Electronics – 96032

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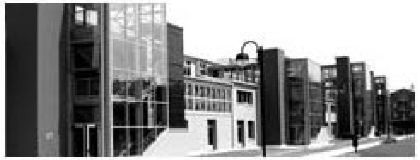



LPFs, GIs and BAs

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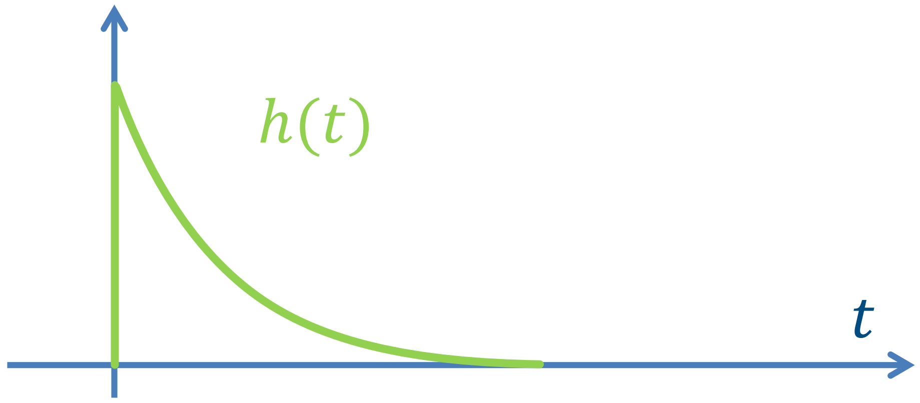
Disclaimer

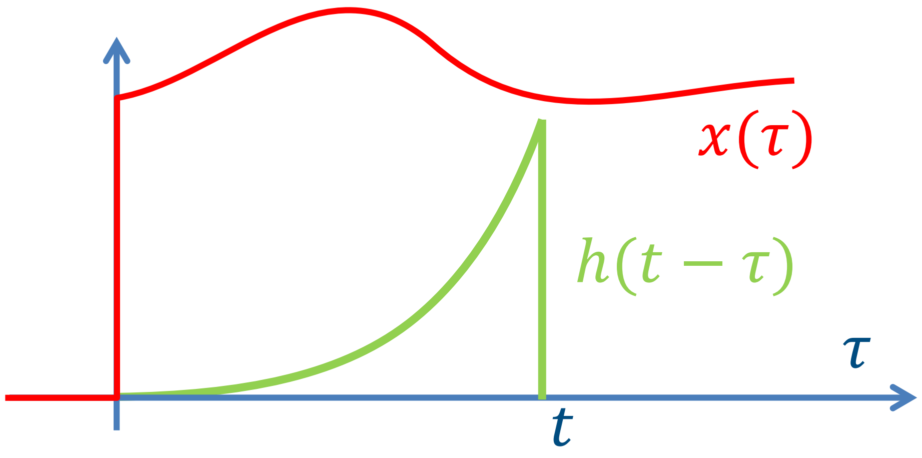
Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes

Outline

- LPF and GI
  - Review
  - Problems
- Boxcar Averager
  - Review
  - Problems

LPF review





- Delta-function response and WF

$$h(t) = \frac{1}{T_F} e^{-\frac{t}{T_F}} \mathrm{u}(t)$$

$$w(t,\tau) = h(t-\tau) = \frac{1}{T_F} e^{-\frac{t-\tau}{T_F}} \mathrm{u}(t-\tau)$$

- Output signal

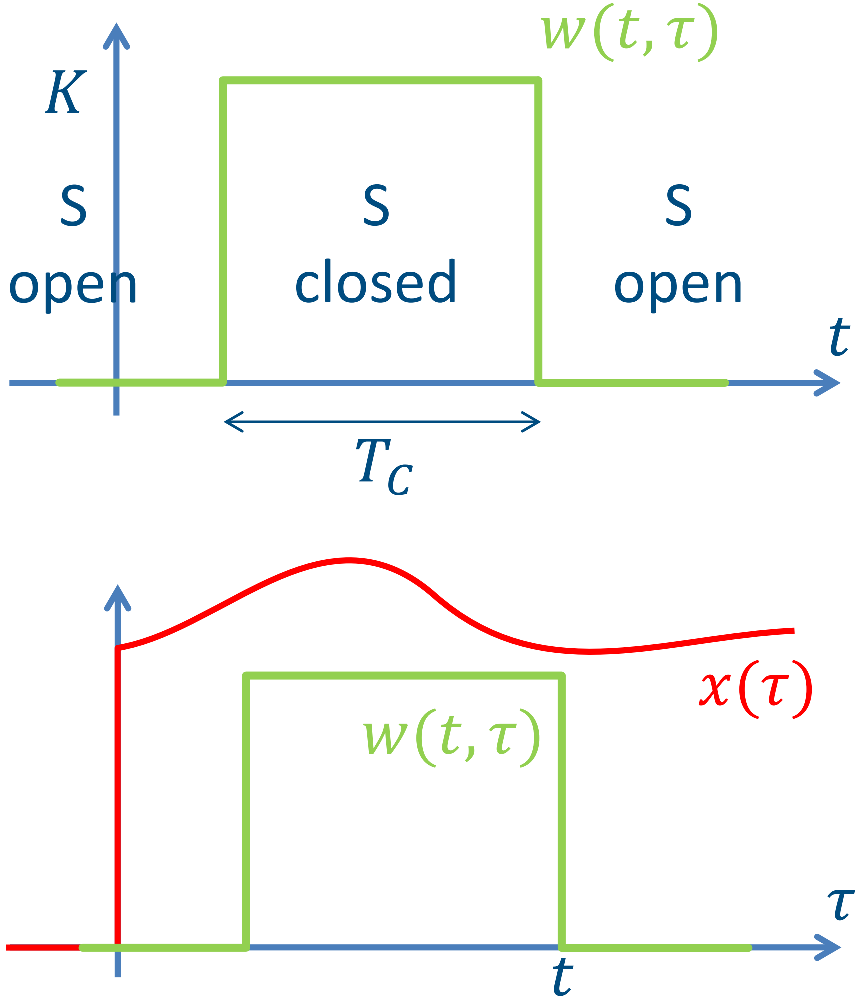
$$y(t) = \int x(\tau) w(t,\tau) d\tau = x(t) * h(t)$$

- Output noise

unilateral

$$\overline{n_y^2} = S_V \frac{\pi}{2} f_p = \frac{S_V}{4T_F}$$

GI review



- Weighting function

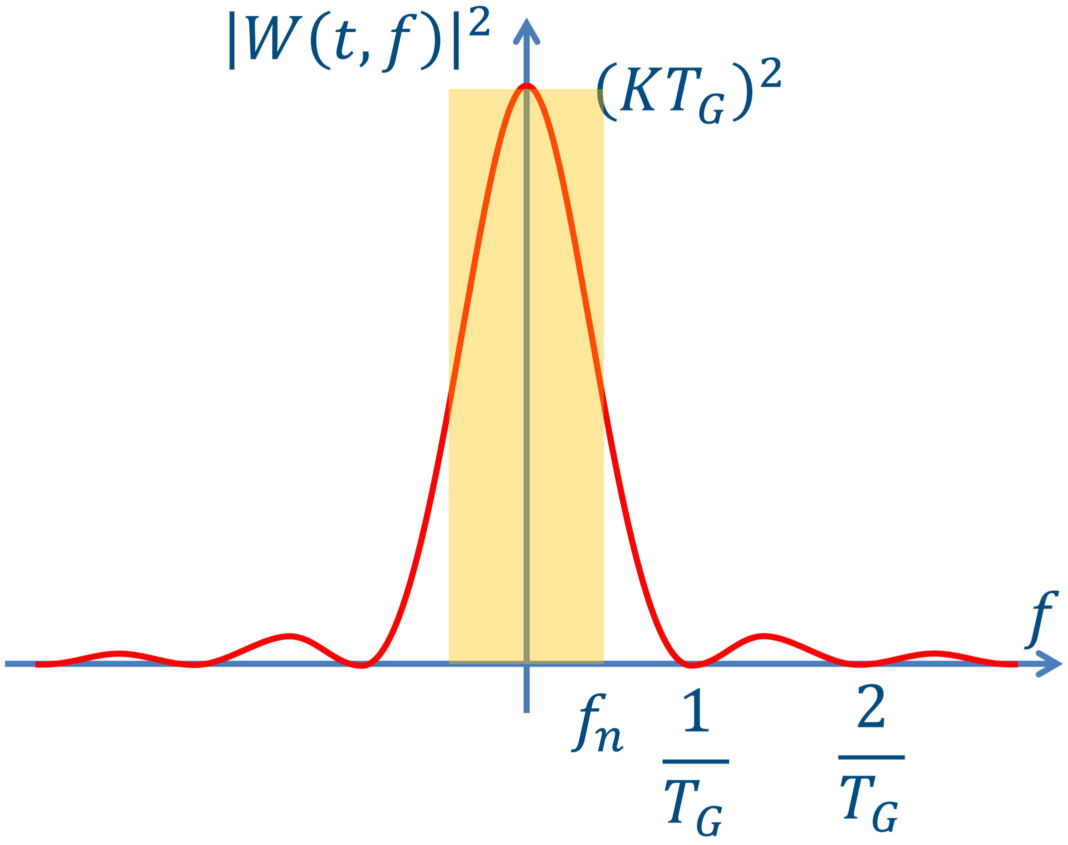
$$w(t,\tau) = K\,\mathrm{rect}(T_C)$$

- Output signal

$$y(t) = \int x(\tau) w(t,\tau) d\tau$$

$$= K \int_{T_C} x(\tau) d\tau$$

GI review: noise



- Remember that the equivalent BW of the GI is

$$f_n = \frac{1}{2T_G}$$

- The output noise is

$$\overline{V_o^2} = \lambda (KT_G)^2 2f_n = K^2 T_G \lambda$$

$$= K^2 T_G \frac{S_V}{2}$$

bilateral

unilateral

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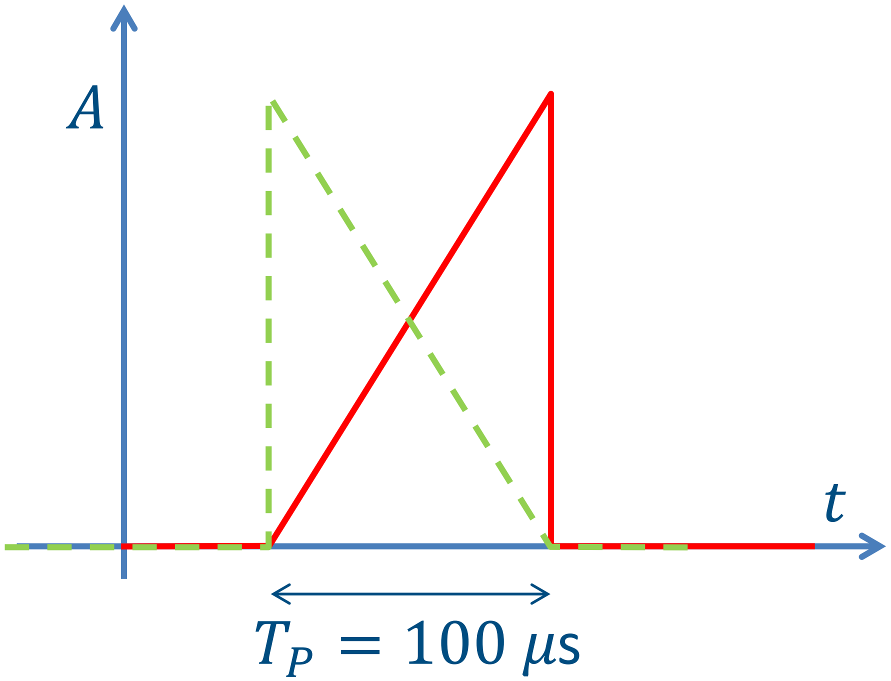
Problem: triangular pulse + WN



WN PSD: $\sqrt{S_V} = 20\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$ (unilateral)

1. Consider an LPF with $R = 1\ \mathrm{k\Omega}$, $C = 100\ \mathrm{nF}$. Evaluate the minimum detectable signal (MDS)
2. Repeat the calculation for the case of a GI
3. Repeat the calculation for a mirrored pulse (dashed-line)

LPF case: signal

- $RC = 100\ \mu s = T_P \Rightarrow$ we cannot say that the signal is unaffected by the LPF. On the contrary, significant filtering will occur
- The output signal (for $t \leq T_P$) must be evaluated as:

$$V_o(t) = \int V_i(\tau) h(t-\tau) d\tau = \int_0^t \frac{A}{T_P} \tau \frac{1}{T_F} e^{-\frac{t-\tau}{T_F}} d\tau = \frac{A}{T_P} e^{-\frac{t}{T_F}} \int_0^t \tau \frac{e^{\frac{\tau}{T_F}}}{T_F} d\tau$$

$$= \frac{A T_F}{T_P} e^{-\frac{t}{T_F}} \int_0^{t/T_F} x e^x dx = \frac{A}{T_P}\left(t - T_F + T_F e^{-\frac{t}{T_F}}\right)$$

Consider $\tau = 0$ at the beginning of the pulse

Alternative calculation

The ramp signal is the integral of a step signal with amplitude A/T_P.

Another way of computing the output is:

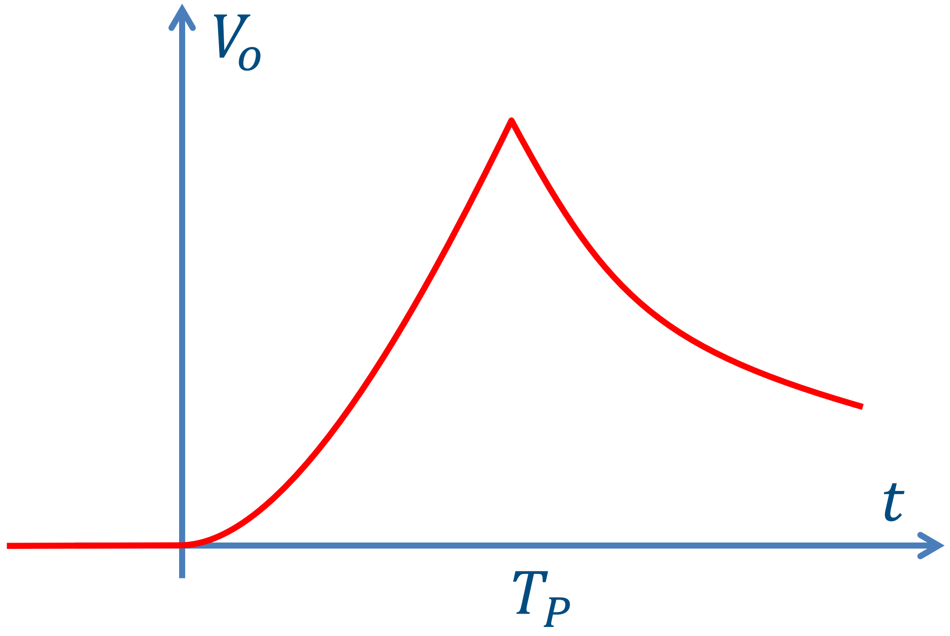
1. Consider the output to the step signal:

$$V_s(t) = \frac{A}{T_P}\left(1 - e^{-\frac{t}{T_F}}\right)$$

2. Integrate the output

$$V_o(t) = \frac{A}{T_P}\int_0^t \left(1 - e^{-\frac{\tau}{T_F}}\right) d\tau = \frac{A}{T_P}\left(t - T_F\left(1 - e^{-\frac{t}{T_F}}\right)\right)$$

Maximum output signal



- For $t > T_P$ there is no input signal and the output is simply an exponential discharge

$$V_o(t) = V_o(T_P)e^{-\frac{(t-T_P)}{T_F}}$$

- The maximum output signal is then

$$V_o^{max} = V_o(T_P) = \frac{A}{e} \approx 0.37\ A$$

LPF: noise and MDS

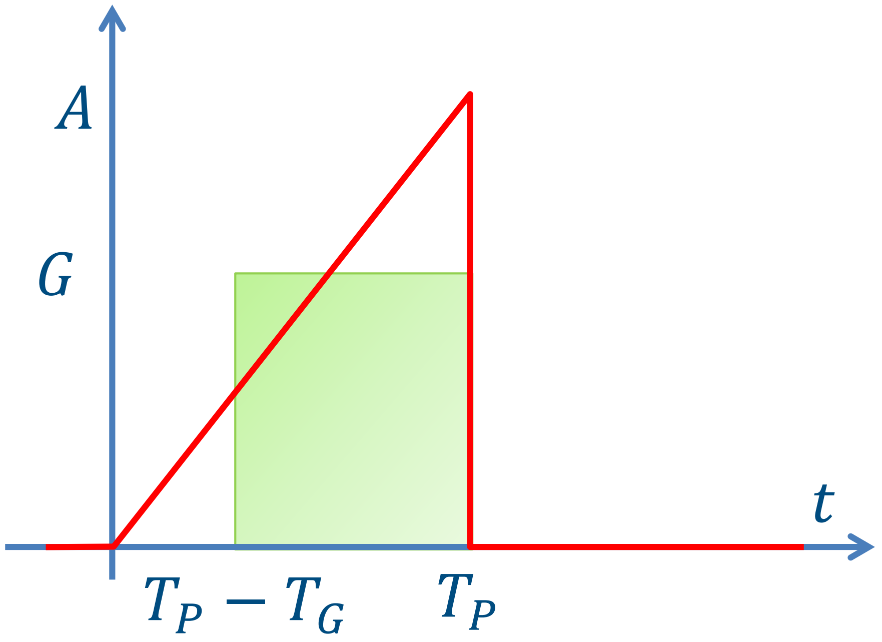
- The output rms noise is

$$\overline{V_o^2} = S_V \frac{\pi}{2} f_p = \frac{S_V}{4T_F} \approx (1\ \mu\text{V})^2$$

- The MDS is given by

$$\frac{S}{N} = \frac{V_o^{max}}{\sqrt{\overline{V_o^2}}} = 1 \Rightarrow A_{min} = 2.7\ \mu\text{V}$$

GI case: signal



- We could simply integrate over the whole pulse. However, let's find what the optimum integration window is
- It is wise to center the integration window T_G around the maximum of the signal

$$V_o = G\int_{T_P-T_G}^{T_P}\frac{A}{T_P}t\,dt = GAT_G\left(1-\frac{T_G}{2T_P}\right)$$

GI case: S/N

- GI output noise is

$$\overline{V_o^2} = G^2 T_G \frac{S_V}{2}$$

- S/N becomes

$$\frac{S}{N} = \frac{GAT_G\left(1-\frac{T_G}{2T_P}\right)}{\sqrt{G^2 T_G \frac{S_V}{2}}} = 2A\sqrt{\frac{T_P}{S_V}}\sqrt{\frac{T_G}{2T_P}}\left(1-\frac{T_G}{2T_P}\right)$$

($x = \frac{T_G}{2T_P}$)

- S/N optimization

$$\frac{d}{dx}\left(\frac{S}{N}\right) = 0 \Rightarrow \frac{d}{dx}\left(\sqrt{x} - x^{3/2}\right) = 0 \Rightarrow x = \frac{1}{3} \Rightarrow T_G = \frac{2}{3}T_P$$

GI case: MDS

- The optimum integration time gives

$$\frac{S}{N} = A\sqrt{\frac{T_P}{S_V}\frac{4}{3\sqrt{3}}} \Rightarrow A_{min} = 2.6\ \mu\text{V}$$

- If we had integrated over the whole pulse, the result would have been

$$\frac{S}{N} = \frac{\frac{GAT_P}{2}}{\sqrt{G^2 T_P \frac{S_V}{2}}} = A\sqrt{\frac{T_P}{S_V}\frac{1}{\sqrt{2}}} \Rightarrow A_{min} = 2.8\ \mu\text{V}$$

Mirrored pulse

- The GI result will not change (if the integration window is moved)
- LPF output signal must be re-computed

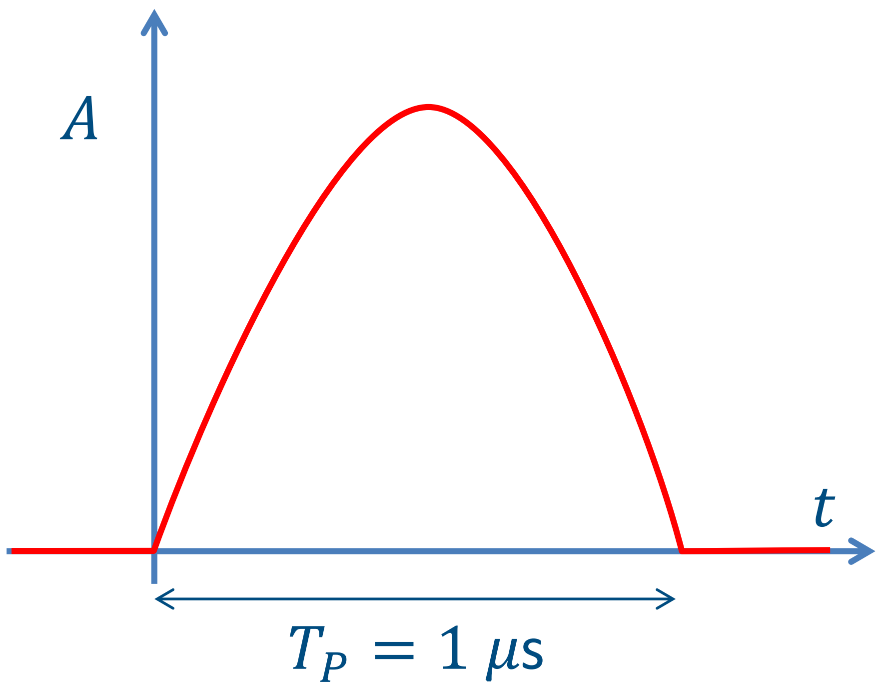
$$V_o(t) = \int_0^t A\left(1-\frac{\tau}{T_P}\right)\frac{1}{T_F}e^{-\frac{t-\tau}{T_F}}d\tau = \int_0^t \frac{A}{T_F}e^{-\frac{t-\tau}{T_F}}d\tau - \text{Prev. res.}$$

$$= \frac{A}{T_P}\left(T_P + T_F - t - (T_P+T_F)e^{-\frac{t}{T_F}}\right)$$

$$\frac{dV_o}{dt} = 0 \Rightarrow t = T_F \ln\left(1+\frac{T_P}{T_F}\right) = T_F \ln 2 \Rightarrow V_o^{max} = T_F(1-\ln 2)$$

$$\Rightarrow A_{min} = 3.25\ \mu\text{V}$$

Problem: sinusoidal pulse + WN

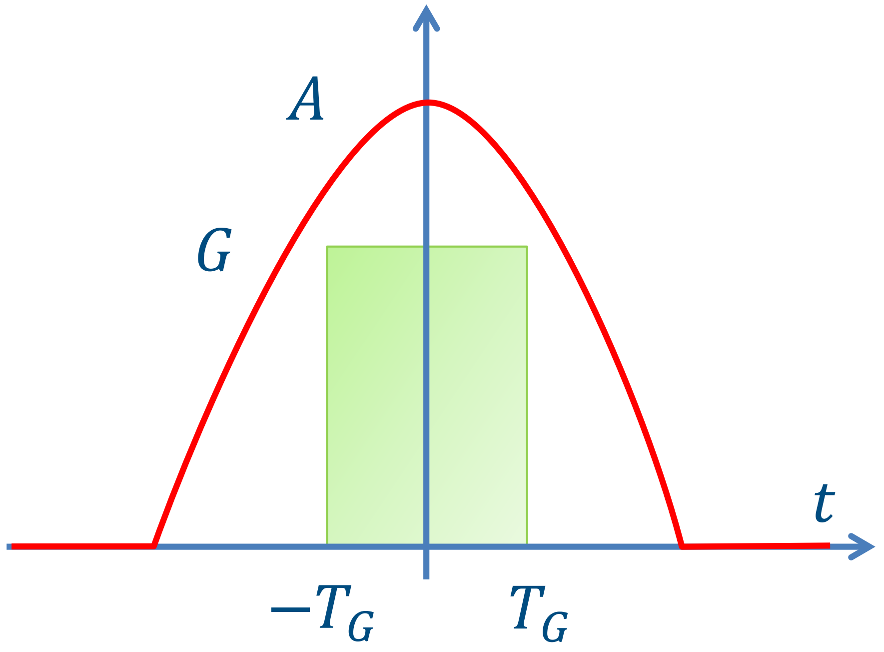


WN PSD: $\sqrt{S_V} = 1\ \mu\text{V}/\sqrt{\text{Hz}}$
(bilateral)

We have a series of sinusoidal pulses with amplitude A and WN PSD S_V

1. What filter would you use to extract the amplitude of every single pulse?
2. Optimize the filter parameters to maximize S/N (note that $\tan x = 2x$ has solution $x \approx 1.17$)

GI: signal



x axis shift: signal is now symmetric and can be written as

$$V_i = A\cos\left(\frac{\pi t}{T_p}\right)$$

A GI must be employed, centered around the maximum of the signal. The output signal is

$$V_o = 2GA\int_0^{T_G}\cos\left(\frac{\pi t}{T_p}\right)dt$$

$$= \frac{2GAT_P}{\pi}\sin\left(\frac{\pi T_G}{T_p}\right)$$

GI case: S/N

- GI output noise is

$$\overline{V_o^2} = G^2 2T_G S_V$$

- S/N becomes

$$\frac{S}{N} = \frac{\frac{2GAT_P}{\pi}\sin\left(\frac{\pi T_G}{T_p}\right)}{\sqrt{2G^2T_GS_V}} = A\sqrt{\frac{2T_P}{\pi S_V}}\frac{\sin\left(\frac{\pi T_G}{T_p}\right)}{\sqrt{\pi T_G/T_P}}$$

x

- S/N optimization

$$\frac{d}{dx}\left(\frac{\sin x}{\sqrt{x}}\right) = 0 \Rightarrow \tan x = 2x \Rightarrow x = 1.17 \Rightarrow T_G = 0.37\ \mu s \Rightarrow \frac{S}{N} = 6.79$$

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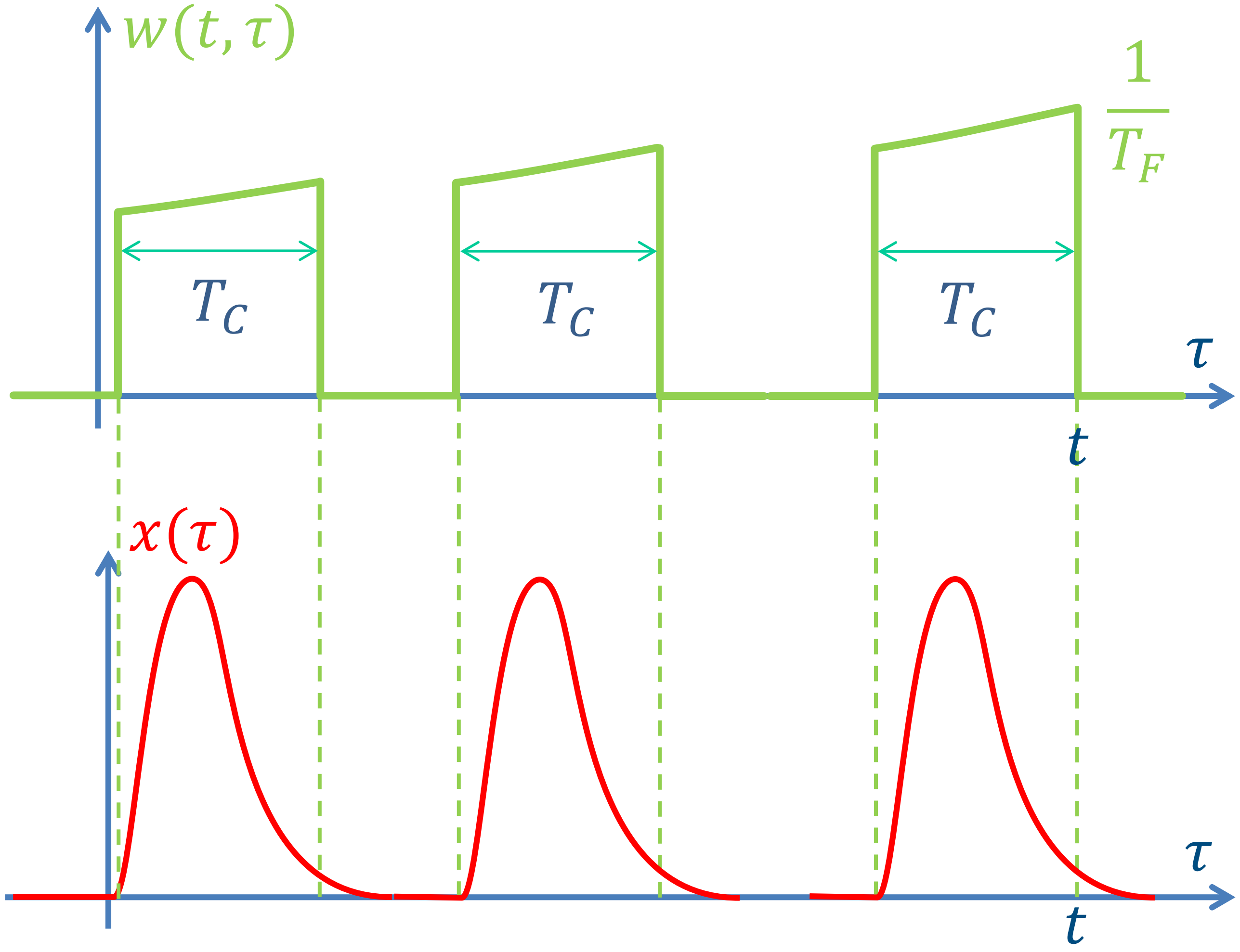
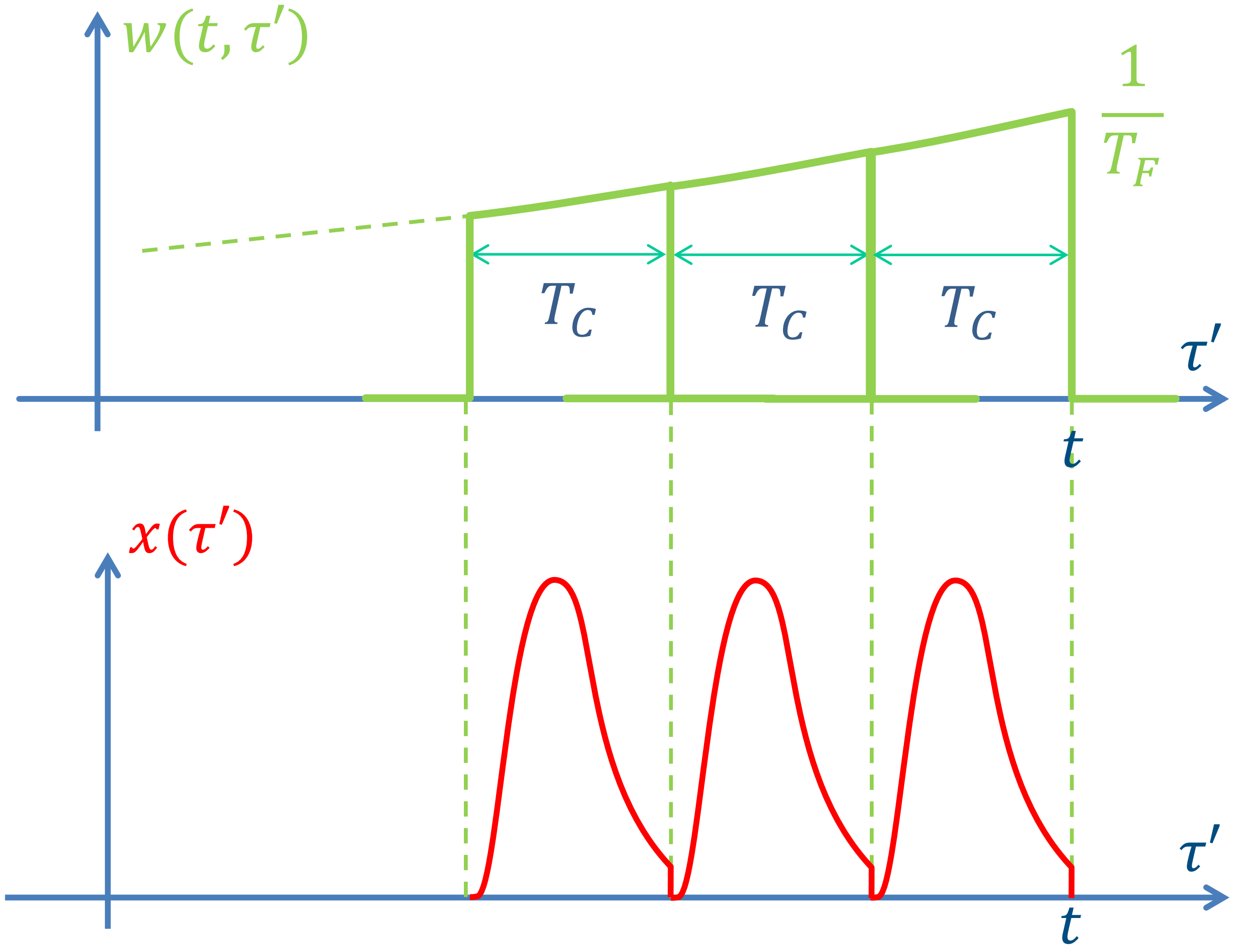
BA review



- Weighting function is the same as the LPF one, broken down into pieces
- Output signal

$$y(t) = \int x(\tau)w(t,\tau)d\tau$$

BA review – equivalent time



- Weighting function is now the same as LPF
- Signal is the replica of $x(\tau)$ over the interval T_C
- Output signal

$$y(t) = \int x(\tau')w(t,\tau')d\tau'$$

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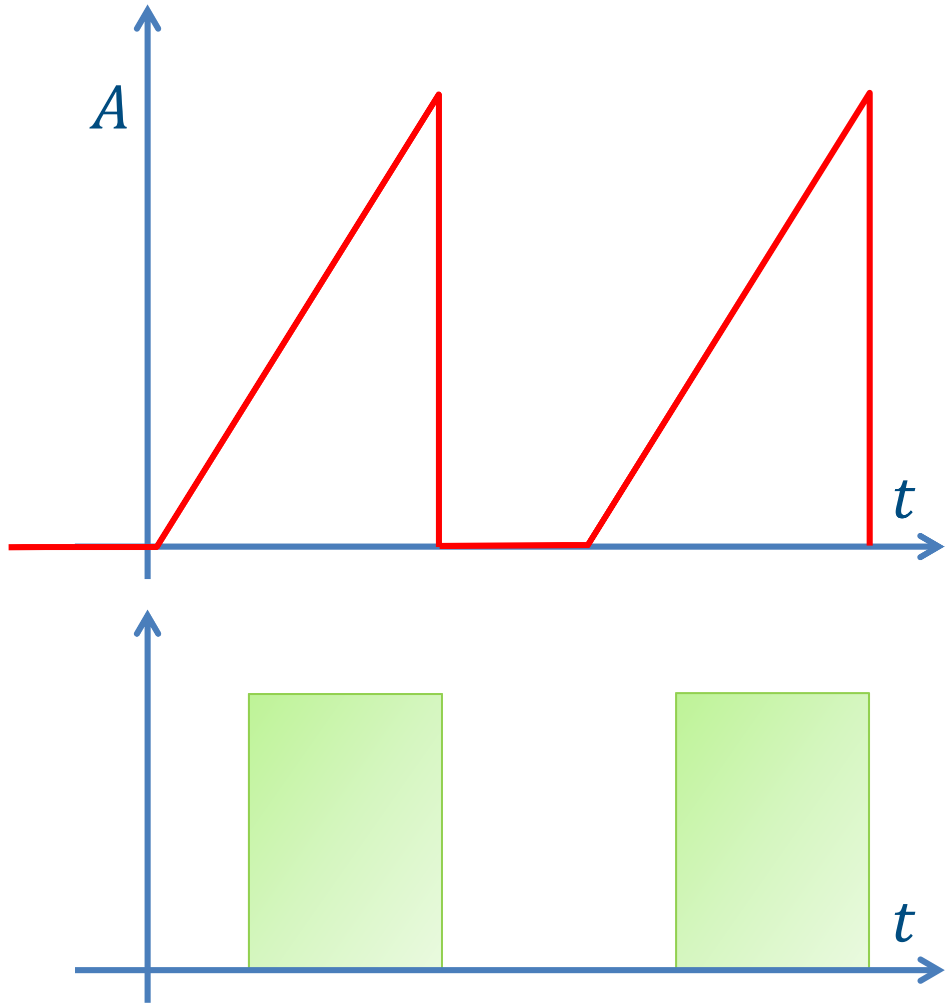
Problem: repetitive triangular pulse + WN



Same triangular pulse as previous problem ($A = A_{min}$), now repetitive with $f_R = 100$ Hz

1. Find the parameters of a filter that can achieve $S/N = 10$
2. Find the new parameters if A can be considered as constant only for a time $T_M = 5$ s

Multiple pulse averaging

If we keep the previous value for T_G (now T_C) and consider $T_F \gg T_C$, the single-pulse result for the BA is the same as for the GI

$$\left(\frac{S}{N}\right)_{sp} = \left(\frac{S}{N}\right)_{GI} = 1 \quad \text{(because } A = A_{min}\text{)}$$

$$\left(\frac{S}{N}\right)_{BA} = \left(\frac{S}{N}\right)_{sp} \sqrt{N_{eq}} \Rightarrow N_{eq} = 100 = \frac{2T_F}{T_C}$$

$$T_F = 50T_C = 3.33 \text{ ms}$$

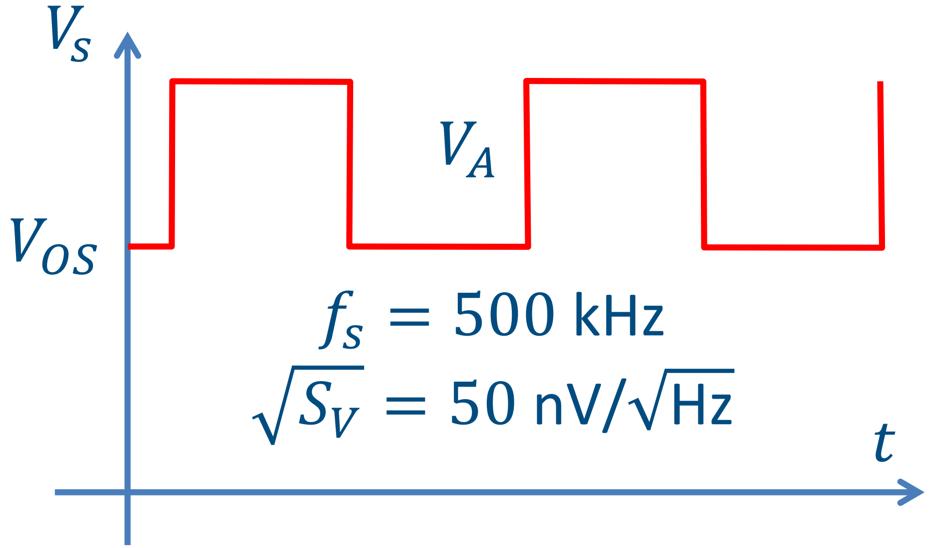
Measurement time

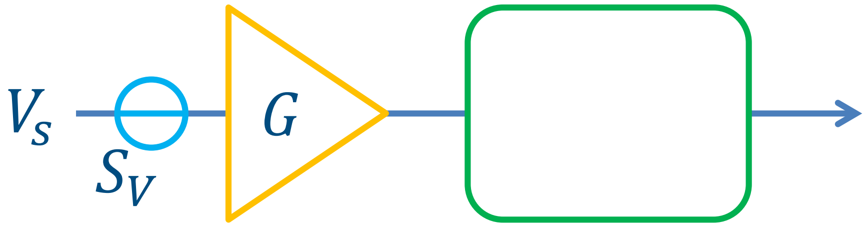
- The total measurement time must now be equal to $T_M \Rightarrow$ the number of pulses that we can process is $N = T_M f_R = 500$
- In the equivalent time, the weighting function must go to zero after N pulses

$$5T_F = NT_C \Rightarrow T_F = 100T_C = 6.67\ \mathrm{ms}$$

$$N_{eq} = \frac{2T_F}{T_C} = 200 \Rightarrow \left(\frac{S}{N}\right)_{BA} = \left(\frac{S}{N}\right)_{sp} \sqrt{N_{eq}} = \sqrt{200} \approx 14$$

Problem: square wave + WN



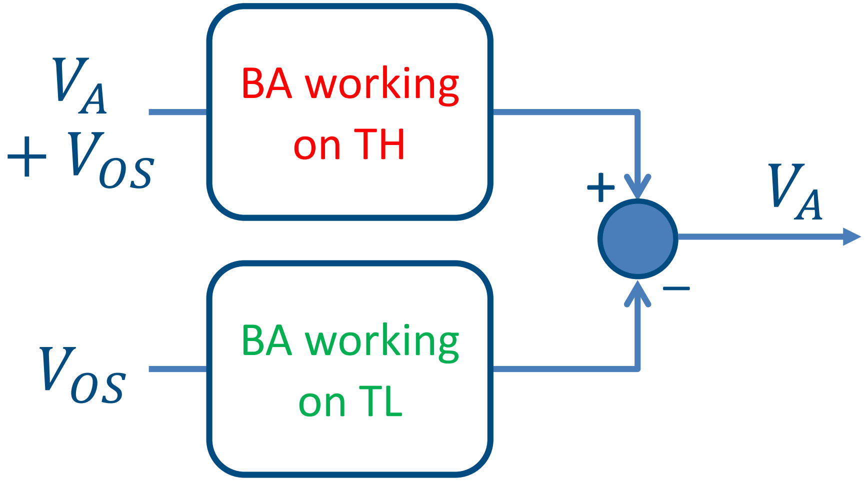


$G = 100$
$\sqrt{S_V} = 50$ nV/$\sqrt{\text{Hz}}$

Square wave with amplitude $V_A \approx 10\ \mu$V and large offset V_{OS}, feeding a large-BW amplifier

1. Devise a scheme for measuring V_A with $S/N = 10$ using BAs and find their parameters
2. Same as above when V_{OS} is a sinusoidal disturb at $3f_s$

Scheme



- G has no effect on S/N
- On a single pulse, output signals are obviously $K(V_A + V_{OS})T_H$ and $KV_{OS}T_H$
- Noise is $\sqrt{K^2 S_V T_H / 2}$

$$\left(\frac{S}{N}\right) = \frac{V_A T_H}{\sqrt{\frac{S_V}{2} T_H}} \sqrt{\frac{2T_F}{T_H}} = V_A \sqrt{\frac{4T_F}{S_V}} = 10 \Rightarrow T_F = \frac{25 S_V}{V_A^2} = 625\ \mu s$$

- In reality, $T_F = 2 \times 625\ \mu s = 1.25$ ms because of the difference operation

Alternative solution

- In the equivalent time frame, BAs behave like LPFs with constant input signals

$$\left(\frac{S}{N}\right) = \frac{V_A}{\sqrt{\frac{S_V}{4T_F}}} = 10 \Rightarrow T_F = \frac{25S_V}{V_A^2} = 625\ \mu s$$

- In reality, $T_F = 2 \times 625\ \mu s = 1.25$ ms because of the difference operation
- If the signal is not constant, this relation still holds if you consider its average value

Sinusoidal disturb – single pulse

- Output of the single-pulse integration:

$$V_H = K\int_0^{T_H} A\sin(3\omega_s t+\phi)dt = \frac{KA}{3\omega_s}(\cos\phi - \cos(3\omega_s T_H + \phi))$$

($3\omega_s T_H = \pi$)

$$= \frac{KA}{3\omega_s} 2\cos\phi$$

(pick always the worst case)

- What about the difference?

$$V_L = K\int_{T_H}^{2T_H} A\sin(3\omega_s t+\phi)dt = -V_H \Rightarrow V_H - V_L = \frac{4KA}{3\omega_s}\cos\phi$$

Final value

- Sum of pulses with exponentially-decaying amplitude ($K = 1/T_F$)

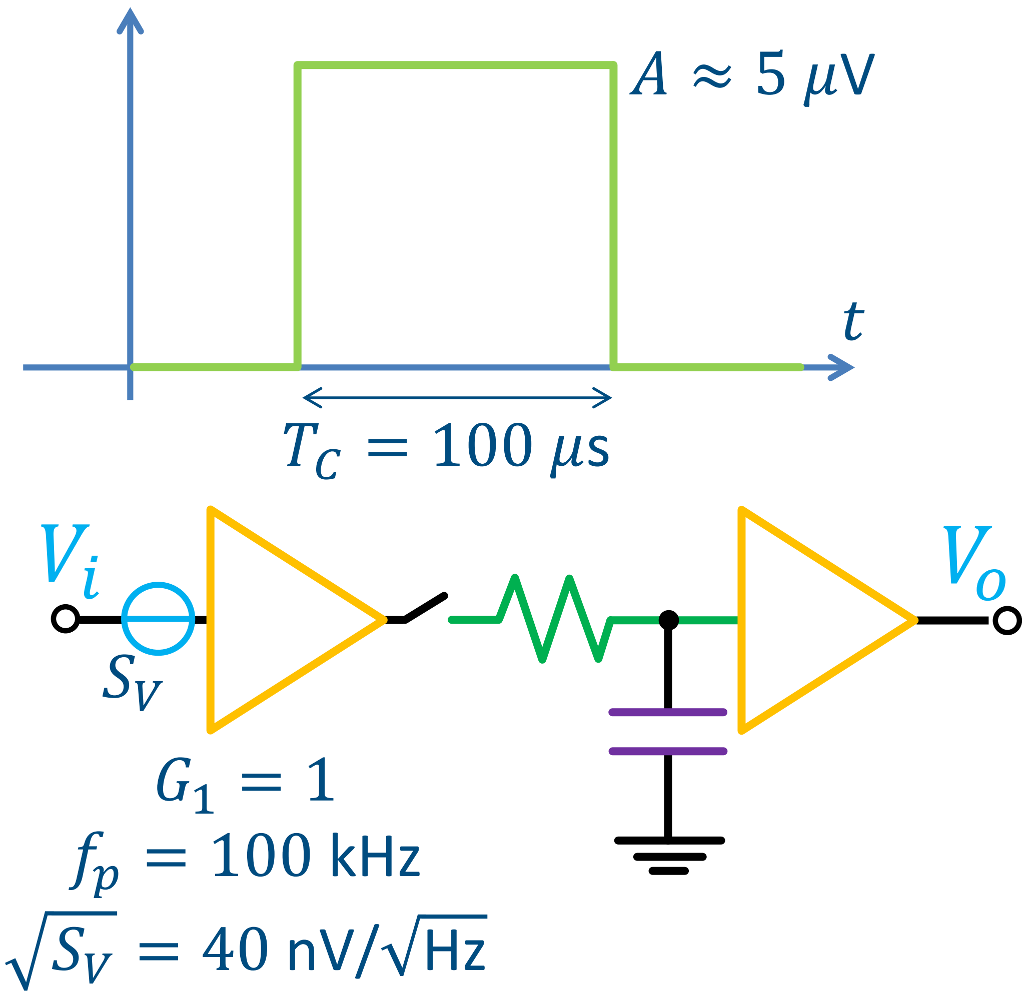
$$V_o = \frac{4KA}{3\omega_s}\sum_n \left(e^{-\frac{T_H}{T_F}}\right)^n = \frac{4KA}{3\omega_s}\frac{1}{1-e^{-\frac{T_H}{T_F}}} \approx \frac{4KA}{3\omega_s}\frac{T_F}{T_H} = \frac{4A}{3\pi}$$

- Alternatively, compute the average value of the signal

$$= \frac{2}{T_H}\int_0^{T_H} V_{OS}(t)dt = \frac{4A}{3\omega_s T_H} = \frac{4A}{3\pi}$$

- In any case, the disturb cannot be eliminated. A possibility would be to change the integration time to $(2/3)T_H$

Problem: BA with discharge



1. Use a GI after G_1 and compute S/N
2. We have repetitive pulses separated by 5 ms. Find R and C that give $S/N = 10$
3. Repeat #2 when the buffer has $R_i = 1\ \text{M}\Omega$
4. Buffer has $I_B = 2.5$ pA. Find values that limit its error to 1%

GI

- The amplifier pole corresponds to a time constant of 1.6 µs $\Rightarrow$ no effect on the signal. The output S/N is then

$$\left(\frac{S}{N}\right)_{GI} = \frac{AT_C}{\sqrt{\frac{S_V}{2}T_C}} = 1.77$$

- Note that at the input of the GI we have an rms noise $\sqrt{S_V f_n} = 15.85\ \mu\text{V}$ and $S/N \approx 0.32$. The improvement in S/N is given by the noise BW ratio:

$$\frac{1.77}{0.32} = \sqrt{\frac{f_n}{BW_n}} = \sqrt{\frac{\frac{\pi}{2}f_p}{\frac{1}{2T_C}}} \approx \sqrt{\frac{157}{5}}$$

BA

- The improvement in S/N goes with $\sqrt{N_{eq}}$:

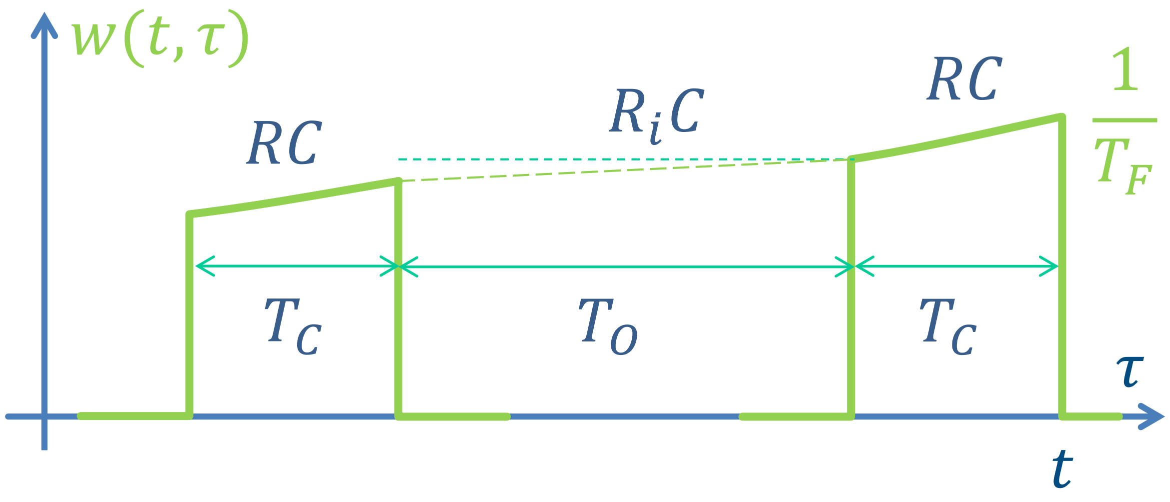
$$\left(\frac{S}{N}\right)_{BA} = \left(\frac{S}{N}\right)_{GI}\sqrt{N_{eq}} = 10 \Rightarrow N_{eq} \approx 32 = \frac{2T_F}{T_C} \Rightarrow T_F = 1.6\text{ ms}$$

- We can now require that the noise of R be lower than S_V

$$4k_BTR \ll S_V \Rightarrow R \ll 96\text{ k}\Omega$$

We can pick – say – $R = 10\text{ k}\Omega$ and $C = 160\text{ nF}$

Effect of R_i



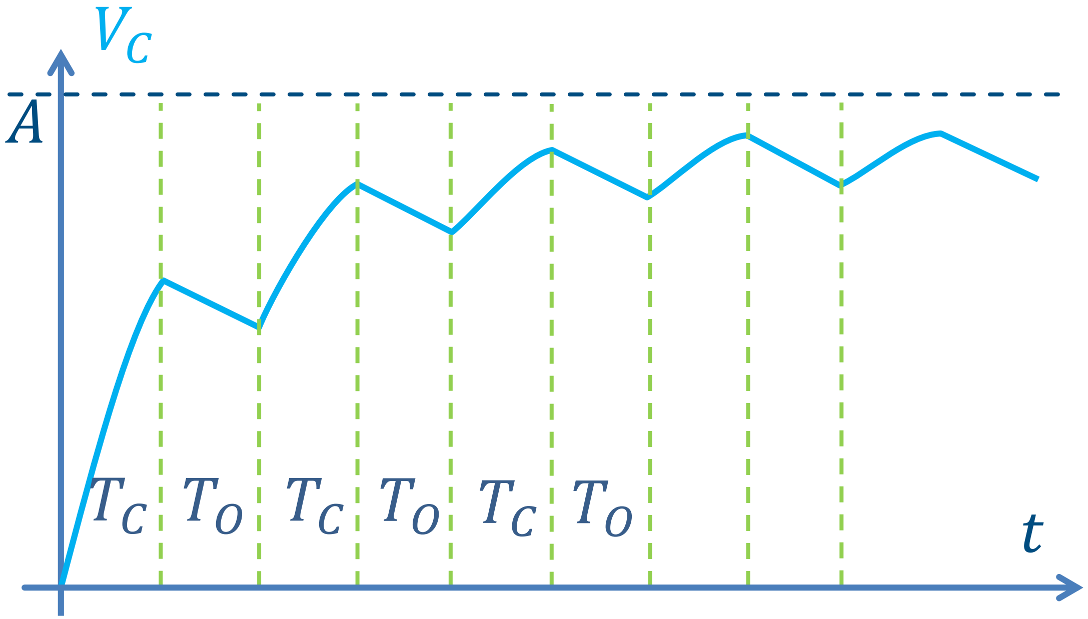
- We now have a discharge during the open-switch phase
- Time constants are different: we cannot use the ratemeter formula

- The decrease in $w(t,\tau)$ from one pulse to the next is

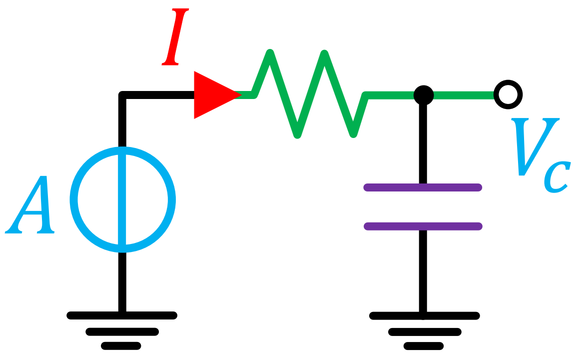
$$\alpha = e^{-\frac{T_C}{RC}} e^{-\frac{T_O}{R_iC}} = e^{-\frac{T_C + T_O(R/R_i)}{RC}} \Rightarrow N_{eq} = \frac{2RC}{T_C + T_O(R/R_i)} = 32$$

$$\Rightarrow RC = 2.4 \text{ ms}$$

Effect of I_B



- Final value will be different from A
- Equilibrium is reached when charging and discharging voltage variations balance each other



- Discharge current: I_B
- Charge current: $I = \dfrac{A - V_c}{R}$

Steady-state balance

- We balance the voltage change (not the currents!)

$$I = C\frac{dV}{dt} \Rightarrow \Delta V = \frac{I}{C}\Delta t$$

$$\frac{I_B}{C}T_O = \frac{A - V_c}{RC}T_C \leq \frac{0.01\ A}{RC}T_C \Rightarrow R \leq \frac{AT_C}{100\ I_B T_O} = 400\ \Omega$$

we can now pick $R = 400\ \Omega$ and $C = 6\ \mu\text{F}$

- The error introduced by I_B during T_C is negligible:

$$\frac{I_B}{C}T_C \approx 42\ \text{pV}$$