Electronics – 96032

POLITECNICO DI MILANO





Signals and Noise in LTI Systems and OpAmps

Alessandro Spinelli

Phone: (02 2399) 4001

alessandro.spinelli@polimi.it spinelli.faculty.polimi.it

Disclaimer

Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes

Purpose of the lesson

- Up to now we have seen Amplifiers and Sensors and we know how to deal with the problems from the viewpoint of the signal
- In reality, noise is always present and can affect the precision or even overshadow the signal
- In the final part of the class, we will discuss techniques for noise reduction. Before doing this, however, we need to learn:
  - how to mathematically describe noise (lesson before the previous);
  - what are the origins of noise (previous lesson);
  - how circuits respond to noise (this lesson);

Outline

- LTI systems response to signal and noise
- Noise in OAs
- Feedback and noise

LTI systems

- Linearity
  - If input $x_i(t)$ produces output $y_i(t)$, then

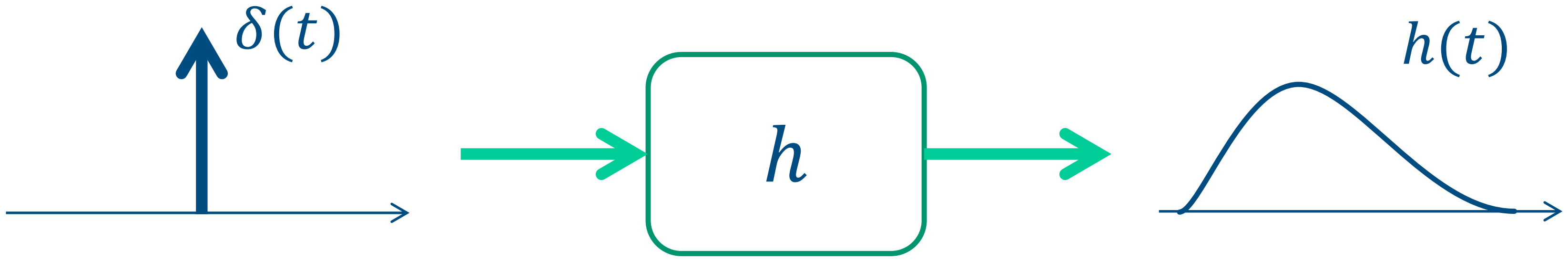
$$\sum_k c_k x_k(t) \Rightarrow \sum_k c_k y_k(t)$$

$$\int c(\tau)x(t,\tau)d\tau \Rightarrow \int c(\tau)y(t,\tau)d\tau$$

- Time invariance

$$x(t+T) \Rightarrow y(t+T)$$

Impulse response



LTI systems are wholly characterized by their impulse response, $h(t)$

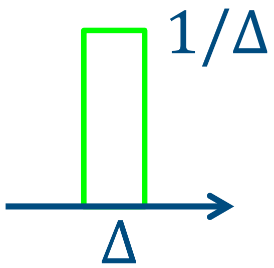
The “sifting” principle

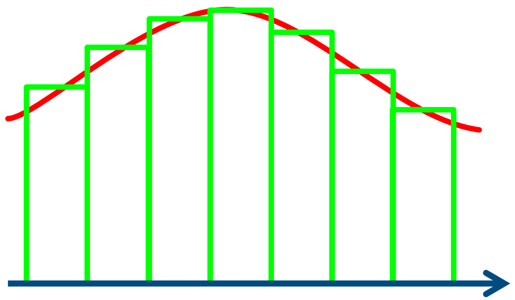
- We consider an approximated δ function (with unit area), δ_Δ
- We approximate a generic signal $x(t)$ as

$$x_\Delta(t) = \sum_k x(k\Delta)\,\delta_\Delta(t - k\Delta)\Delta$$

- For $\Delta \to 0$ we have

$$\lim_{\Delta \to 0} x_\Delta(t) = \int x(\tau)\delta(t-\tau)d\tau = x(t)$$





System response

- The LTI output to x_Δ is the superposition of the responses to $\delta_\Delta(t-k\Delta)$, $h_\Delta(t-k\Delta)$

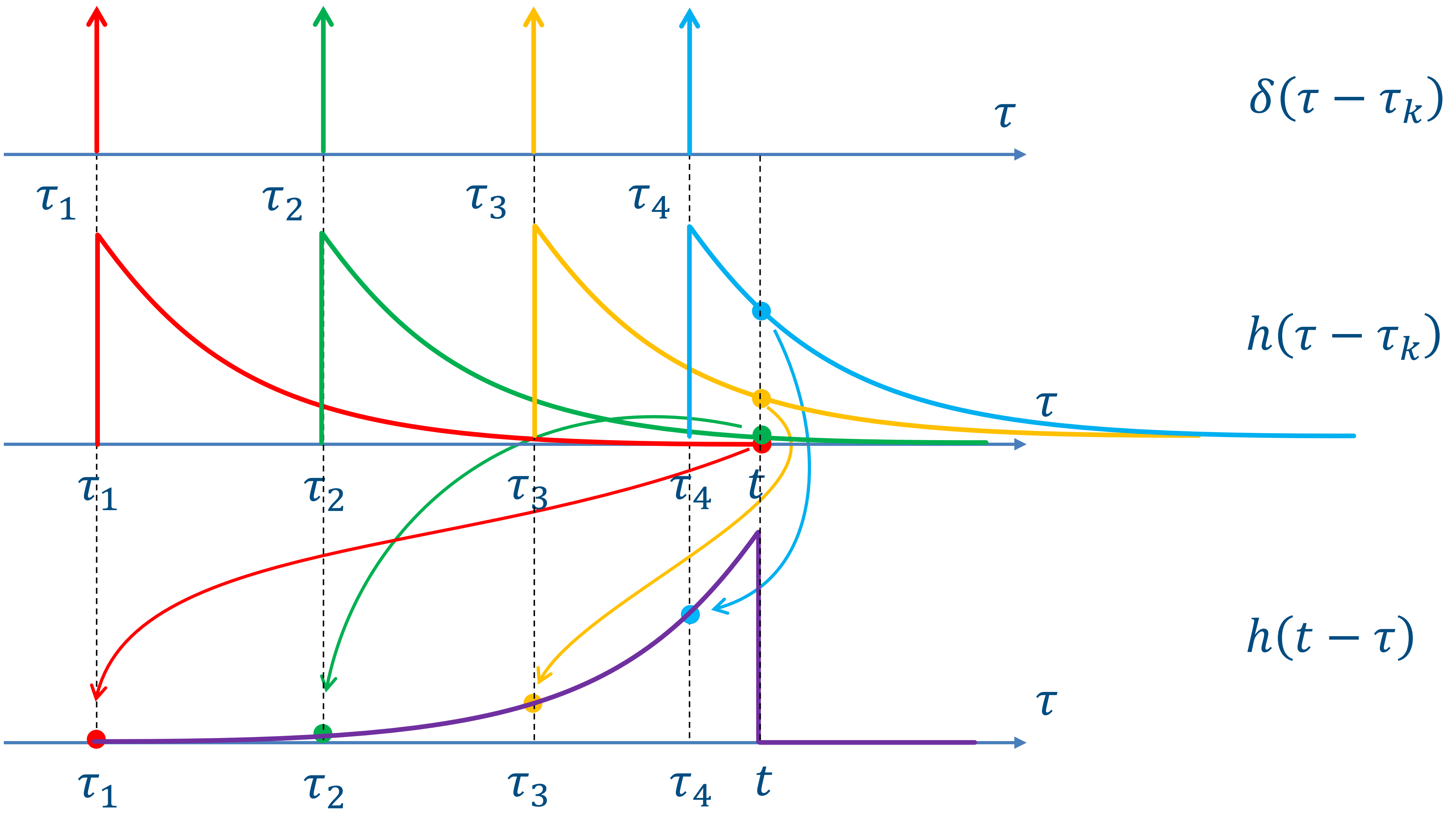
$$y_\Delta(t) = \sum_k x(k\Delta)\, h_\Delta(t-k\Delta)\Delta$$

- For $\Delta \to 0$ we have

$$y(t) = \lim_{\Delta\to 0} y_\Delta(t) = \int x(\tau)h(t-\tau)\, d\tau = x(t) * h(t)$$

Weighting function of the filter

Weighting function



Response to noise

$$R_{yy}(t_1,t_2) = \overline{y(t_1)y(t_2)} = \overline{\iint x(\alpha)h(t_1-\alpha)\, x(\beta)h(t_2-\beta)\, d\alpha d\beta}$$

$$= \iint R_{xx}(\alpha,\beta)h(t_1-\alpha)h(t_2-\beta)d\alpha d\beta$$

$$= \int h(t_2-\beta)d\beta \int R_{xx}(\alpha,\beta)h(t_1-\alpha)d\alpha = \int h(t_2-\beta)R_{xx}(t_1,\beta) * h(t_1)d\beta$$

$$= \int R_{xx}(t_1,\beta)h(t_2-\beta)d\beta * h(t_1) = R_{xx}(t_1,t_2) * h(t_2) * h(t_1)$$

Stationary noise

$$R_{yy}(\tau) = \overline{y(t)y(t+\tau)} = \iint R_{xx}(\beta - \alpha)h(t-\alpha)h(t+\tau-\beta)d\alpha d\beta$$

$$= \iint R_{xx}(\gamma)h(t-\alpha)h(t+\tau-\gamma-\alpha)d\alpha d\gamma$$

$$= \int R_{xx}(\gamma)d\gamma \int h(t-\alpha)h(t-\alpha+\tau-\gamma)d\alpha = \int R_{xx}(\gamma)d\gamma \int h(z)h(z+\tau-\gamma)dz$$

$$= \int R_{xx}(\gamma)k_{hh}(\tau-\gamma)d\gamma = R_{xx}(\tau) * k_{hh}(\tau)$$

Output noise rms value

- For stationary input noise

$$\overline{n_y^2} = R_{yy}(0) = \int R_{xx}(\gamma) k_{hh}(\gamma) d\gamma$$

- If the input noise is non-stationary, so is the output, and there is no simple expression for it

$$\overline{n_y^2(t)} = \iint R_{xx}(\alpha, \beta) h(t-\alpha) h(t-\beta) d\alpha d\beta$$

Frequency domain

- For the signal we have

$$Y(f) = X(f)H(f)$$

- For the (stationary) noise:

$$S_y(f) = S_x(f)|H(f)|^2$$

$$\overline{n_y^2} = \int S_x(f)|H(f)|^2 df$$

Also from the expression in the time domain via Parseval theorem

The case of white stationary noise

- Time domain: $R_{xx}(\tau) = \lambda\delta(\tau)$

$$R_{yy}(\tau) = R_{xx}(\tau) * k_{hh}(\tau) = \lambda k_{hh}(\tau)$$

$$\overline{n_y^2} = R_{yy}(0) = \lambda k_{hh}(0) = \lambda \int h^2(t)dt$$

- Frequency domain: $S_x(f) = \lambda$

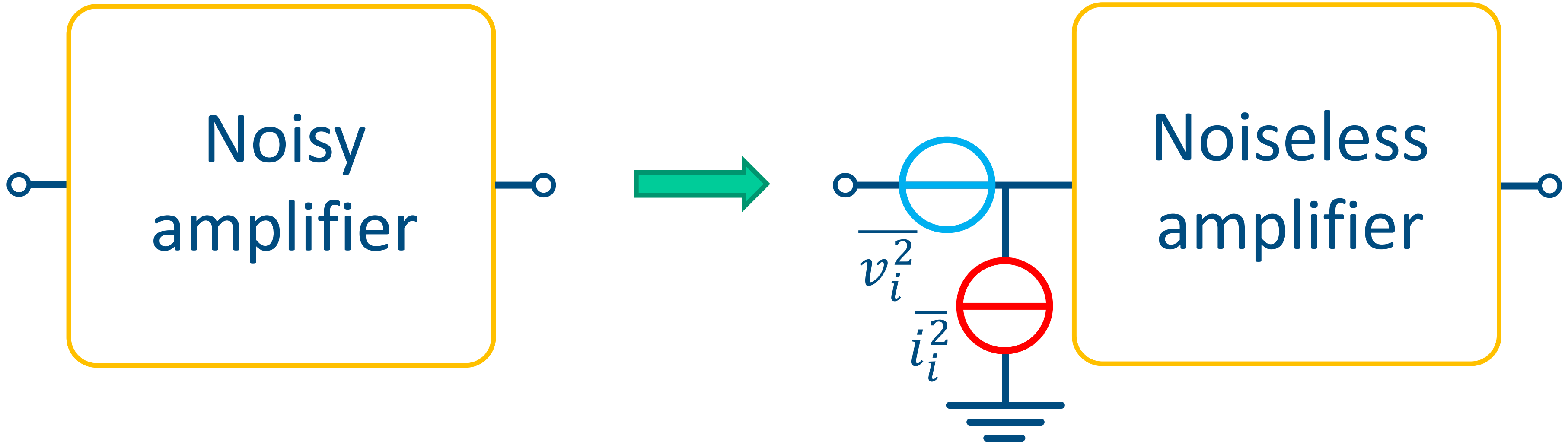
$$S_y(f) = S_x(f)|H(f)|^2 = \lambda|H(f)|^2$$

$$\overline{n_y^2} = \int S_x(f)df = \lambda \int |H(f)|^2 df$$

Outline

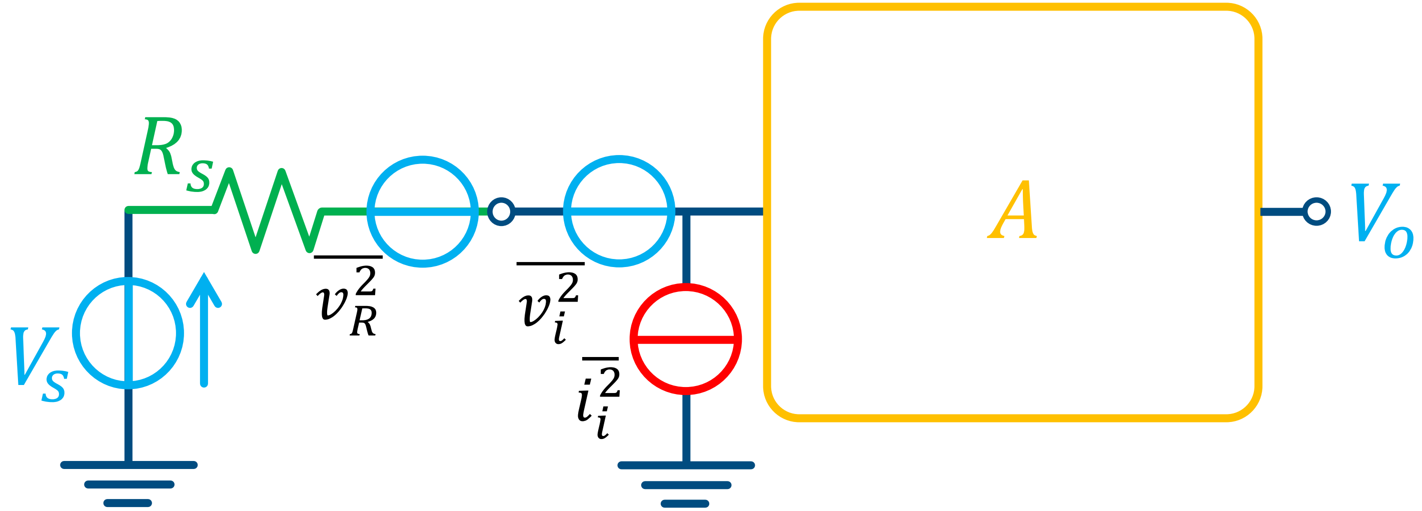
- LTI system response to signal and noise
- Noise in OAs
- Feedback and noise

Noise in amplifiers



- Equivalent input noise voltage and current are used to get non-zero output noise for every input condition
- Correlation between the sources is neglected

Output signal and noise



We consider a sinusoidal signal and a frequency interval Δf:

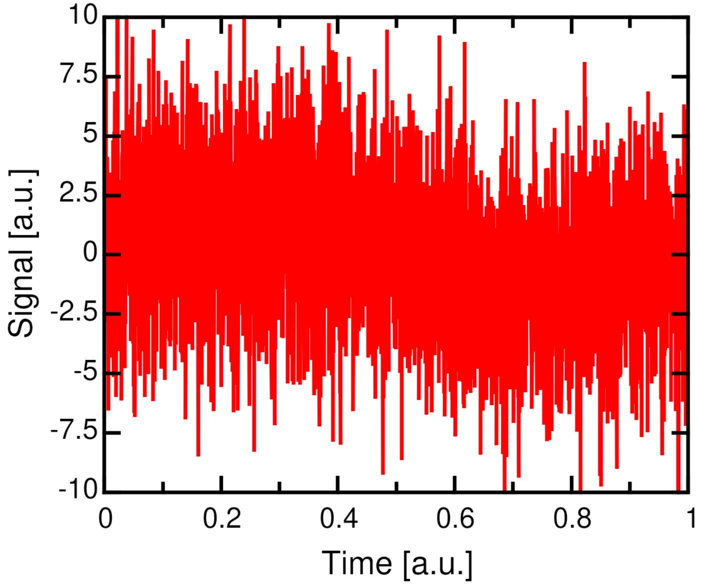
$$V_o = AV_s \cos \omega t$$

$$\overline{V_o^2} = S_{V_o}\Delta f = \left(S_{V_R} + S_V + S_I R_S^2\right)A^2\Delta f$$

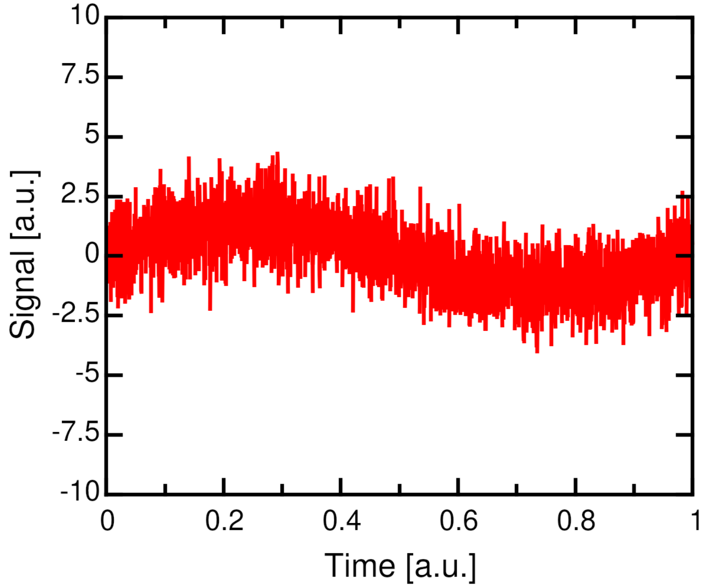
S/N ratio

$$\left(\frac{S}{N}\right)_{out} = \frac{\text{signal amplitude}}{\text{noise rms amplitude}} = \frac{V_o}{\sqrt{\overline{V_o^2}}}$$

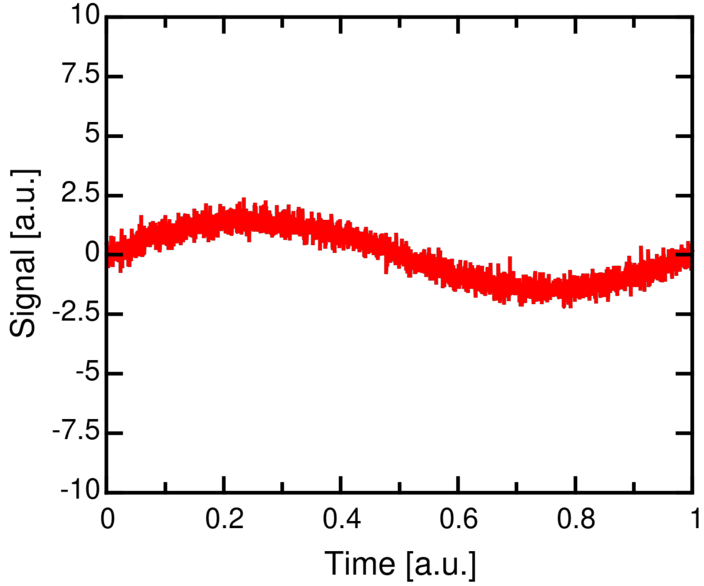
$S/N = 0.1$



$S/N = 1$



$S/N = 10$



Effect of the amplifier

$$\left(\frac{S}{N}\right)^2_{out} = \frac{V_S^2}{\left(S_{V_R} + S_V + S_I R_S^2\right)\Delta f}$$

$$\left(\frac{S}{N}\right)^2_{in} = \frac{V_S^2}{S_{V_R}\Delta f}$$

S/N is lower at the amplifier output, because of the noise of the amplifier itself

Noise factor and figure

Noise factor $F = \frac{\left(\frac{S}{N}\right)^2_{in}}{\left(\frac{S}{N}\right)^2_{out}} = 1 + \frac{S_V(f) + S_I(f) R_S^2}{S_{V_R}}$

Amplifier noise

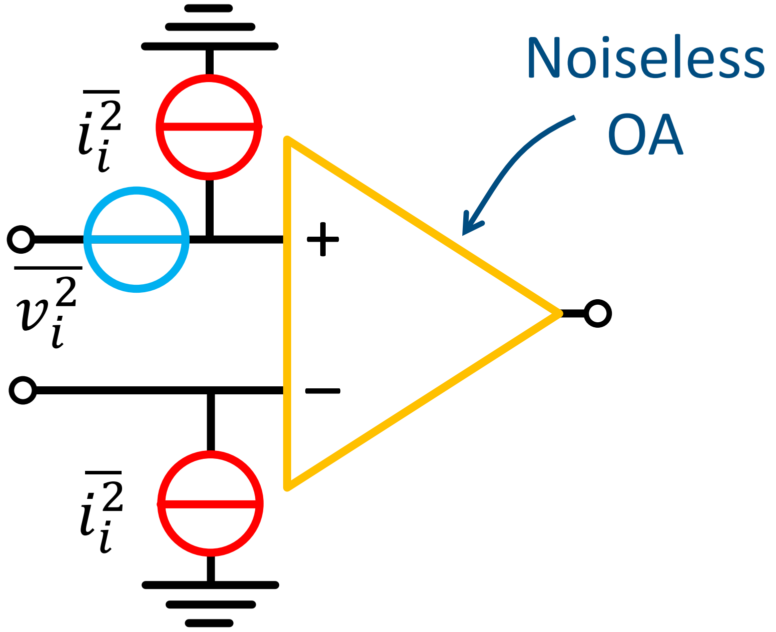
Source noise $(4 k_B T R_s)$

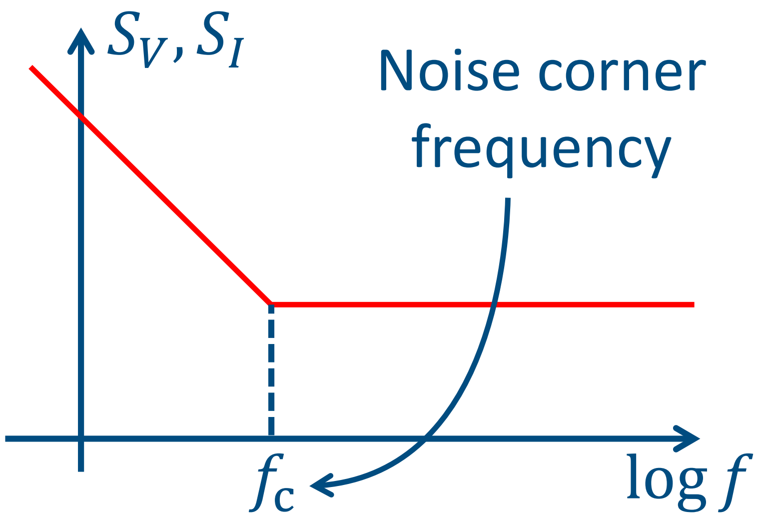
- F is minimum for $R_{s,opt}^2 = \frac{S_V(f)}{S_I(f)}$
- The noise figure is commonly used:

$$NF = 10 \log_{10} F$$

(very useful in RF applications)

Noise in OAs



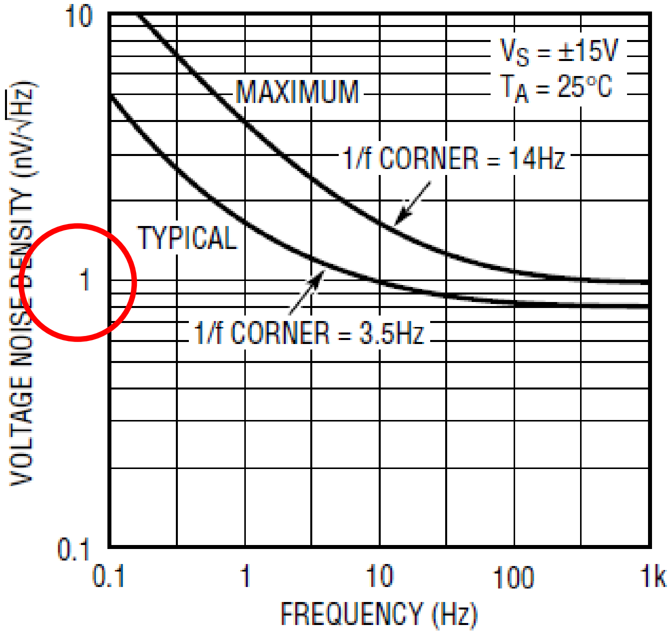


- Noise is referred to the input via the equivalent input noise voltage and currents
- All contain white and flicker components

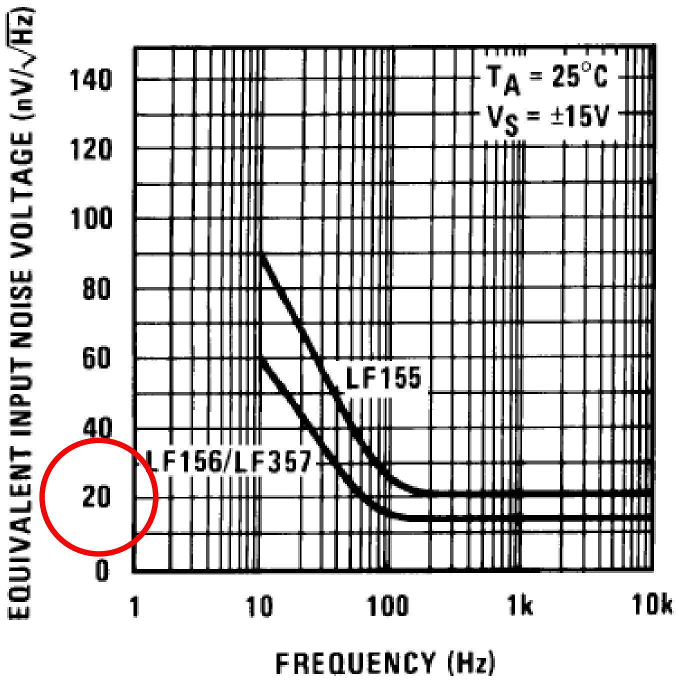
Indicative values

- Voltage noise (white component)
  - BJT: $1 \text{ to } 10 - 20\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$
  - JFET/MOS: usually higher, $20 - 30\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$
- Current noise (white component)
  - BJT: a few $\mathrm{pA}/\sqrt{\mathrm{Hz}}$
  - JFET: a few $\mathrm{fA}/\sqrt{\mathrm{Hz}}$; can be even less for CMOS
- Noise corner frequency
  - BJT/JFET: $1 - 100\ \mathrm{Hz}$
  - MOS: a few kHz or even much higher, but improving (can find some in the 50 Hz range)

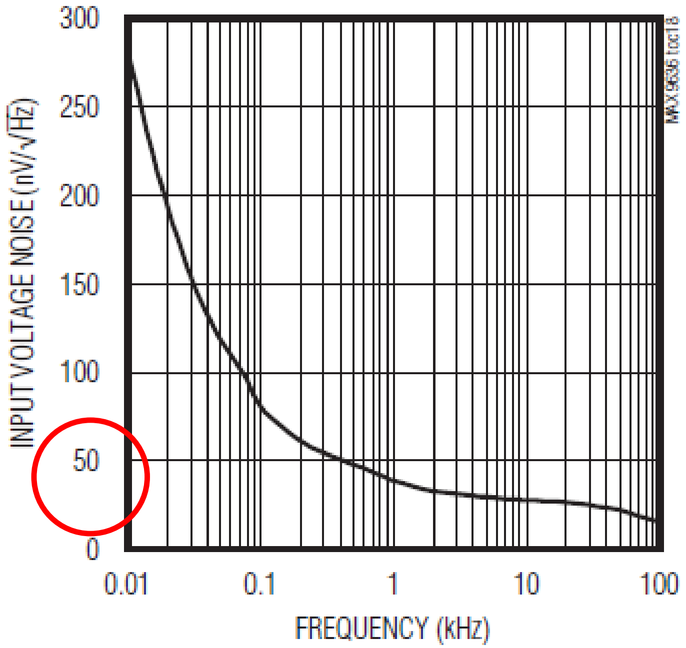
Input noise voltage PSD



BJT

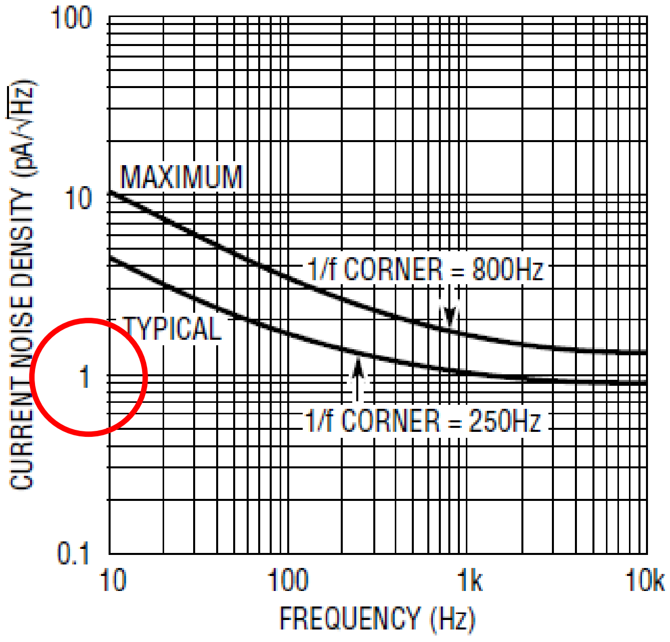


JFET

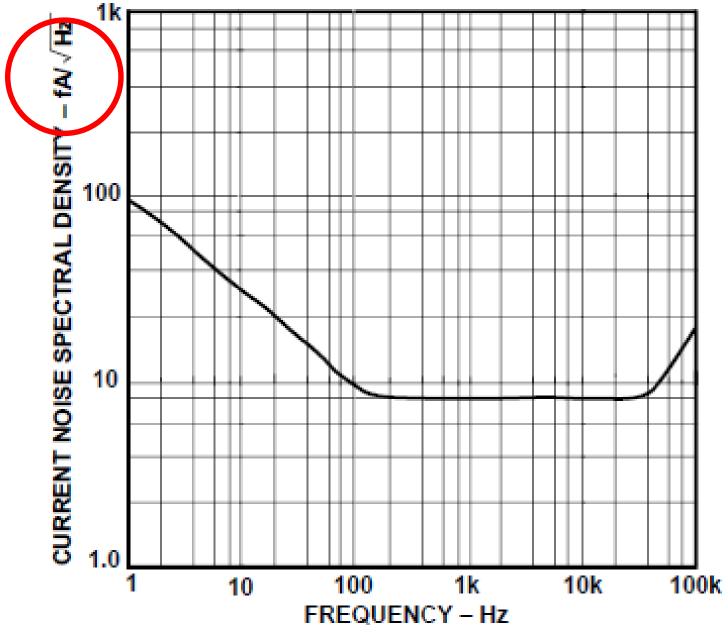


CMOS

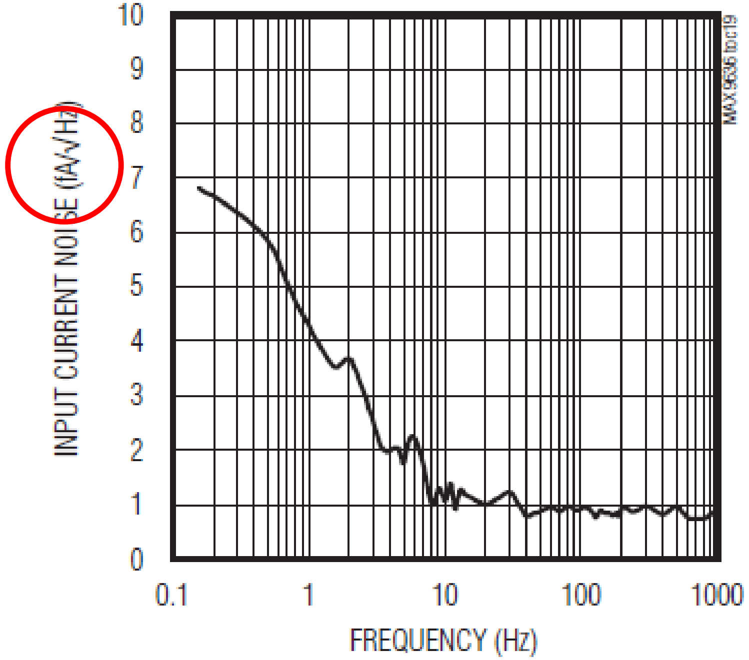
Input noise current PSD



BJT

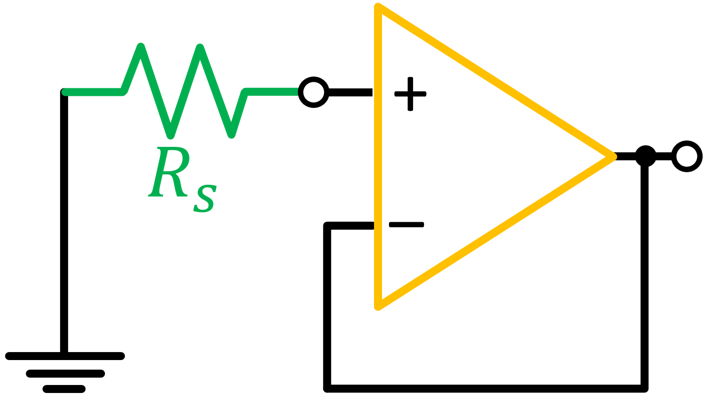


JFET



CMOS

OA comparison



$$S_{V_o} = S_V + S_I R_S^2 + 4k_B T R_S$$

From [1]

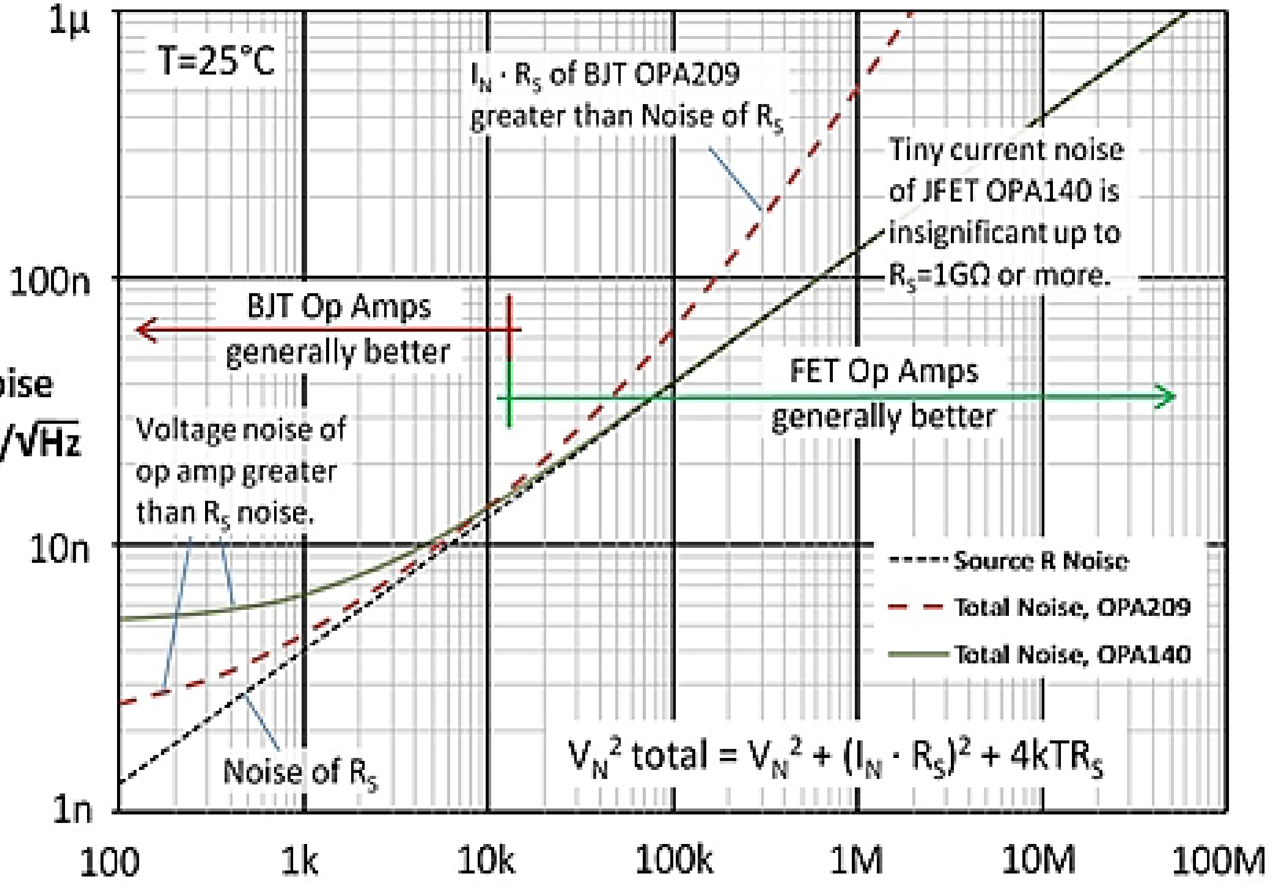
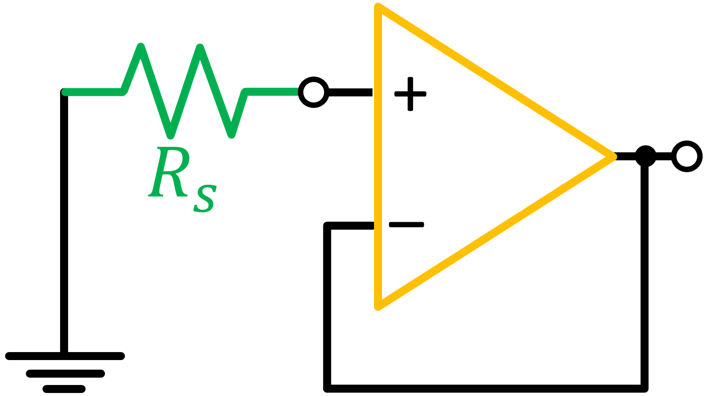


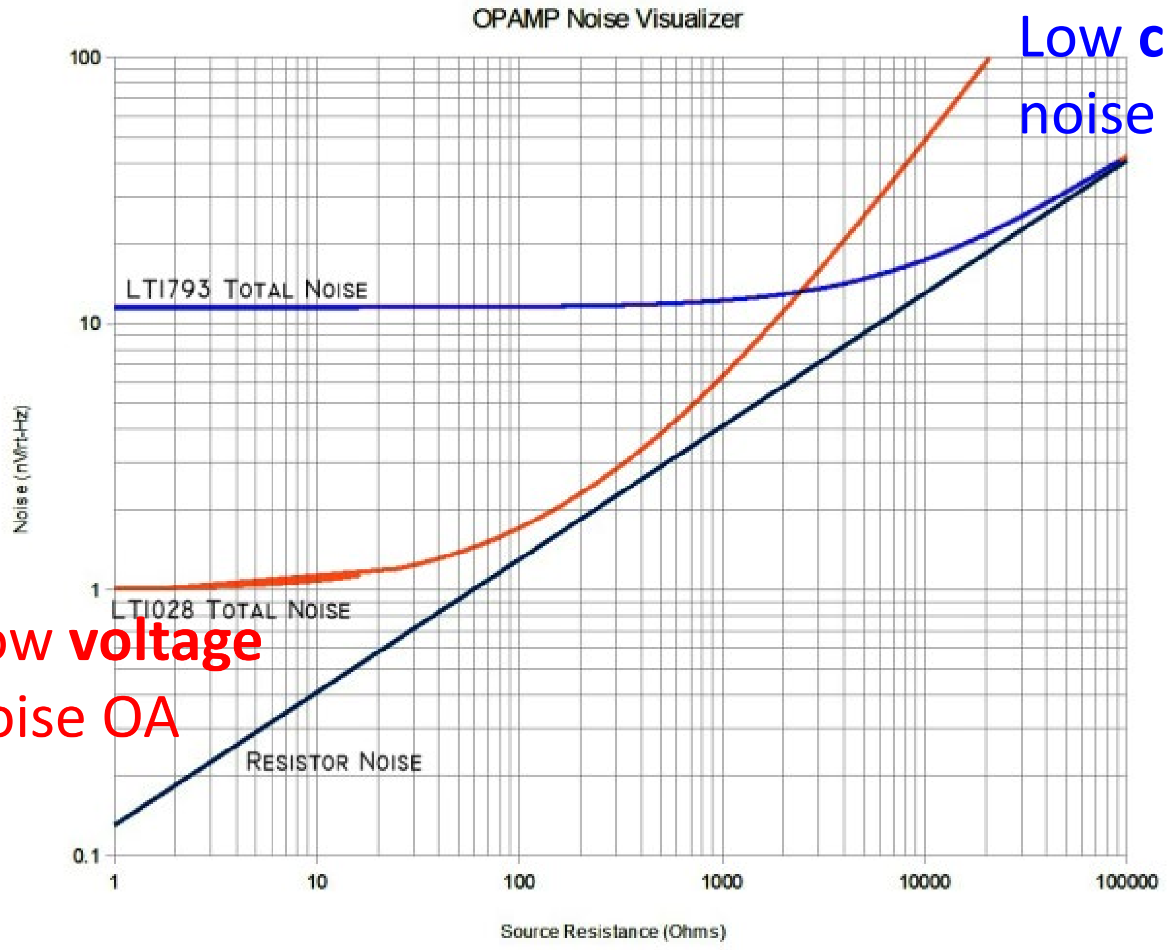
Figure 2.

Low-noise OAs



$$S_{V_o} = S_V + S_I R_S^2 + 4k_B T R_S$$

From [2]



Low **current** noise OA

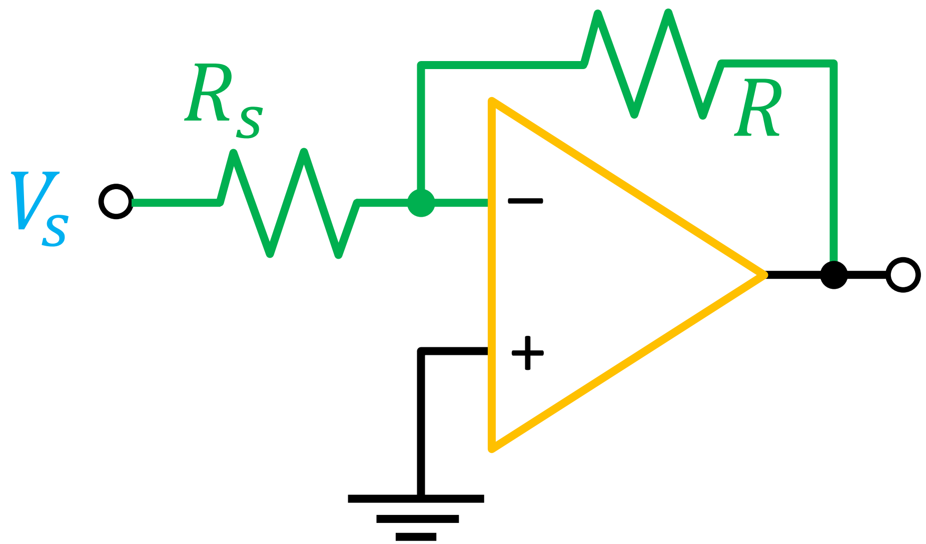
Low **voltage** noise OA

Outline

- LTI system response to signal and noise
- Noise in OAs
- Feedback and noise

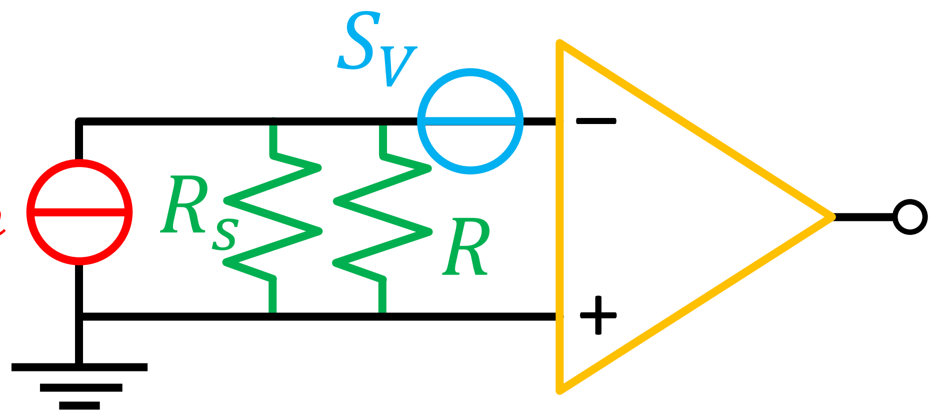
Feedback and noise

- We want to assess the effect of negative feedback on the noise performance, measured via F or S/N
- We consider a simple inverting amplifier example:



F – open-loop case

$$\frac{4k_BT}{R_s} + \frac{4k_BT}{R} + S_I = S_{I_n}$$

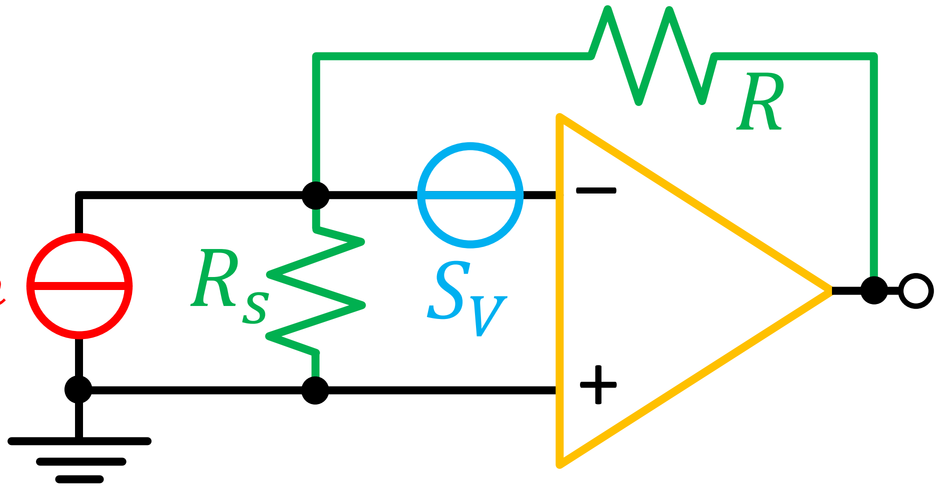


$$S_{V_o} = \left(S_{I_n}(R_s \parallel R)^2 + S_V\right)|A(s)|^2 \quad S_{V_o}^{source} = \frac{4k_BT}{R_s}(R_s \parallel R)^2|A(s)|^2$$

$$F^{OL} = 1 + \left(\frac{4k_BT}{R} + S_I + \frac{S_V}{(R_s \parallel R)^2}\right)\frac{R_s}{4k_BT}$$

F – closed-loop case

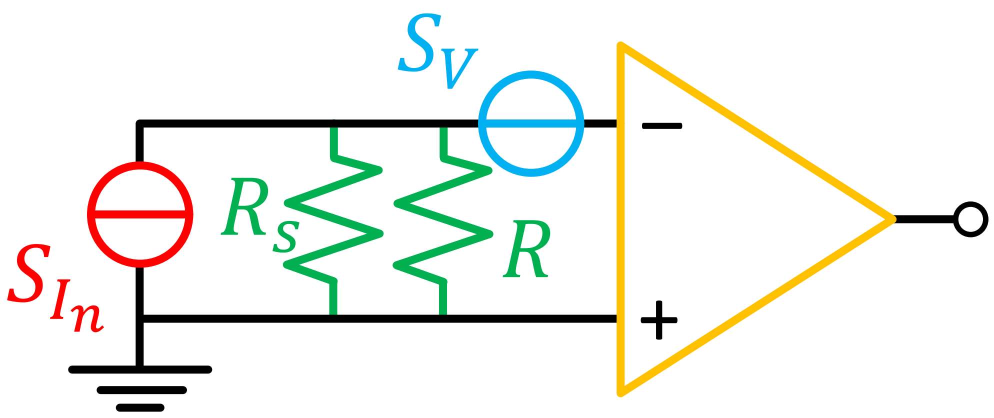
$$\frac{4k_BT}{R_s} + \frac{4k_BT}{R} + S_I = S_{I_n}$$

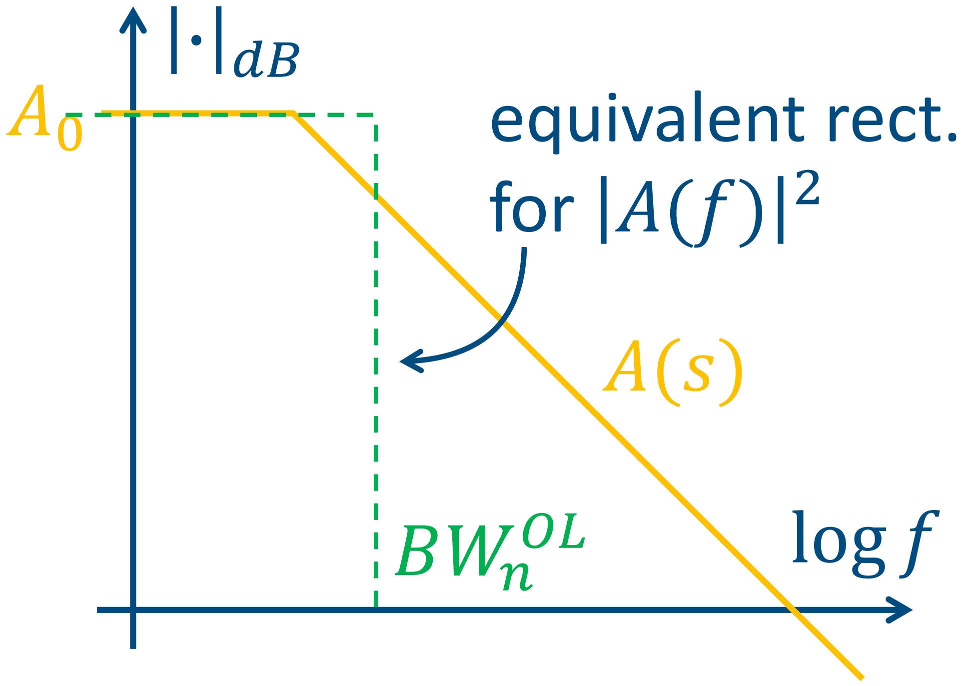


$$S_{V_o} = \frac{(\text{open} - \text{loop PSD})}{\left|1 - G_{loop}\right|^2} \qquad S_{V_o}^{source} = \frac{(\text{open} - \text{loop PSD})}{\left|1 - G_{loop}\right|^2}$$

$$F^{CL} = F^{OL}$$

S/N – open-loop case



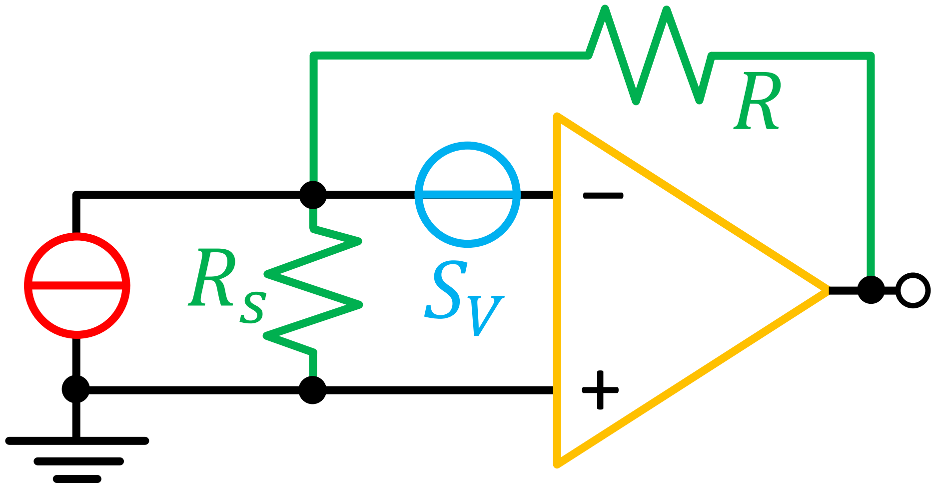


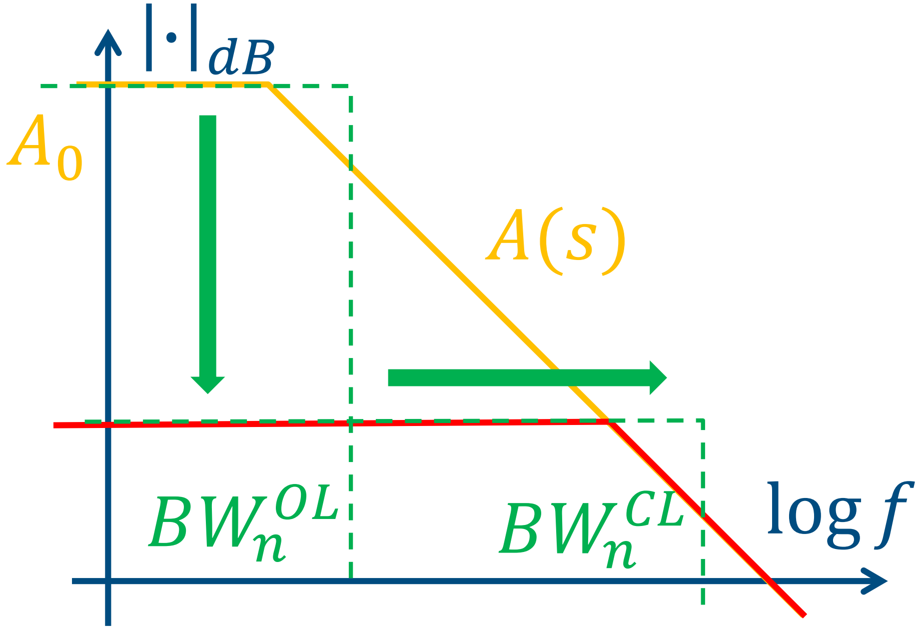
$$V_o = A_0 \frac{R}{R + R_s} V_s$$ (step voltage input, steady-state value)

$$\overline{V_o^2} = \int_0^\infty S_{V_o}(f)df = \left(S_{I_n}(R_s \parallel R)^2 + S_V\right)\int_0^\infty |A(f)|^2 df$$

$$= \left(S_{I_n}(R_s \parallel R)^2 + S_V\right)A_0^2\, BW_n^{OL}$$

S/N – closed-loop case





$$V_o = (\text{open} - \text{loop transfer})/(1 - G_{loop}(0))$$

$$\overline{V_o^2} = \int_0^\infty \frac{\text{open} - \text{loop PSD}}{|1 - G_{loop}(f)|^2} df = \left(S_{I_n}(R_s \parallel R)^2 + S_V\right) \int_0^\infty \left|\frac{A(f)}{1 - G_{loop}(f)}\right|^2 df$$

$$= \left(S_{I_n}(R_s \parallel R)^2 + S_V\right) \frac{A_0^2}{\left(1 - G_{loop}(0)\right)^2} BW_n^{CL}$$

S/N – summary

$$\left(\frac{S}{N}\right)^{OL} = \frac{V_s \frac{R}{R+R_s}}{\sqrt{S_{I_n}(R_s \parallel R)^2 + S_V}} \frac{1}{\sqrt{BW_n^{OL}}}$$

this term is $\frac{1}{1-G_{loop}(0)} \ll 1$

$$\left(\frac{S}{N}\right)^{CL} = \frac{V_s \frac{R}{R+R_s}}{\sqrt{S_{I_n}(R_s \parallel R)^2 + S_V}} \frac{1}{\sqrt{BW_n^{CL}}} = \left(\frac{S}{N}\right)^{OL} \sqrt{\frac{BW_n^{OL}}{BW_n^{CL}}}$$

Wrap-up

- Negative feedback modifies all open-loop transfers by the same factor $\Rightarrow F$ is unaffected
- Negative feedback widens the bandwidth, collecting more noise $\Rightarrow S/N$ is degraded
- For this reason, you should never use extra BW beyond what is needed by the signal
- If BW is independently set by the requirements, negative feedback does not modify the noise performance

References

1. http://www.edn.com/electronics-blogs/the-signal/4404375/Op-Amp-Noise-the-non-inverting-amplifier
2. http://www.edn.com/electronics-blogs/the-practicing-instrumentation-engineer/4410043/Visualize-op-amp-noise