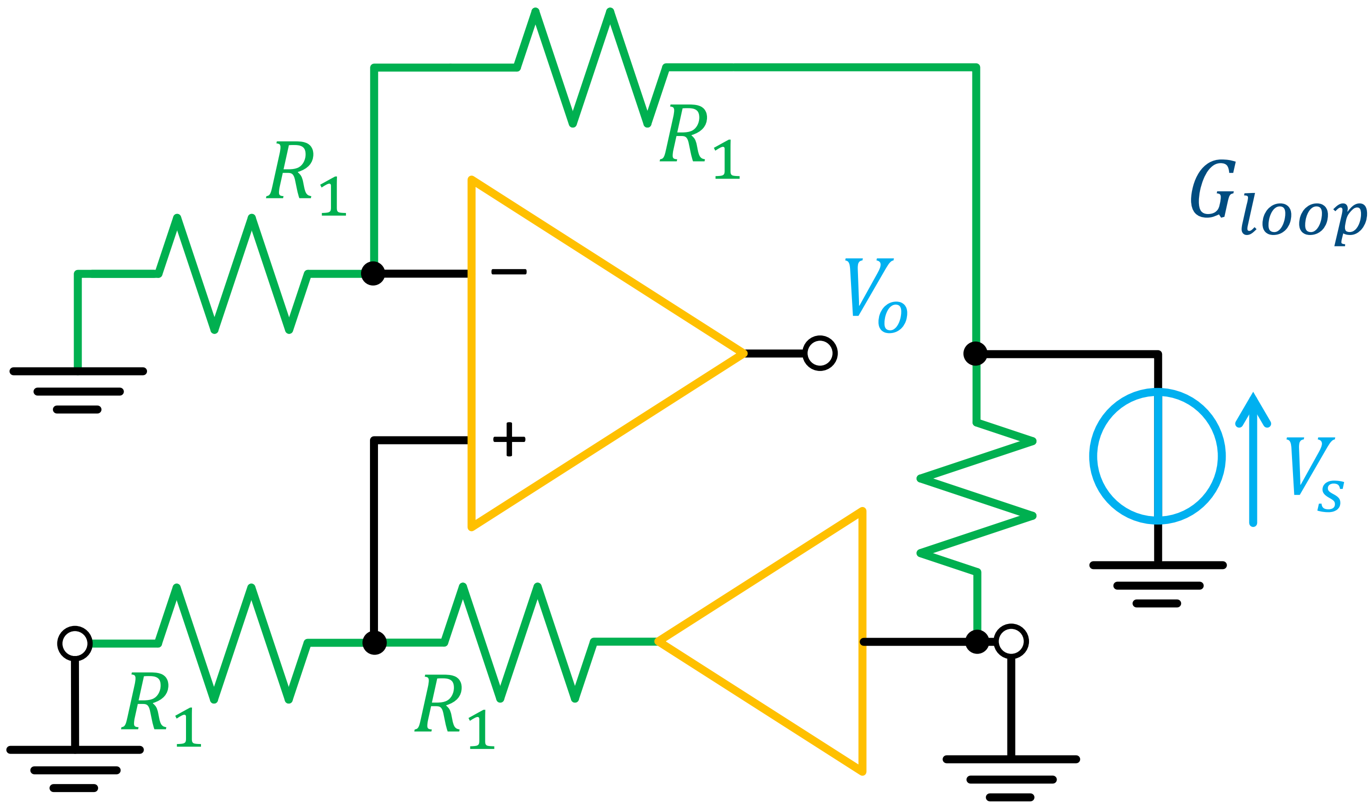
Open-loop impedance



$Z_{id} = \infty$
$Z_{OL} = R$

Always disconnect the load when computing Z_{out}!

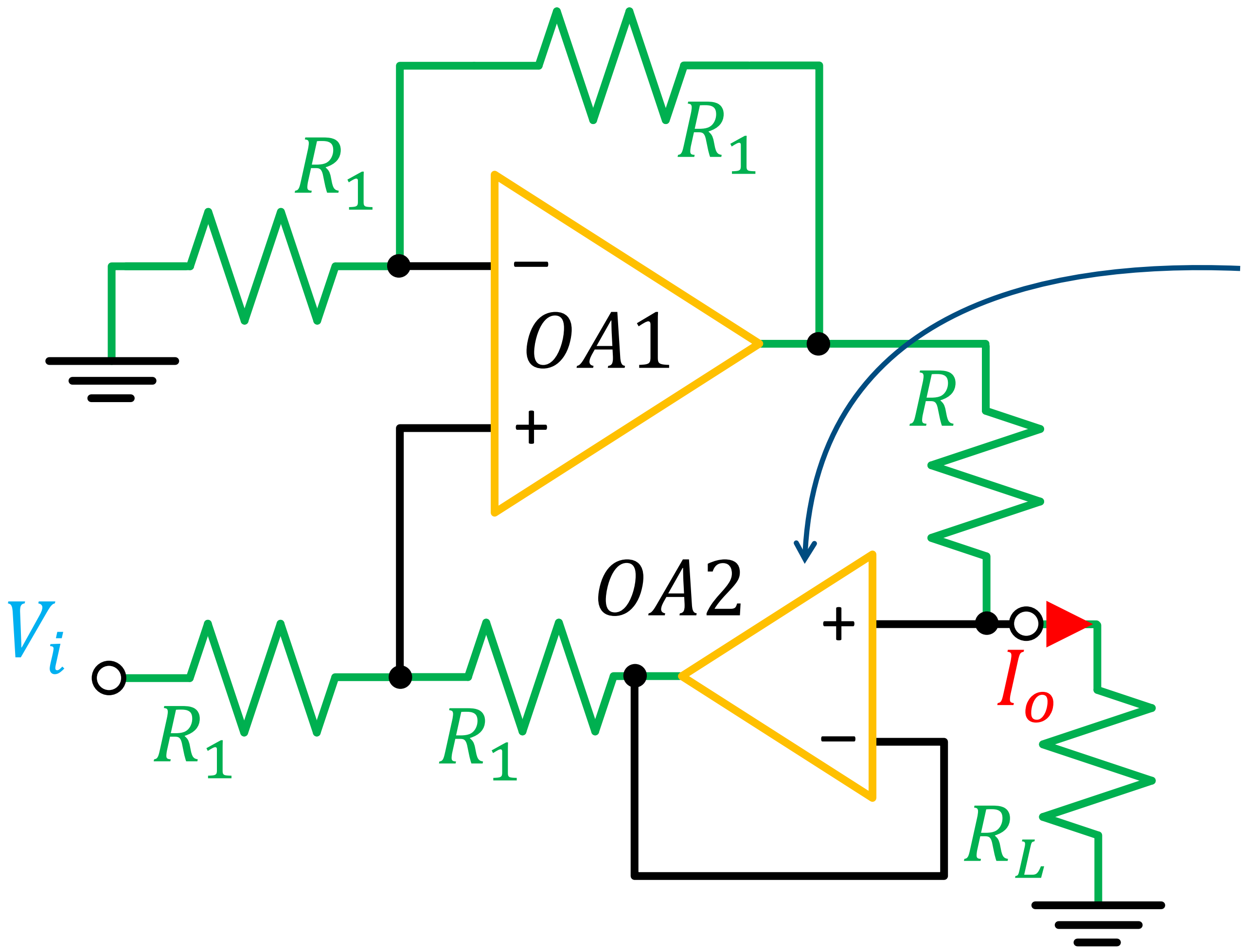
Loop gain



$$G_{loop} = -\frac{A(s)}{2}$$

$$Z_{out} = R\left(1+\frac{A(s)}{2}\right) = 10\ \text{G}\Omega \text{ at LF}$$

Inner loop example



Transfer function becomes $1/(1+s\tau_c)$, with a pole at $GBWP$

Homework

1. Compute the input impedance for the HP amplifier stage in slide #5
2. Compute the output impedance for the scheme in slide #32 when OA1 is ideal and OA2 is not

Outline

- HP amplifier
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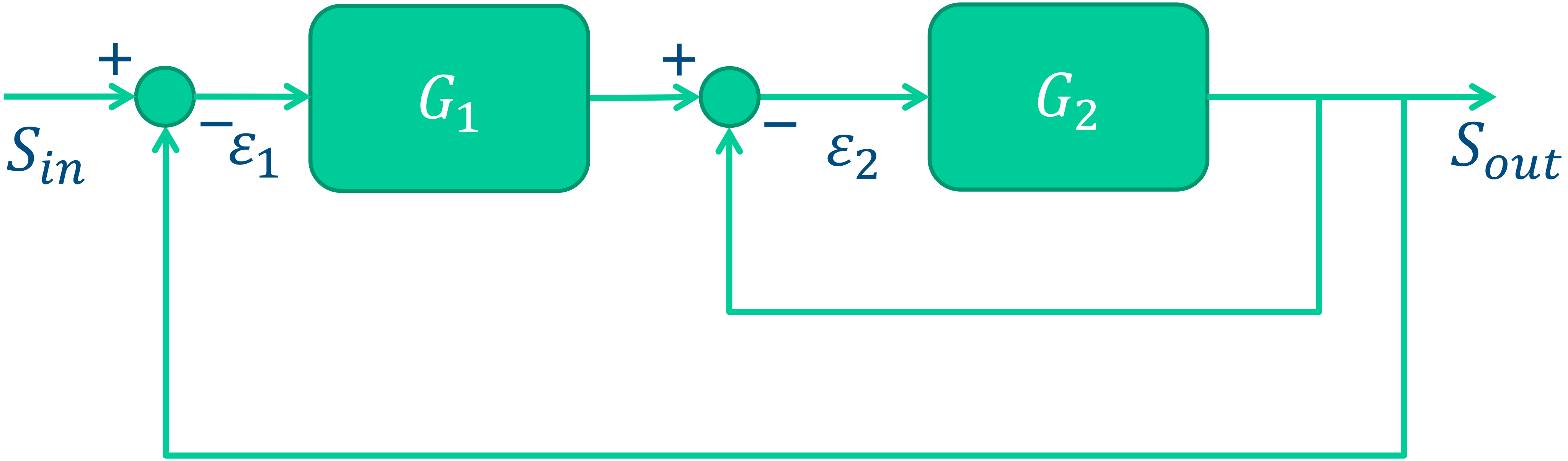
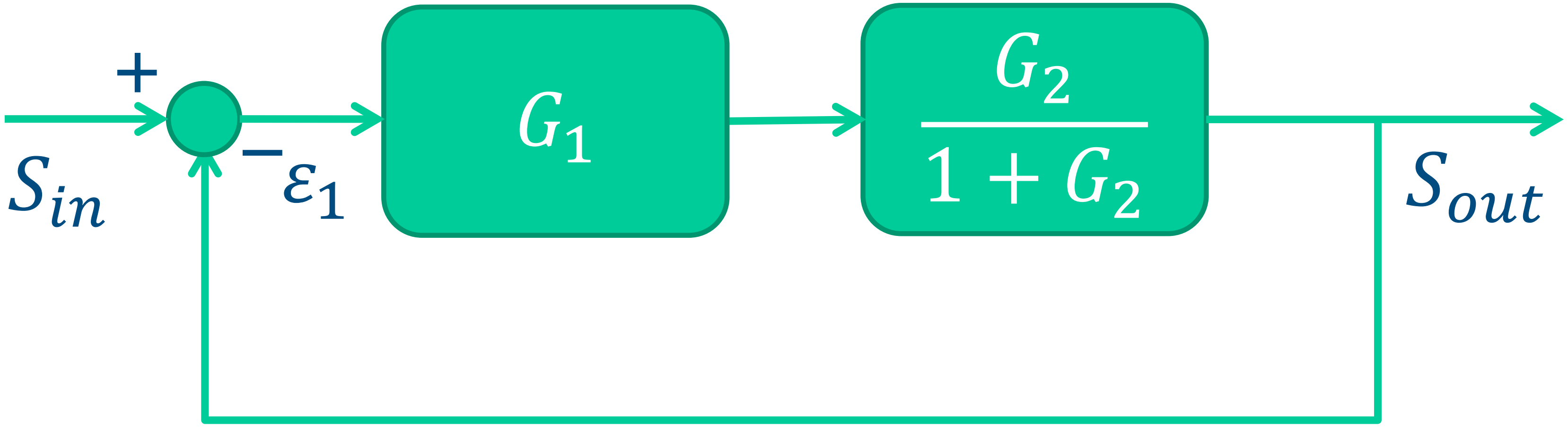
Simplified scheme



What is the relation between G_{id}, G_{OL} and G_{loop}?

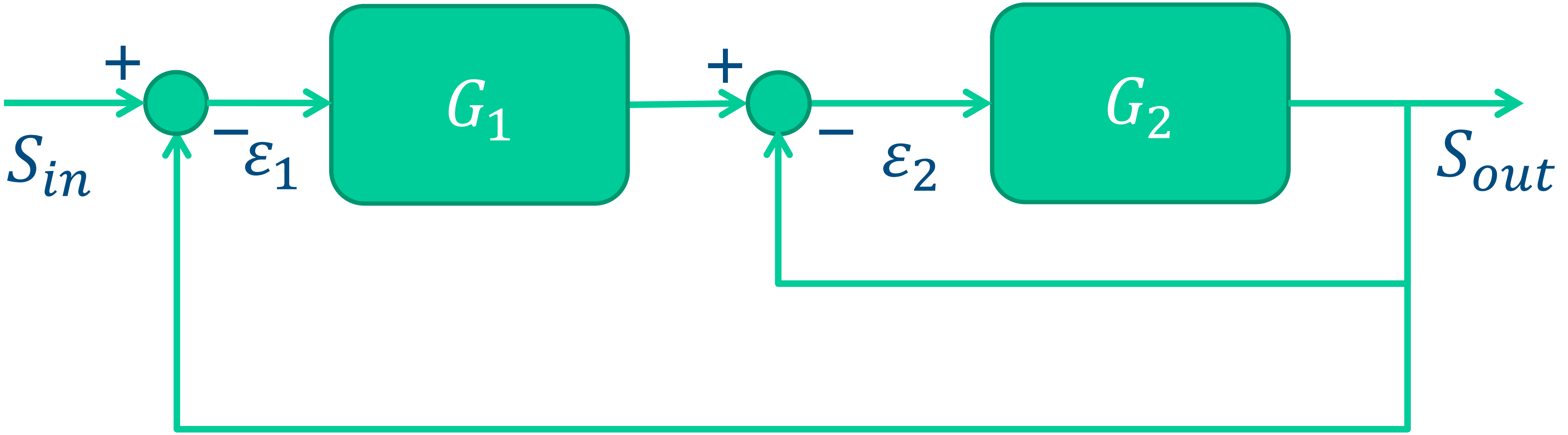
Approach 1: solving the inner loop



$$G_{loop} = -\frac{G_1 G_2}{1+G_2} \qquad G_{OL} = \frac{G_1 G_2}{1+G_2} \qquad G_{id} = 1$$

$$G = \frac{G_{OL}}{1-G_{loop}} = \frac{G_1 G_2}{1+G_2+G_1 G_2} = \frac{G_{id}}{1-\frac{1}{G_{loop}}} = \frac{1}{1+\frac{1+G_2}{G_1 G_2}}$$

Approach 2: cutting both loops



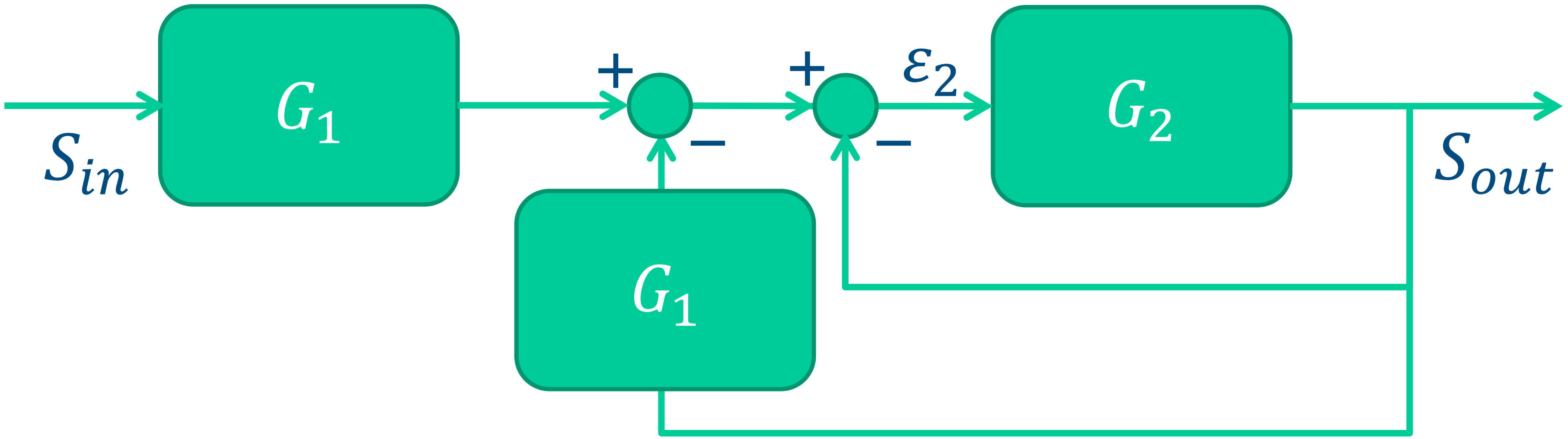
$$G_{loop} = -G_2(1+G_1) \qquad G_{OL} = G_1 G_2 \qquad G_{id} = 1$$

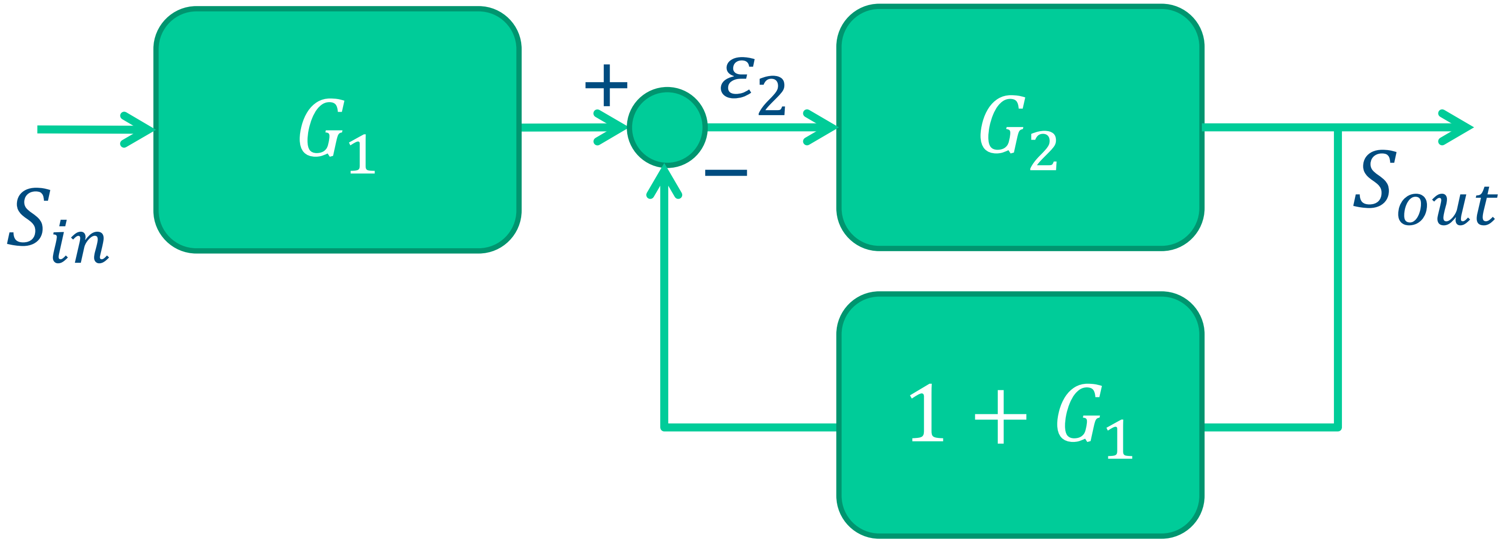
$$G = \frac{G_{OL}}{1 - G_{loop}} = \frac{G_1 G_2}{1 + G_2 + G_1 G_2} \neq \frac{G_{id}}{1 - \frac{1}{G_{loop}}} = \frac{1}{1 + \frac{1}{G_2(1+G_1)}}$$

Discussion

- From the viewpoint of stability, both approaches return the same condition
- The relation among G_{id}, G_{OL} and G_{loop} is only valid for a single-loop system
- To understand this point, remember that computing G_{id} means assuming $\varepsilon = 0$
  - In the first case, the error signal is ε_1, which is the error that controls the $S_{out} - S_{in}$ relation
  - When we are cutting both loops, the error signal is different

Loop simplification





$$G_{loop} = -G_2(1+G_1)$$

$$G_{OL} = G_1 G_2$$

$$G_{id} = \frac{G_1}{1+G_1} = -\frac{G_{OL}}{G_{loop}}$$