Electronics – 96032

POLITECNICO DI MILANO

Amplifiers and Feedback Theory

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Disclaimer

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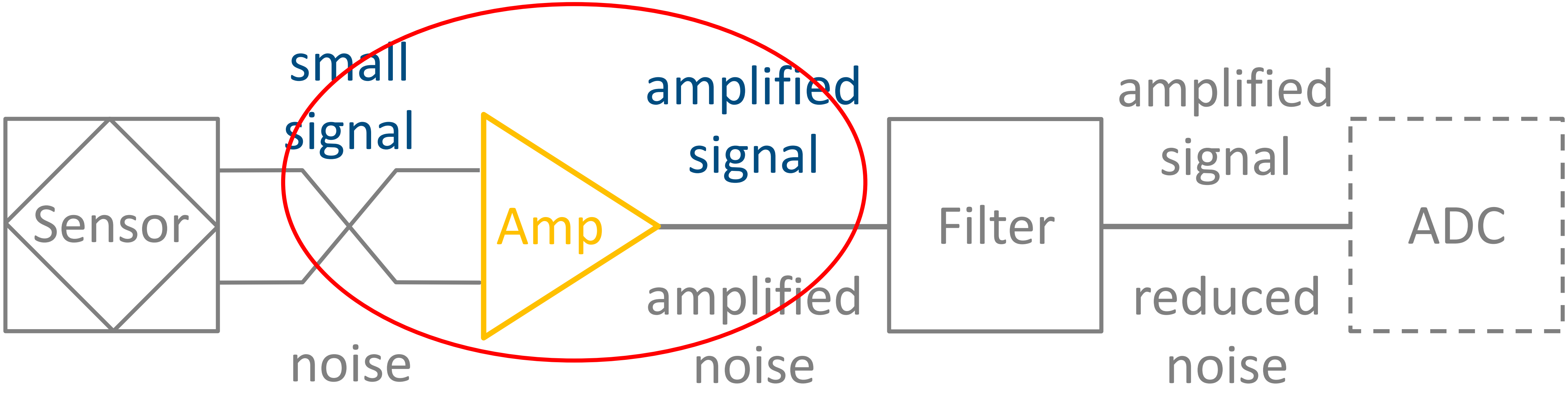
Acquisition chain



next lessons

Purpose of the lesson

- We begin our study with the analysis and design of simple amplifiers
- Next lessons will deal with
  - Basic amplifier principles and the feedback amplifier concept (this lesson)
  - Linear applications of OpAmps
  - Feedback amplifier properties
  - Stability of feedback amplifiers
  - Instrumentation amplifiers and OpAmp parameters

Outline

- Review: equivalent circuits
- Amplifiers
- Negative feedback
- Operational amplifiers

The origin

Voltage source equivalent circuit

Current source equivalent circuit

Hermann von Helmholtz (1821-1894)

1853

Lèon Charles Thévenin (1857-1926)

1883

Hans Ferdinand Mayer (1895-1980)

1926

Edward Lawry Norton (1898-1983)

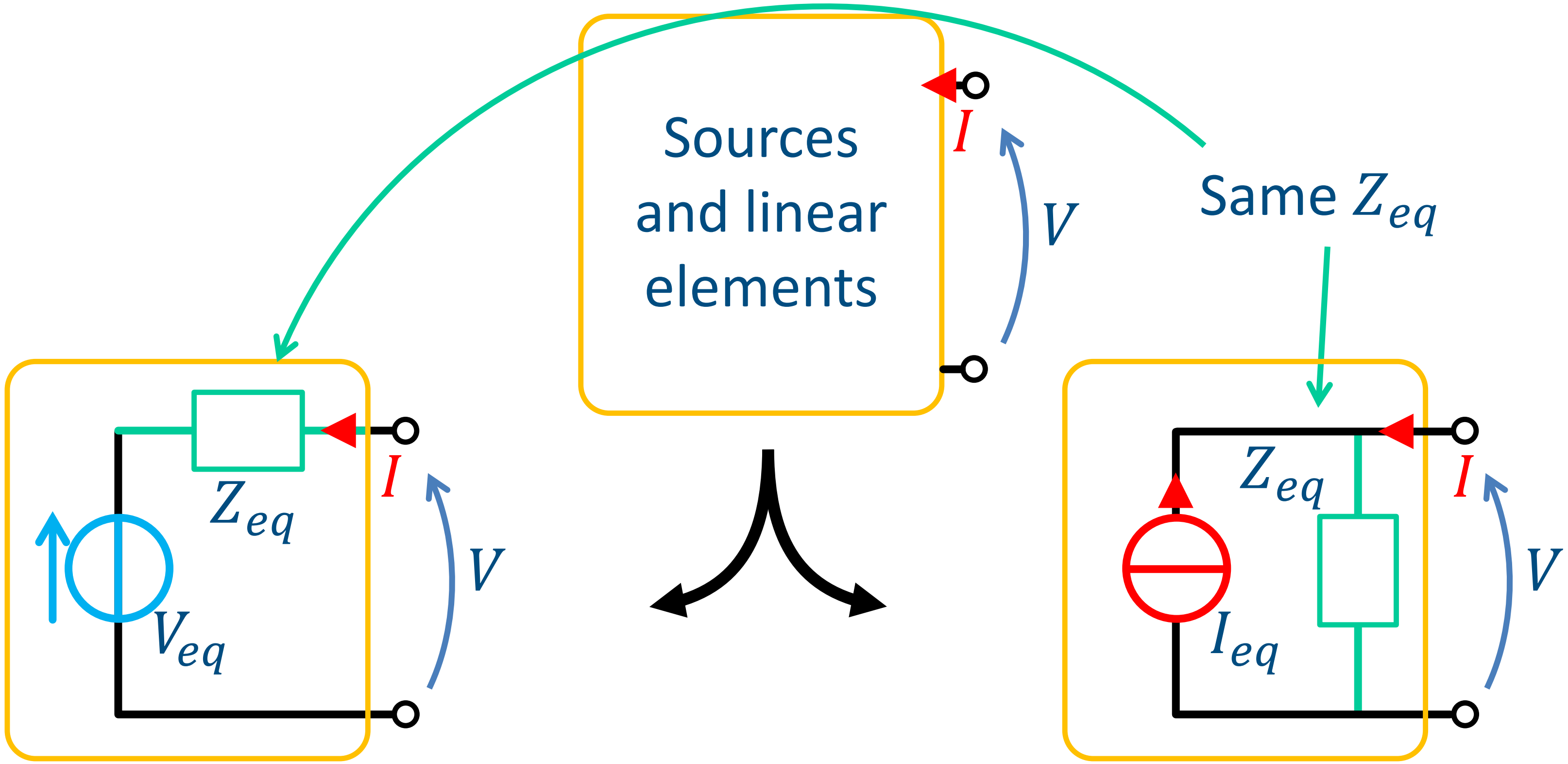
1926

From [1]

Equivalent circuits

- Linear network: R, L, C with parameters not dependent on I or V and V/I sources either constant or linearly dependent on other voltages or currents
- Every linear network «seen» between any pair of terminals behaves as if composed by a source and an impedance only
  - Thévenin equivalent circuit: voltage source with impedance in series
  - Norton equivalent circuit: current source with impedance in parallel

Equivalent circuits



Equivalence is only from the viewpoint of the external load.

Power dissipation, for example, is not equal

Element calculations

- V_{eq} is the open-circuit voltage at the terminals
- I_{eq} is the short-circuit current through the terminals
- $Z_{eq} = V_{eq}/I_{eq}$, or equivalently
- Z_{eq} is the impedance between the terminals when
  - Independent voltage sources are replaced by short-circuits
  - Independent current sources are replaced by open circuits

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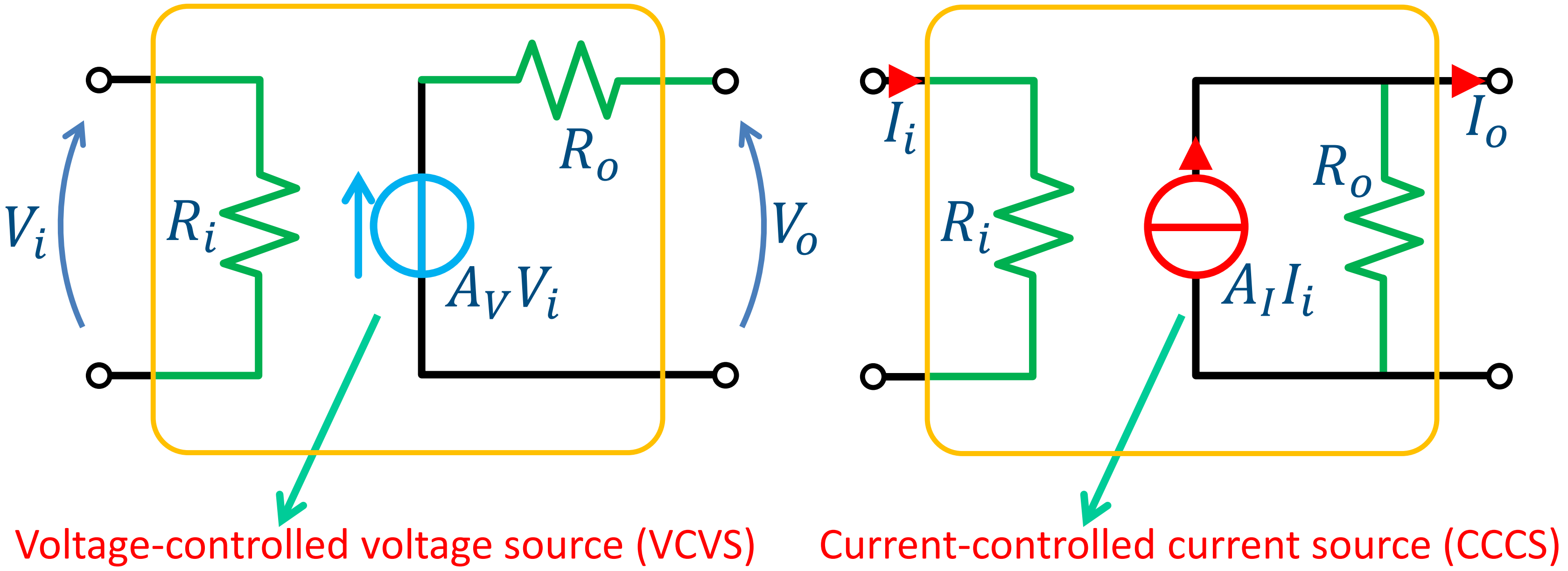
Amplifiers

- We consider a «black box» approach with equivalent circuits
- Four kinds can be identified:

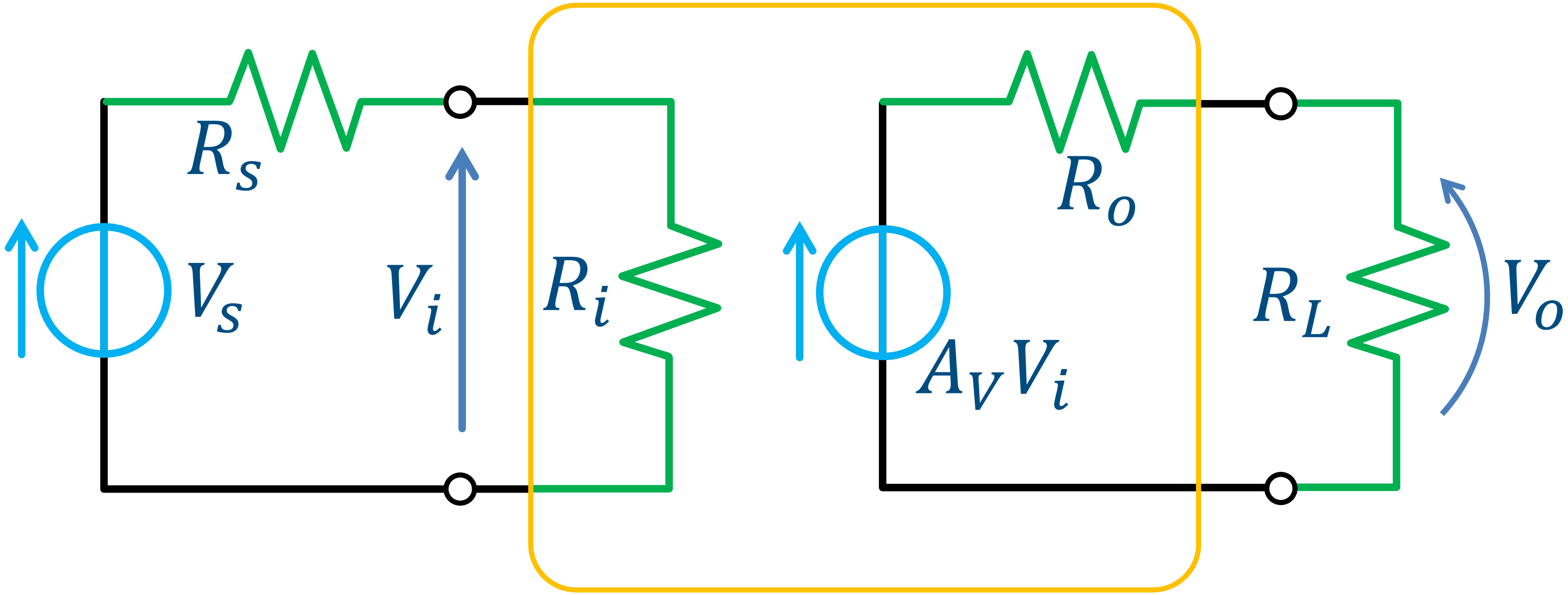
In	Out	Type
V	V	Voltage ampl.
I	I	Current ampl.
V	I	Transconductance ampl.
I	V	Transresistance ampl.

Voltage/current amplifiers



- One-directional amplifiers (no reverse transfer from output to input)
- Resistors will be considered for simplicity, though complex impedances can be assumed

Source and load resistors (VA)



$$V_i = V_S \frac{R_i}{R_i + R_S}$$

$$V_o = A_V V_i \frac{R_L}{R_o + R_L}$$

Voltage gain

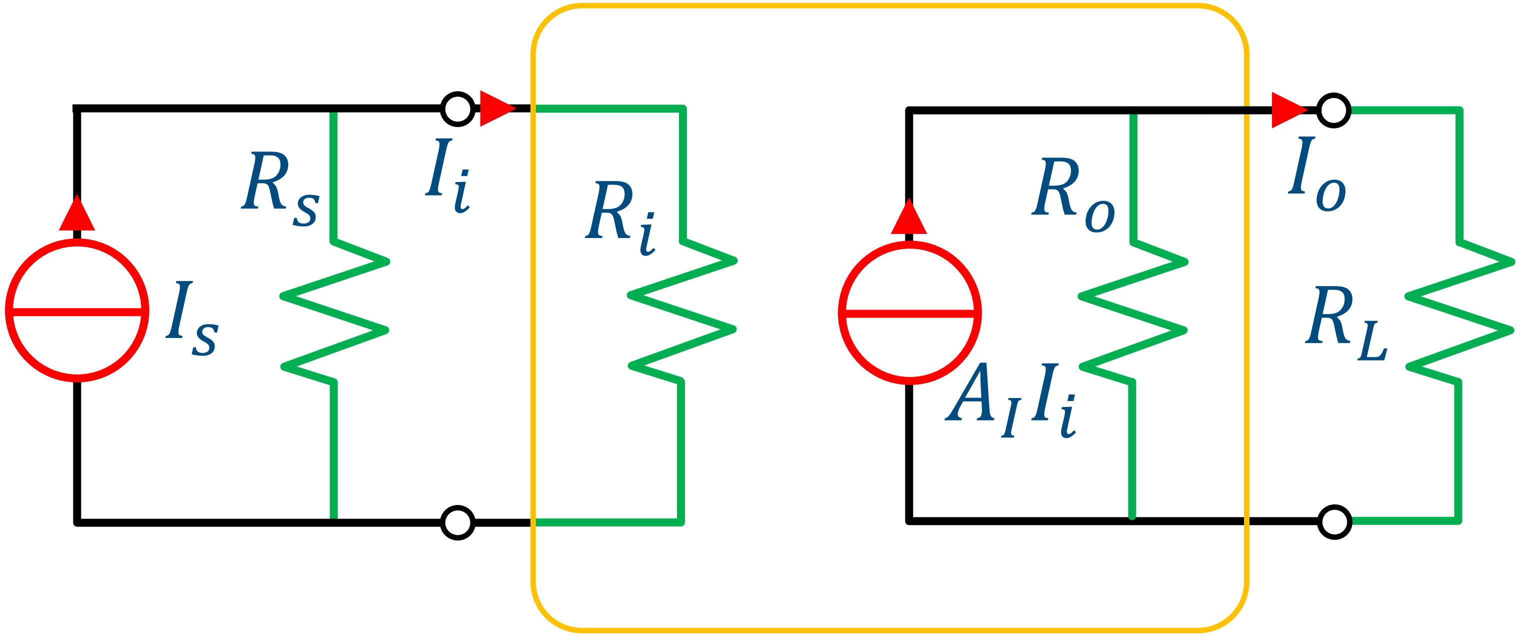
$$\frac{V_o}{V_S} = A_V \frac{R_L}{R_o + R_L} \frac{R_i}{R_i + R_S}$$

- Total gain is less than A_V
- Gain is dependent on R_S and R_L
- To avoid these drawbacks, a voltage amplifier should have:

  $R_i = \infty$ (very high input impedance)

  $R_o = 0$ (very low output impedance)

Source and load resistors (CA)



$$I_i = I_S \frac{R_S}{R_i + R_S}$$

$$I_o = A_I I_i \frac{R_o}{R_o + R_L}$$

Current gain

$$\frac{I_o}{I_S} = A_I \frac{R_o}{R_o + R_L} \frac{R_S}{R_i + R_S}$$

- Total gain is less than A_I
- Gain is dependent on R_S and R_L
- To avoid these drawbacks a current amplifier should have:

  $R_i = 0$ (very low input impedance)

  $R_o = \infty$ (very high output impedance)

Summary

Type	R_i	R_o
Voltage amplifier	∞	0
Current amplifier	0	∞
Transconductance ampl.	∞	∞
Transresistance ampl.	0	0

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American telephone lines…

- First transcontinental telephone line built in 1914 (announced 1915), upgraded in 1921 to three channels and using twelve amplifiers
- Second line built in 1923 with four channels and twenty amplifiers
- A further increase in the number of channels was very, very challenging…

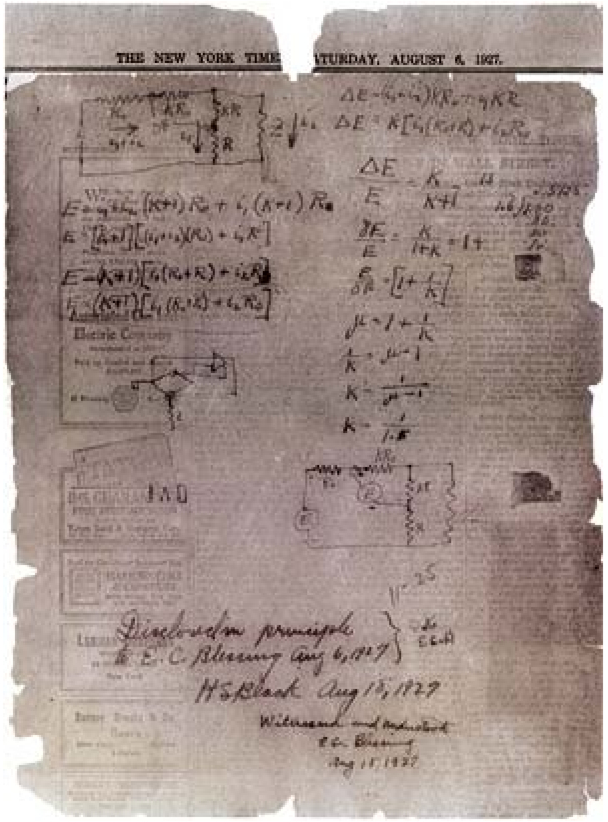
The amplifier problem

- Signal is attenuated as it propagates along the wires and must be regenerated
- Vacuum-tube amplifiers were costly $\Rightarrow$ minimum number of amplifiers with high gain
- Gain changes with plate voltage, temperature, humidity, aging,...
- Non-linearity creates intermodulation distorsion in multi-channel systems

$$\sin \omega t \longrightarrow \boxed{x^2} \longrightarrow \sin^2 \omega t \approx \sin 2\omega t$$

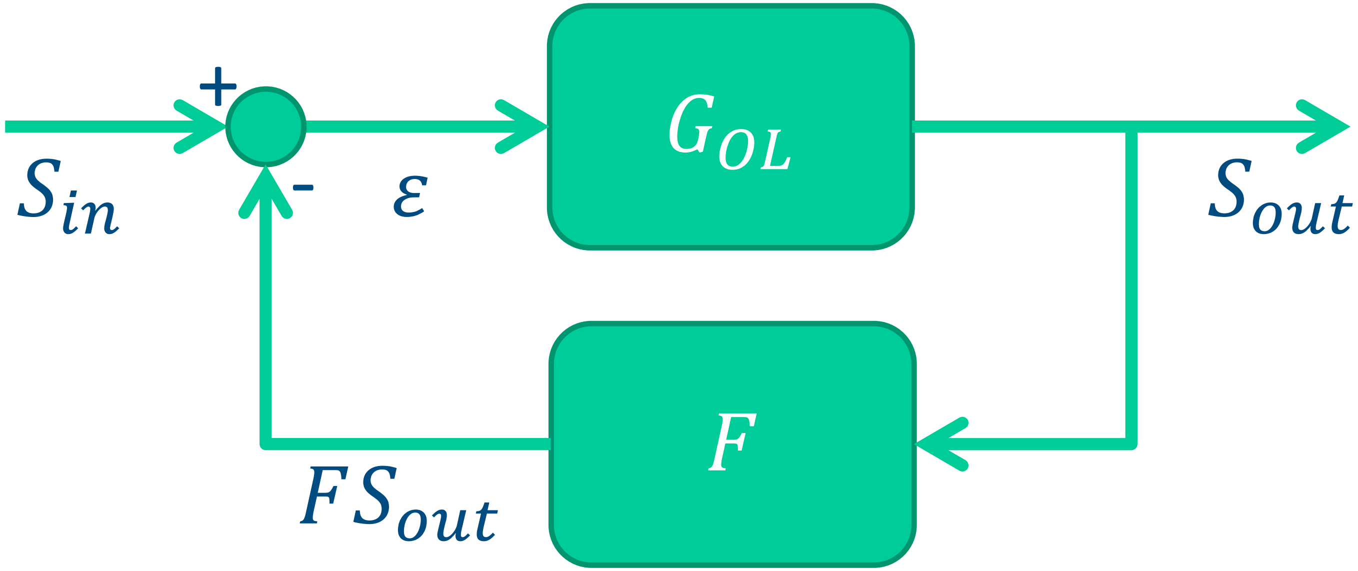
Negative-feedback concept

Harold S. Black (1898-1983)



From [2]

The theory



$$\varepsilon = S_{in} - FS_{out}$$
$$S_{out} = G_{OL}\,\varepsilon$$

$$\frac{S_{out}}{S_{in}} = G = \frac{G_{OL}}{1 + G_{OL}F}$$

Closed-loop gain

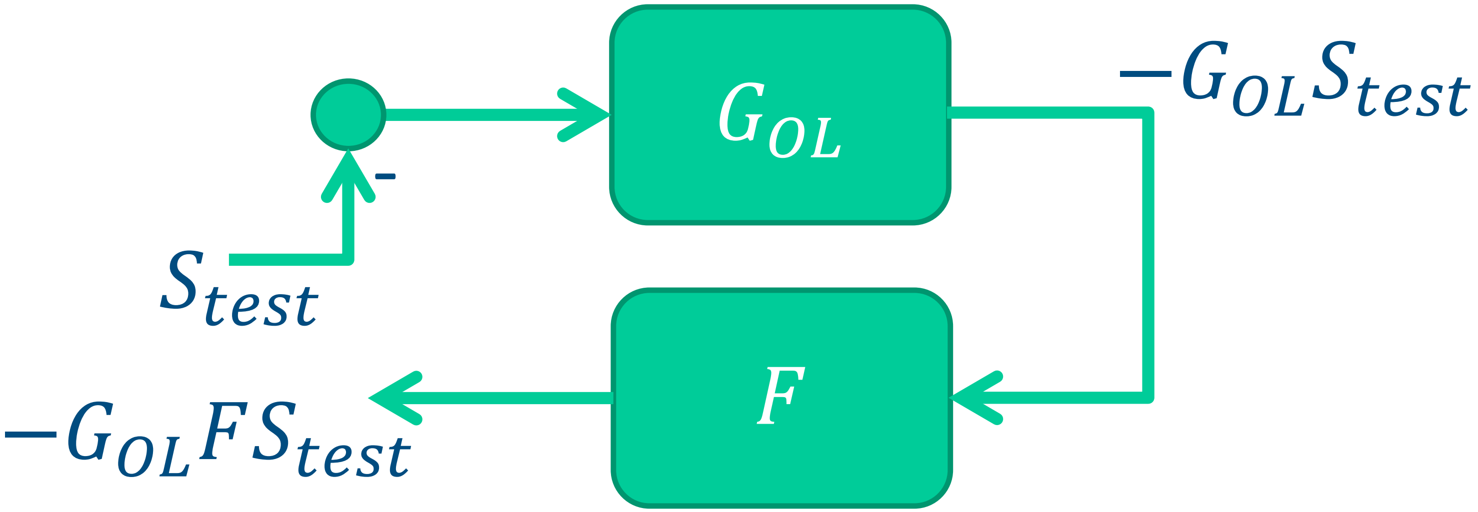
$$|G_{OL}F| \ll 1 \;\rightarrow G = \frac{G_{OL}}{1 + G_{OL}F} \sim G_{OL}$$

Open-loop gain, no feedback

$$|G_{OL}F| \gg 1 \;\rightarrow G = \frac{G_{OL}}{1 + G_{OL}F} \sim \frac{1}{F} = G_{id}$$

Ideal gain, independent of G_{OL}

Loop gain – calculation



- $G_{loop} = -G_{OL}F$ measures the strength of the feedback
- The result is independent of the breaking point
- A good feedback system has $G_{loop} < 0$ and $|G_{loop}| \gg 1$

Loop gain – interpretation

$$G = \frac{G_{OL}}{1 + G_{OL}F} = \frac{1/F}{1 + 1/G_{OL}F} = \frac{G_{id}}{1 - 1/G_{loop}}$$

- Example:

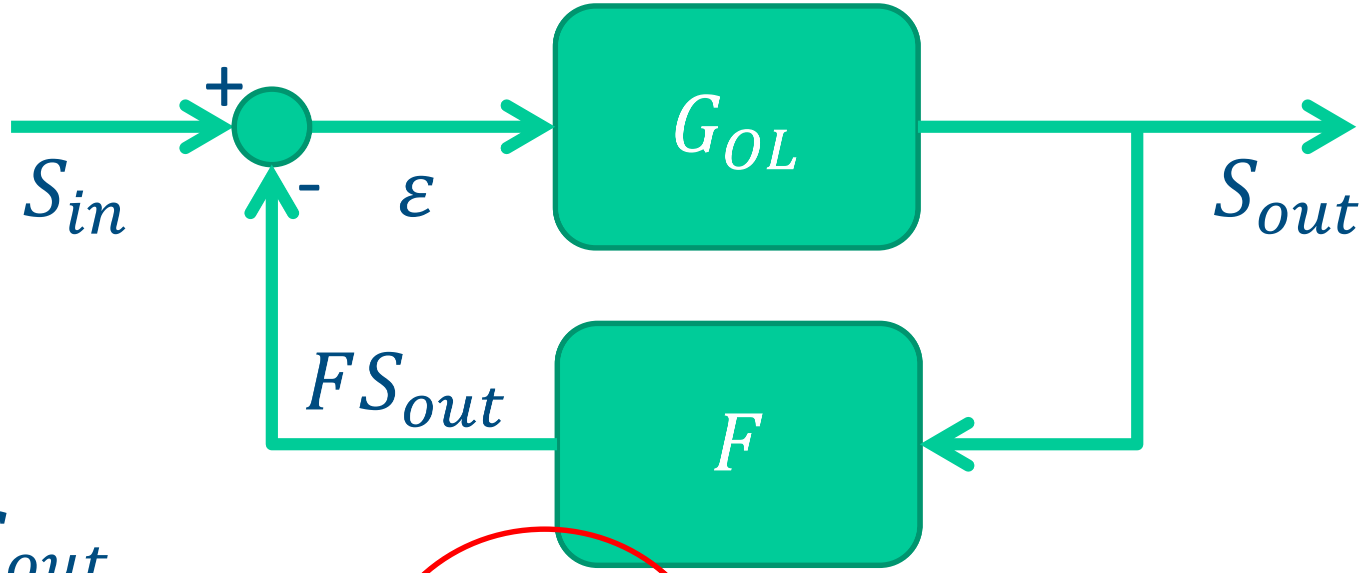
$$G_{OL} = 10^5, F = 10^{-2}$$

$$\Rightarrow G_{loop} = -1000,\ \ G_{id} = 100,\ \ \ G = 99.9$$

- The relative error between G and G_{id} is

$$\varepsilon_{rel} = \frac{G_{id} - G}{G_{id}} = \frac{100 - 99.9}{100} = 0.001 = \frac{1}{|G_{loop}|}$$

Error signal



$$\varepsilon = S_{in} - FS_{out}$$

$$\frac{\varepsilon}{S_{in}} = 1 - FG = \frac{G_{id} - G}{G_{id}}$$

$$\frac{\varepsilon}{S_{in}} = 1 - FG = 1 - \frac{1}{1 - \frac{1}{G_{loop}}} = \frac{1}{1 - G_{loop}} \sim \frac{1}{|G_{loop}|}$$

Sensitivity to G_{OL}

$$\frac{dG}{dG_{OL}} = \frac{1}{(1+G_{OL}F)^2} = \frac{G}{G_{OL}}\frac{1}{1-G_{loop}}$$

$$\frac{dG}{G} = \frac{dG_{OL}}{G_{OL}}\frac{1}{1-G_{loop}} \ll 1$$

$$G_{OL} = 10^5, F = 0.01 \quad \Rightarrow \quad G = 99.9$$

$$G_{OL} = 2\times 10^5, F = 0.01 \quad \Rightarrow \quad G = 99.95$$

Sensitivity to F

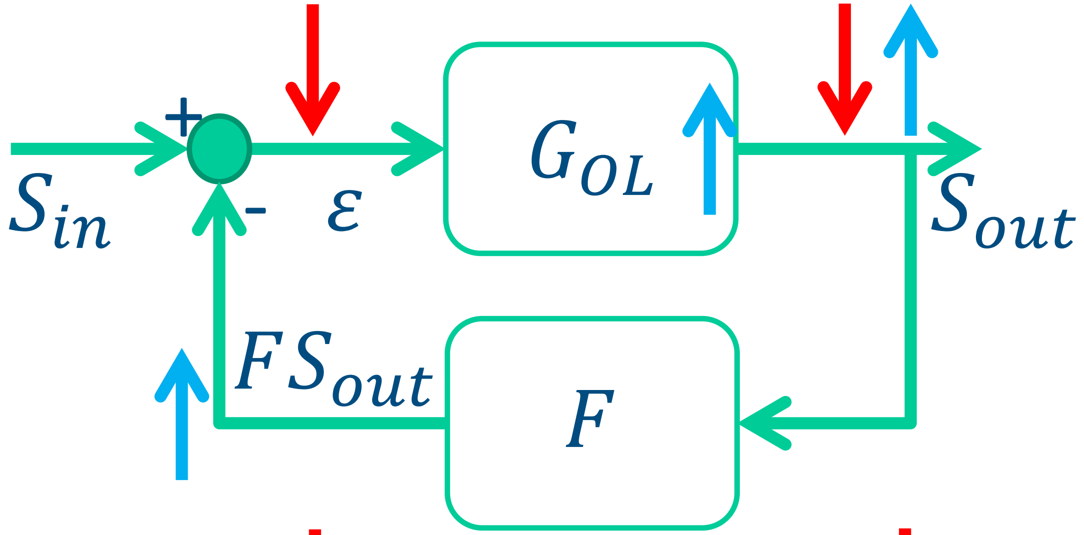
$$\frac{dG}{dF} = -\frac{{G_{OL}}^2}{(1 + G_{OL}F)^2} = -G^2$$

$$\frac{dG}{G} = \frac{dF}{F}\frac{G_{loop}}{1 - G_{loop}} \approx -1$$

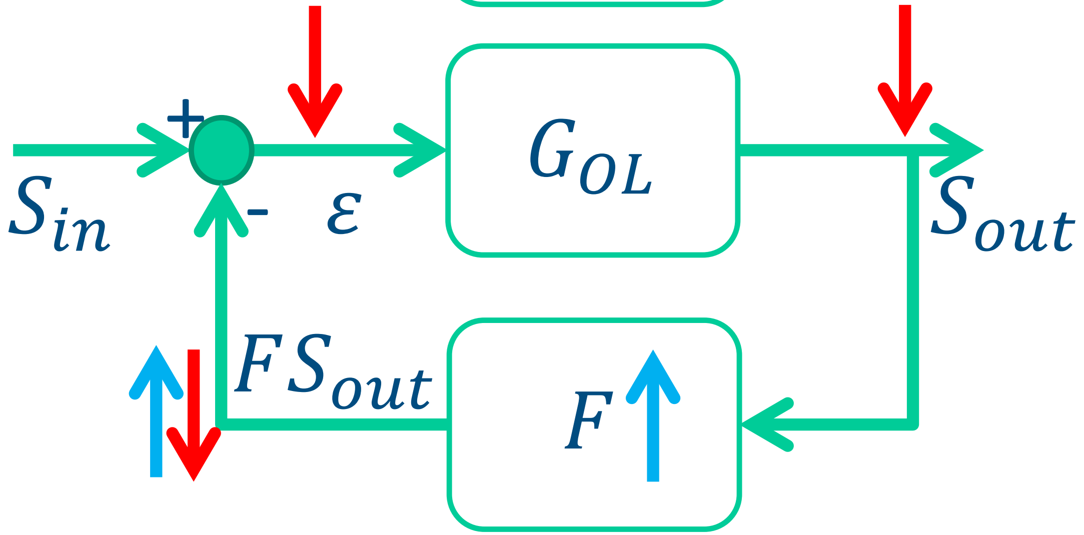
$$G_{OL} = 10^5, F = 0.01 \quad \Rightarrow \quad G = 99.9$$

$$G_{OL} = 10^5, F = 2 \times 0.01 \quad \Rightarrow \quad G = 49.98$$

Qualitative interpretation



- Changes in G_{OL} are nulled by the feedback loop



- Changes in F cannot be compensated

Feedback amplifier design

- Forward (open-loop) block G_{OL} must have high gain, to ensure that $|G_{loop}| \gg 1$.
  All active elements are placed here even if gain is not stable – their fluctuations are reduced by $1/|G_{loop}|$
- Feedback block F must be stable, to ensure a stable closed-loop gain $\Rightarrow$ usually made with passives

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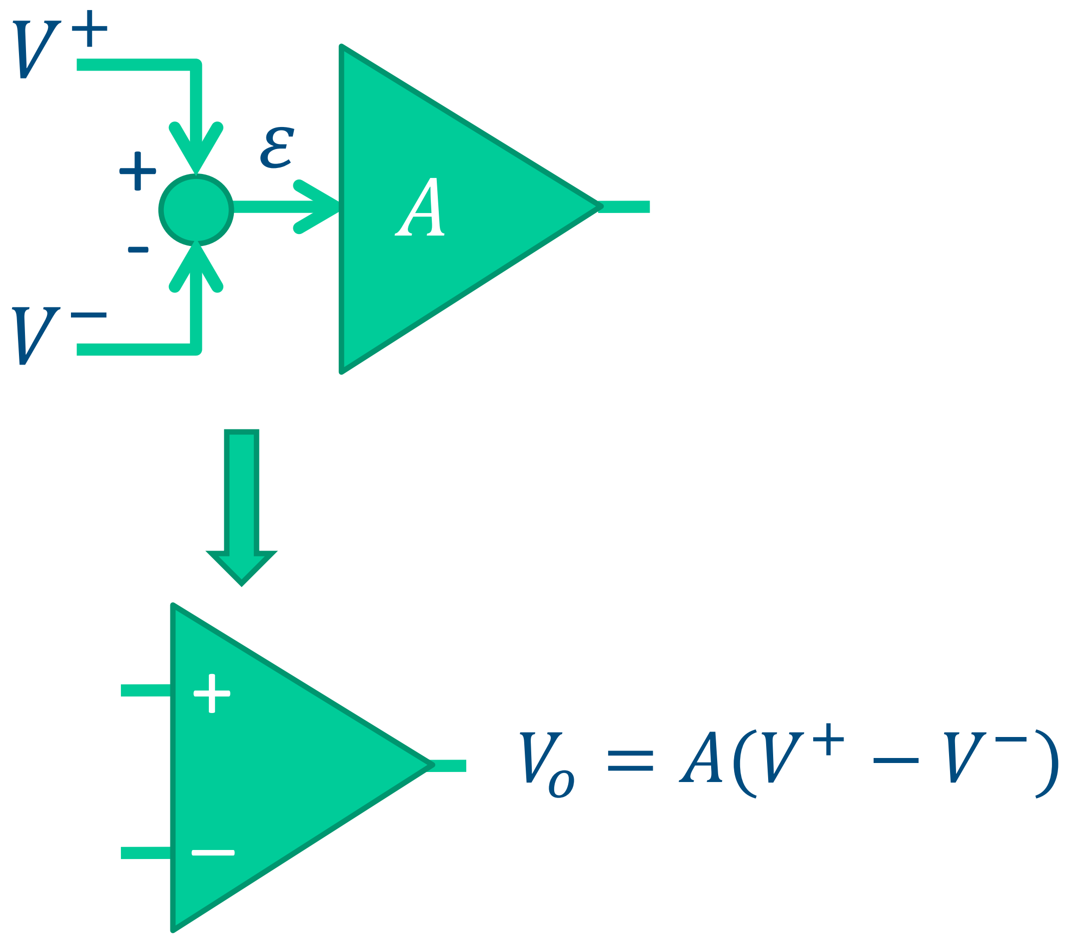
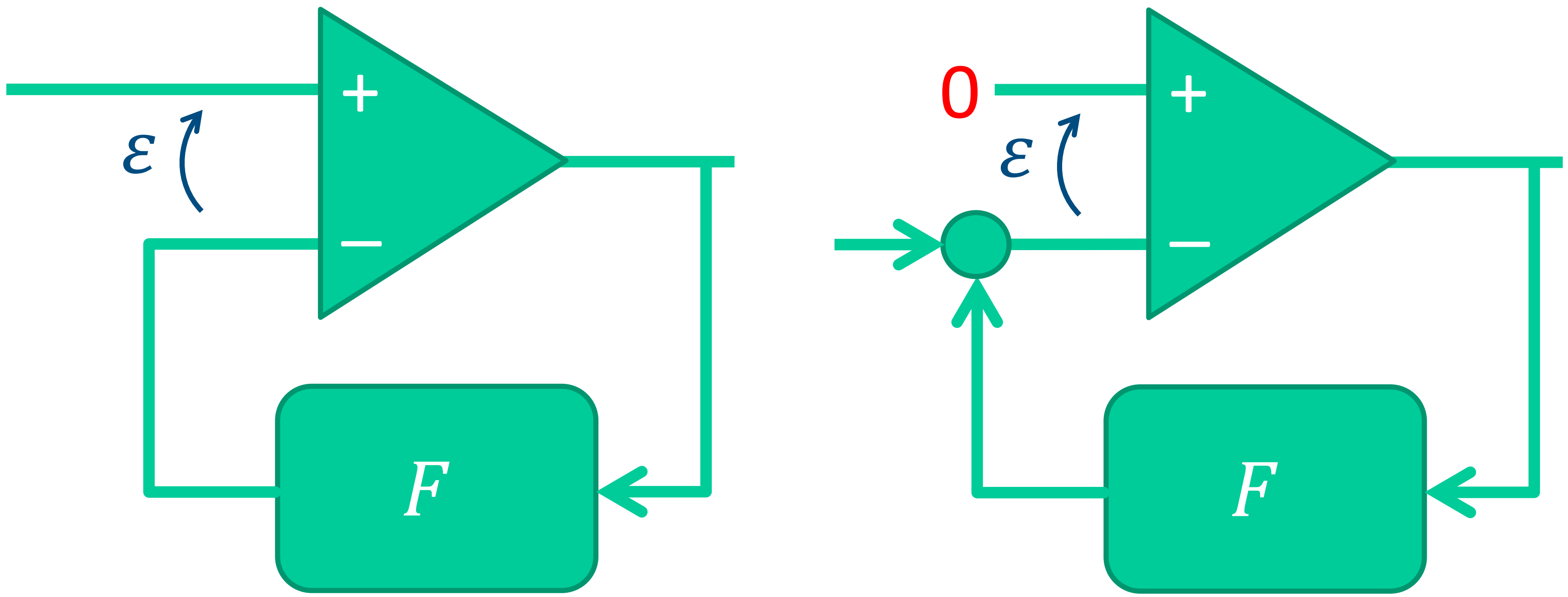
Operational amplifiers (OAs)

- Integrated voltage amplifiers used as forward gain blocks in feedback circuits

- The ideal OA has
  - $A = \infty \ (10^5 - 10^6)$
  - $R_i = \infty \ (10^6 - 10^9 \Omega)$
  - $R_o = 0 \ (\approx 100 \ \Omega)$



Typical circuit arrangements



Feedback loop is on the negative pin

In ideal feedback loops, $\varepsilon = 0 \Rightarrow$ ideal OAs keep $V^+ - V^- = 0$

$\Rightarrow V^+ = V^-$

References

1. http://tcts.fpms.ac.be/cours/1005-01/equiv.pdf
2. https://www.wpi.edu/News/Transformations/2005Summer/timecapsule.html