Electronics – 96032

POLITECNICO DI MILANO

Flicker noise and BLR/CDS

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Disclaimer

Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes

Outline

- Flicker noise
  - Review
  - Problems
- BLR and CDS
  - Problems
- Appendix

Flicker noise

- Output noise (unilateral PSD K/f)

$$\overline{n_y^2} = \int_0^{\infty} \frac{K}{f} df = \infty$$

- In reality

$$\overline{n_y^2} = \int_{f_L}^{f_H} \frac{K}{f} df = K \ln\left(\frac{f_H}{f_L}\right)$$

with f_H and f_L set by the min/max observation time

Filtered flicker noise

- Filter with WF $W(t,f)$:

$$\overline{n_o^2} = \int_0^\infty \frac{K}{f} |W(t,f)|^2 df$$

- We employ a rectangular approximation for $|W|^2 \approx |W(0)|^2 \text{rect}(f_H)$

$$\overline{n_o^2} = \int_{f_L}^\infty \frac{K}{f} |W(0)|^2 \text{rect}(f_H) df = K|W(0)|^2 \ln\left(\frac{f_H}{f_L}\right)$$

f_H: set by the filter (see Appendix for examples)

f_L: set by the working time

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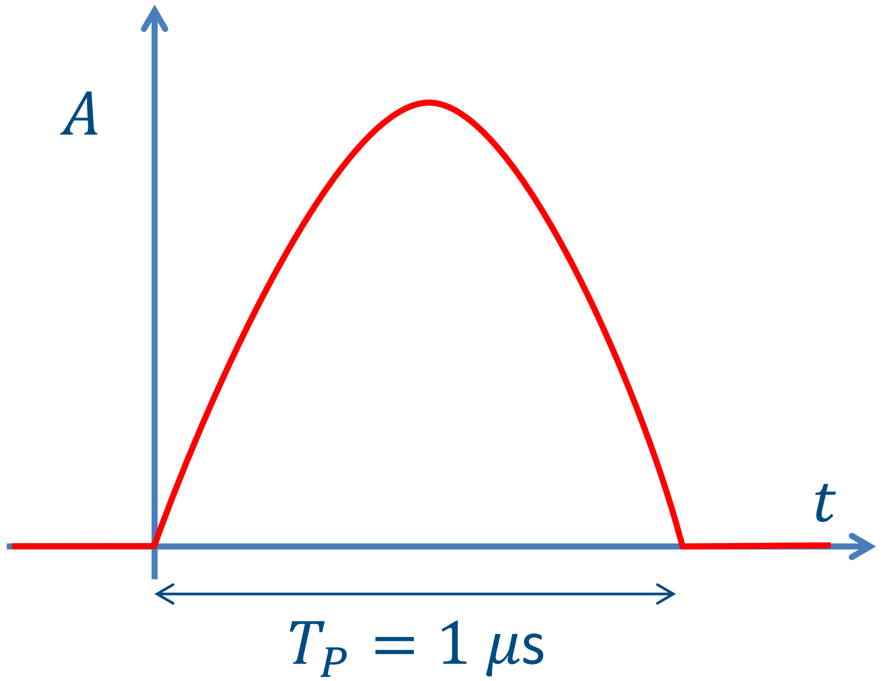
Problem: sinusoidal pulse + WN + FN



WN PSD: $\sqrt{\lambda} = 1\ \mu V/\sqrt{Hz}$ (bilat.)
FN: $f_{nc} = 10$ kHz

1. Evaluate the impact of the FN when the filter is running for 8 h
2. Compare the result with the WN one

FN noise contribution

- We can take as a reference the optimum integration window (for WN only)

$$T_G = 0.74\ \mu s \Rightarrow w(t,\tau) = \frac{G}{T_G}\text{rect}(T_G) \Leftrightarrow W(t,f) = G\ \text{sinc}(\pi f T_G)$$

- The (unilateral) FN magnitude is

$$\frac{K}{f_{nc}} = 2\lambda \Rightarrow K = 2\times 10^{-8}\ \text{V}^2$$

- Output FN noise contribution is

$$\overline{n_o^2} = \int_{f_L}^{\infty} \frac{K}{f} G^2 \text{sinc}^2(\pi f T_G) df \approx K G^2 \ln\left(\frac{f_H}{f_L}\right)$$

Comparison with WN

- For FN we can take (assume $G = 100$)

WN BW (see Appendix)

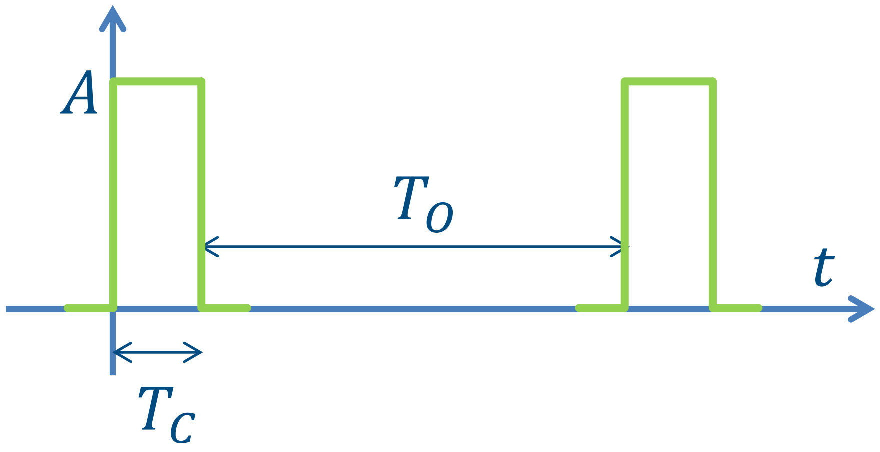
$$f_L = \frac{1}{8\text{h}} = 34.7\ \mu\text{Hz}, \quad f_H = \frac{1}{2T_G} = 676\ \text{kHz}$$

$$\overline{n_o^2} = KG^2 \ln\left(\frac{f_H}{f_L}\right) = 2 \times 10^{-4} \times 23.7 = 47.4 \times 10^{-4}\ \text{V}^2 = (68.8\ \text{mV})^2$$

- The WN contribution is

$$\overline{n_o^2} = \lambda \left(\frac{G}{T_G}\right)^2 T_G = 1.35 \times 10^{-2}\ \text{V}^2 = (116.2\ \text{mV})^2$$

Problem: BA with WN and FN



$A = 100\ \mu$V
$T_C = 10$ ns, $f_p = 100$ kHz
WN PSD: $\lambda = 10^{-15}$ V²/Hz (bilat.)

1. Find the parameters that give $S/N = 10$
2. Find the maximum FN corner frequency that does not degrade S/N if the filter works for 1 h
3. Size a CR filter that eliminates the FN

BA time constant

- We can use the LPF formula:

$$\left(\frac{S}{N}\right)_{BA} = \frac{A}{\sqrt{\frac{\lambda}{2T_F}}} = 10 \Rightarrow T_F = \frac{50\lambda}{A^2} = 5\ \mu s$$

- Or the single-pulse expression

$$\left(\frac{S}{N}\right)_{sp} = A\sqrt{\frac{T_C}{\lambda}} \approx 0.32 \Rightarrow 0.32\sqrt{N_{eq}} = 10 \Rightarrow N_{eq} = 1000 \Rightarrow$$

$$T_F = 5\ \mu s$$

FN noise contribution

- The open-switch time is $T_O = 10\ \mu s - T_C \approx 10\ \mu s$
- The envelope time constant is

$$T_{env} \approx T_F \frac{T_O}{T_C} = 5\ \text{ms} \Rightarrow f_{max} = \frac{1}{2\pi T_{env}} \approx 32\ \text{Hz}$$

- The (unilateral) noise contributions are

$$\frac{S_{WN}}{4T_F} \gg K \ln\left(\frac{f_{max}}{f_{min}}\right) \Rightarrow f_{nc} \ll \frac{1}{4T_F \ln\left(\frac{f_{max}}{f_{min}}\right)} = 4.3\ \text{kHz}$$

K: $S_{WN} f_{nc}$

f_{min}: $\frac{1}{3600} = 2 \times 10^{-4}\ \text{Hz}$

HPF time constant

- Solutions like $f_p = f_{nc}$ or $f_p = f_{env}$ do not work: the time constant is too large, leading to strong pile-up
- The requirements on the time constant T_{HP} are

$$T_C \ll T_{HP} \ll T_O$$

- In our case, a solution could be $T_{HP} = 1\ \mu s \Rightarrow f_p = 160$ kHz

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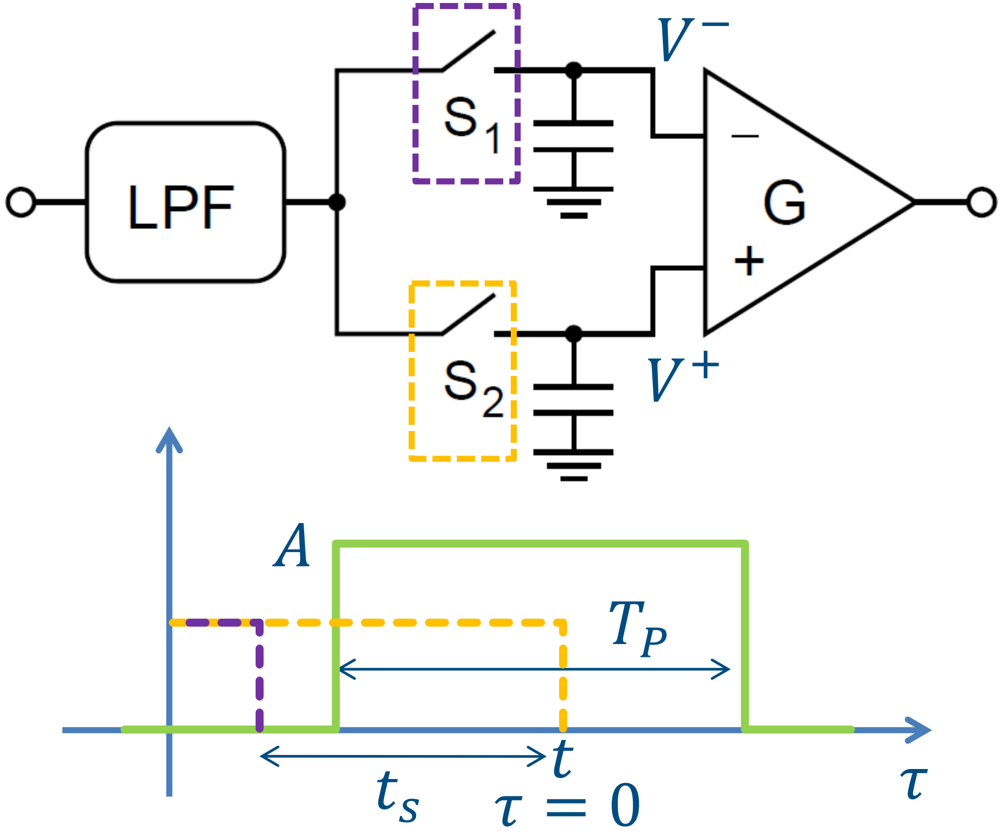
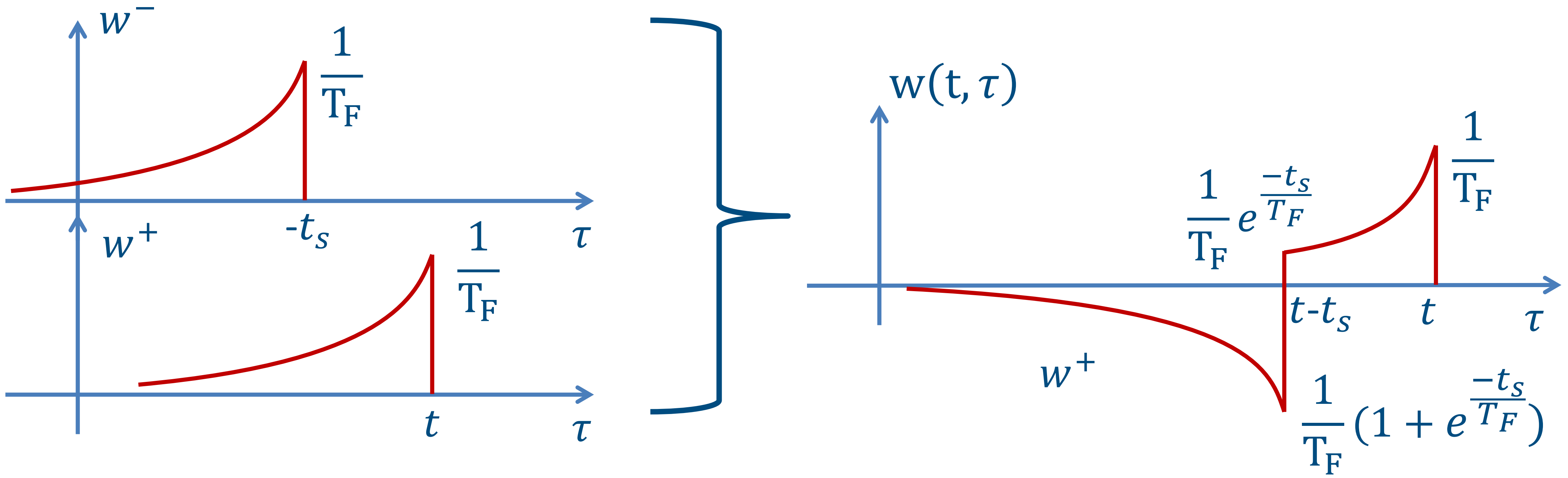
Problem: Baseline restorer



$T_P = 1\ \mu s$
$G = 1$
WN PSD: S_V (unilateral)

1. Draw and compute $w(t, \tau)$
2. Size T_F and t_s and compute $\overline{n_o^2}$
3. Repeat 2. with 1/f noise (approximate filter behavior around f=0)
4. Filter behavior on white noise of G

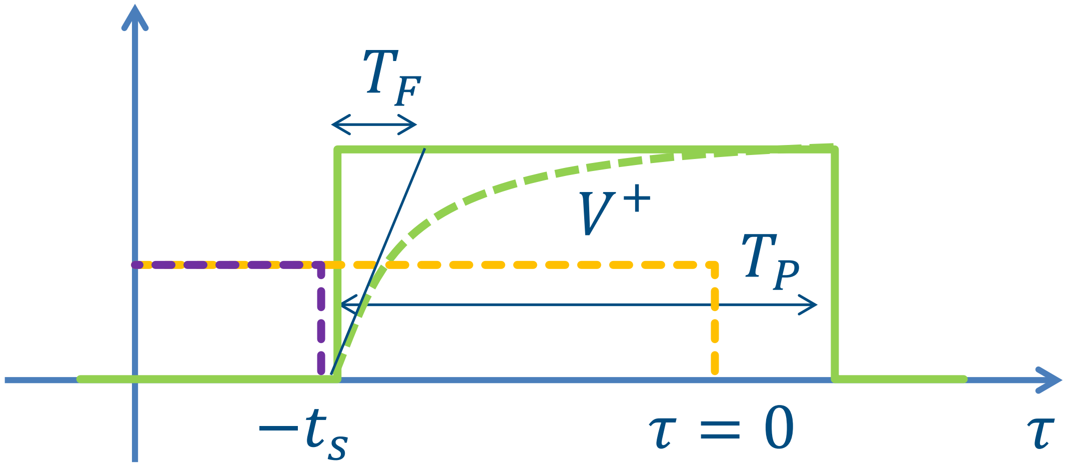
CDS + LPF weighting function



- Filter implements a correlated double sampling preceded by an LPF

$$w(t,\tau) = w^+(t,\tau) - w^-(t,\tau) = \frac{1}{T_F} e^{\frac{\tau - t}{T_F}} u(t-\tau) - \frac{1}{T_F} e^{\frac{\tau-(t-t_s)}{T_F}} u(t-t_s-\tau)$$

T_F and t_s sizing



- We must choose $T_F \ll T_P$, let's set $T_F = T_P/10$ = 100ns
- t_s is the delay of signal sampling (S2 opens) w.r.t. the baseline. V^- must be at regime so $t_s > 5T_F$ → $t_s = 7T_F$ = 700ns

Output noise (WN case)

- Output noise power is $\overline{n_o^2} = \lambda k_{w_{tt}}(0) = \frac{S_V}{2} \int \mathrm{w}(\mathrm{t}, \tau)^2 d\tau =$

$$= \frac{S_V}{2}\left(\int_{-\infty}^{-t_s} \frac{1}{T_F}\left(e^{\frac{\tau}{T_F}} - e^{\frac{\tau+t_s}{T_F}}\right)^2 d\tau + \int_{-t_s}^{0} \frac{1}{T_F}\left(e^{\frac{\tau}{T_F}}\right)^2 d\tau\right) = \frac{S_V}{2T_F}\left(1 - e^{\frac{-t_s}{T_F}}\right)$$

- The result makes sense because
  - $t_s \to 0 : \overline{n_o^2}$ goes to 0 because the noise samples are correlated
  - $t_s \to \infty : \overline{n_o^2} \to \frac{S_V}{2T_F} = 2 \cdot \frac{S_V}{4T_F}$, i.e. twice the noise of an LPF

Output noise (flicker noise case)

- Flicker noise forces us to work in the frequency domain:

$$|W(t,\omega)|^2 = \left|\frac{1}{1-j\omega T_F}\left(1-e^{-j\omega t_s}\right)\right|^2 = \underbrace{\frac{1}{1+\omega^2 T_F^2}}_{\text{LPF part}} \cdot \underbrace{2(1-\cos(\omega t_s))}_{\text{correlated double sampling}}$$

- The noise power of the flicker noise component is then:

$$\overline{n_o^2} = \int_0^{+\infty} \frac{k}{f} \overbrace{\frac{1}{1+(2\pi f T_F)^2}}^{\simeq 1} \cdot \overbrace{2(1-\cos(2\pi f t_s))}^{\simeq 1-(2\pi f t_s)^2/2} df \simeq \int_0^{f_{LP}} \frac{k}{f}(2\pi f t_s)^2 df$$

→ $\overline{n_o^2} = \mathrm{k}(2\pi t_s)^2 \cdot \frac{f_{LP}^2}{2}$ (15% error vs. exact calculation)

$$f_{LP} = \frac{1}{2\pi T_F}$$

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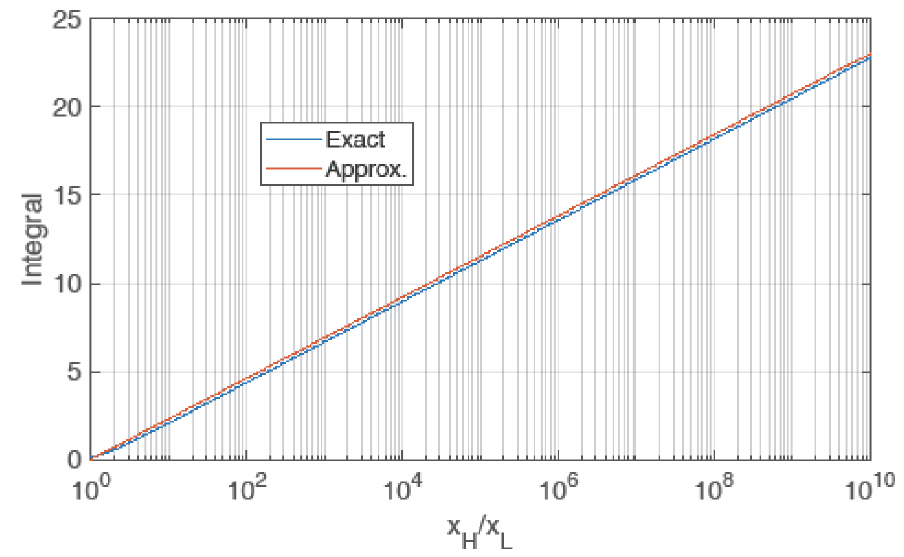
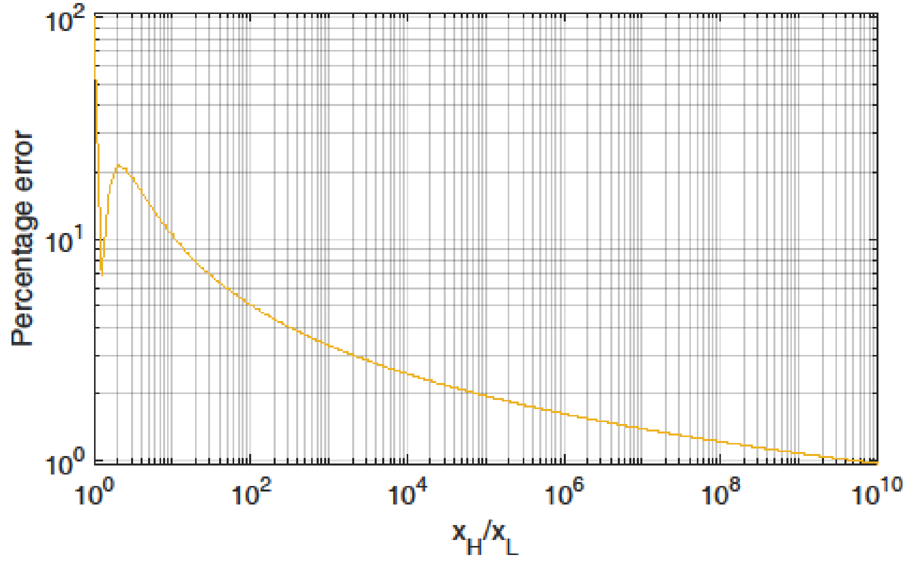
FN BW for a GI





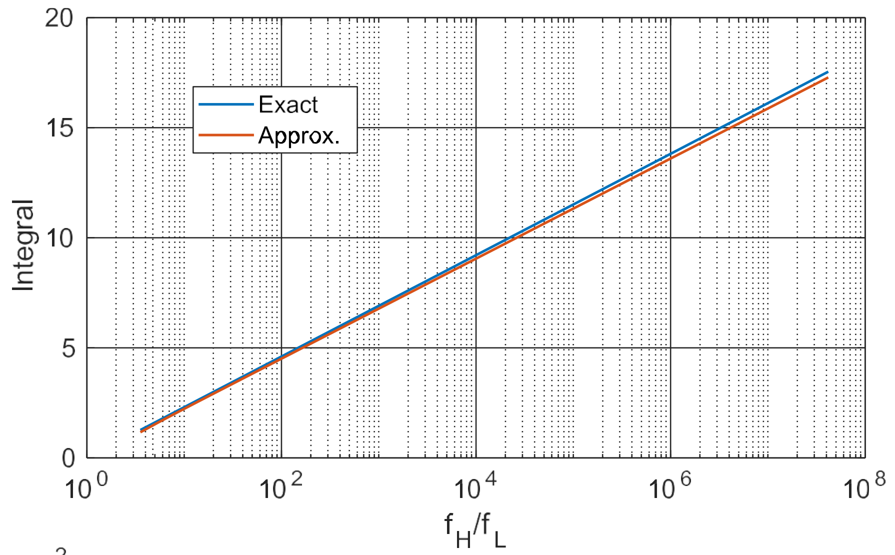
$$\overline{n_o^2} = \int_{f_L}^{\infty} \frac{K}{f} G^2 \mathrm{sinc}^2(\pi f T_G) df$$

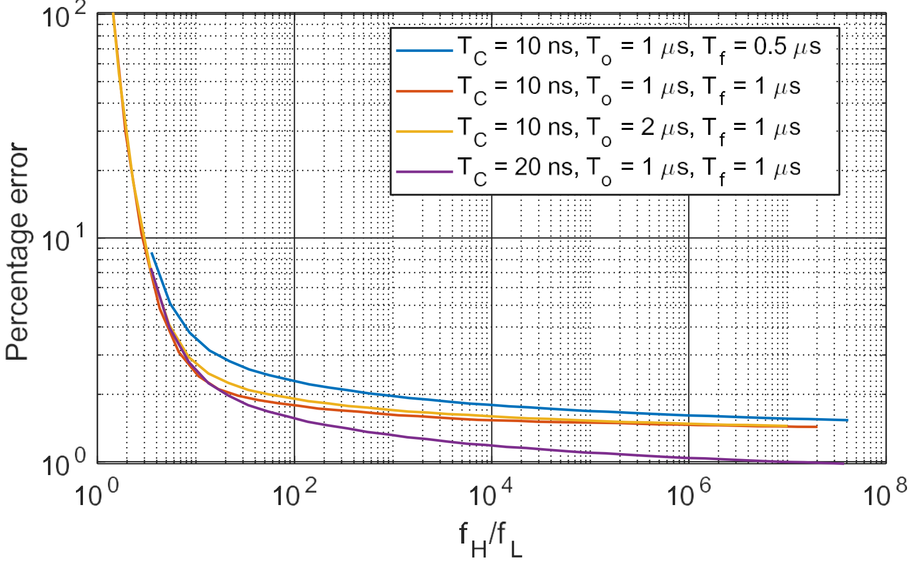
$$= K G^2 \int_{x_L}^{\infty} \frac{\sin^2 x}{x^3} dx \approx K G^2 \ln\left(\frac{x_H}{x_L}\right)$$

For the WN BW, $f_H = \frac{1}{2T_G} \Rightarrow x_H = \frac{\pi}{2}$

The approx. works fine, with relative error smaller than 10% when $f_H/f_L > 10$

FN BW for a BA





$$\overline{n_o^2} = \int_{f_L}^{\infty} \frac{K}{f} |W(t,f)|^2 df \approx K \ln\left(\frac{f_H}{f_L}\right)$$

We take the envelope time constant

$$T_{env} = T_F \frac{T_o + T_c}{T_c} \Rightarrow f_H = \frac{1}{2\pi T_{env}}$$

The approx. works fine, with relative error smaller than 10% when $\frac{f_H}{f_L} > 3 - 4$