Electronics – 96032

POLITECNICO DI MILANO





White Noise Filtering: LPF

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Disclaimer

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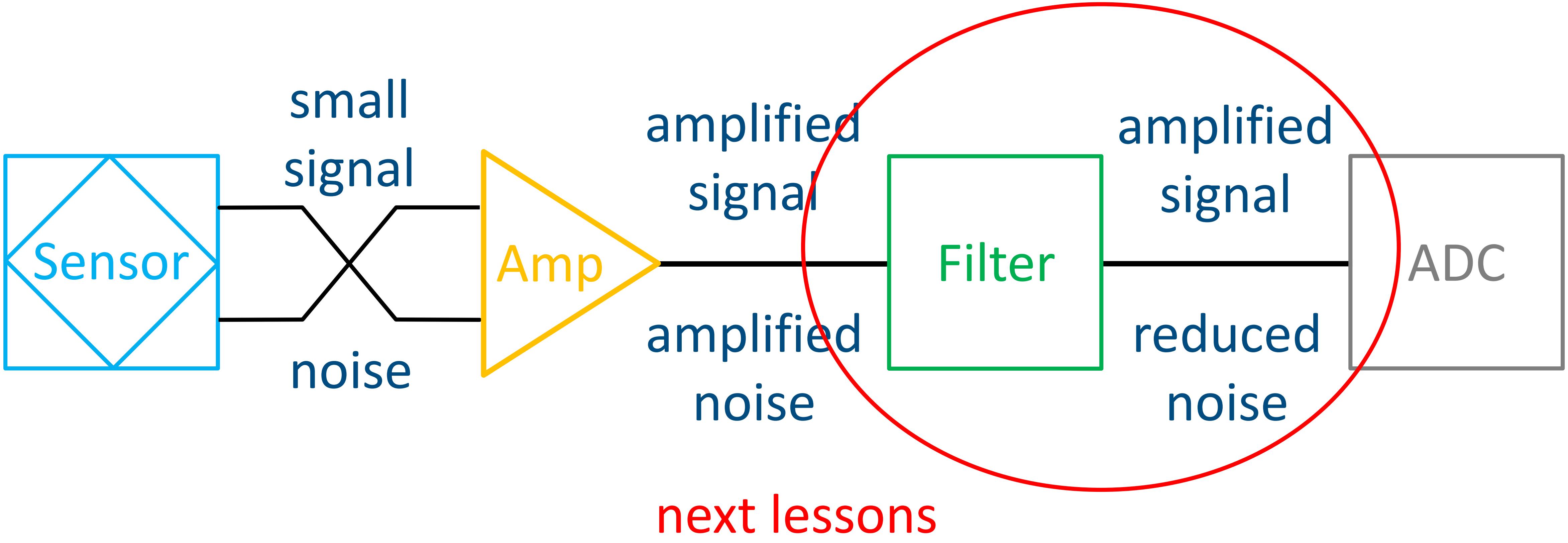
Acquisition chain



Purpose of the lesson

- It is now time to begin a discussion on the techniques for improving S/N
- Noise-reduction techniques obviously depend on the type of signal and of noise:

		Noise	
		HF (white)	LF (flicker)
Signal	LF (constant)	this lesson	next lessons
	HF (pulse)	next lesson	next lessons

Outline

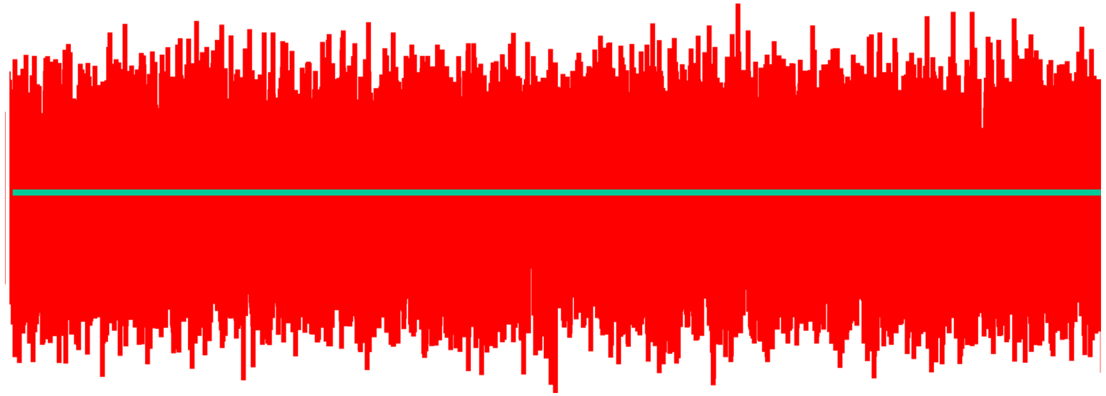
- Intro – WN filtering
- LPF output signal and noise in the time domain
- LPF output signal and noise in the frequency domain

Signal recovery stage

- Usually placed after the sensor and the first amplification stage
- Its fundamental function is to increase the signal-to-noise ratio
- Two basic approaches
  - Time domain
  - Frequency domain

White noise filtering: concept

- Consider a constant signal plus white noise



- We know that the average value of noise is zero $\Rightarrow$ to reduce noise, we can compute the time average (ergodic process)
- Note that averaging does not affect the (constant) signal $\Rightarrow$ we improve S/N

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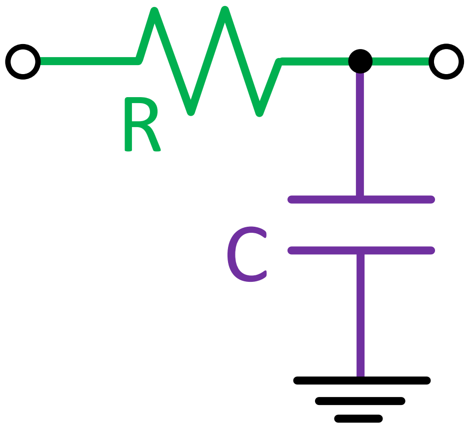
LP filter

- Delta-function response

$$h(t) = \frac{1}{T_F} e^{-\frac{t}{T_F}} \mathrm{u}(t)$$



- Transfer function

$$H(s) = \frac{1}{1 + sT_F}$$

- What kind of average are we performing here?

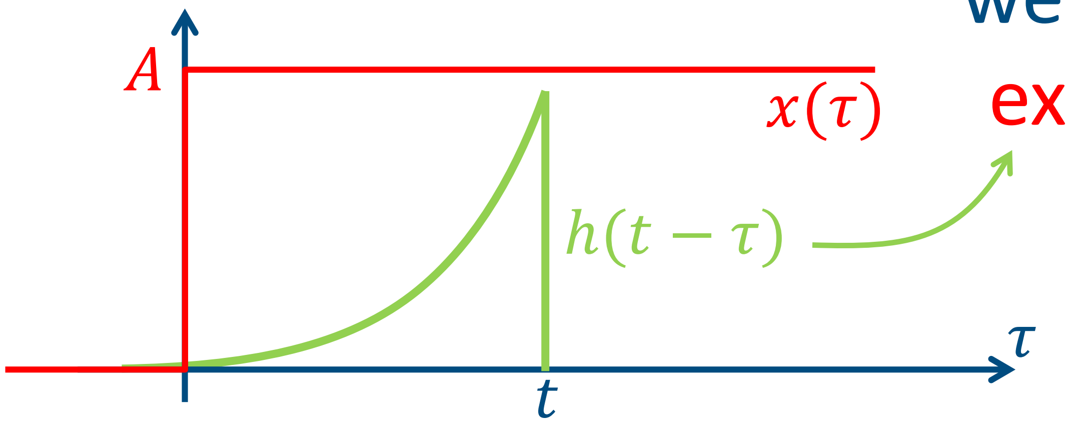
Signal (time domain)

Step response, $x(t) = A\mathrm{u}(t)$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{t} x(\tau)h(t-\tau)d\tau = \int_{0}^{t} A\frac{1}{T_F}e^{-\frac{t-\tau}{T_F}}d\tau$$

$$= A\left(1 - e^{-\frac{t}{T_F}}\right)$$

weighting function ⇒ exponential average

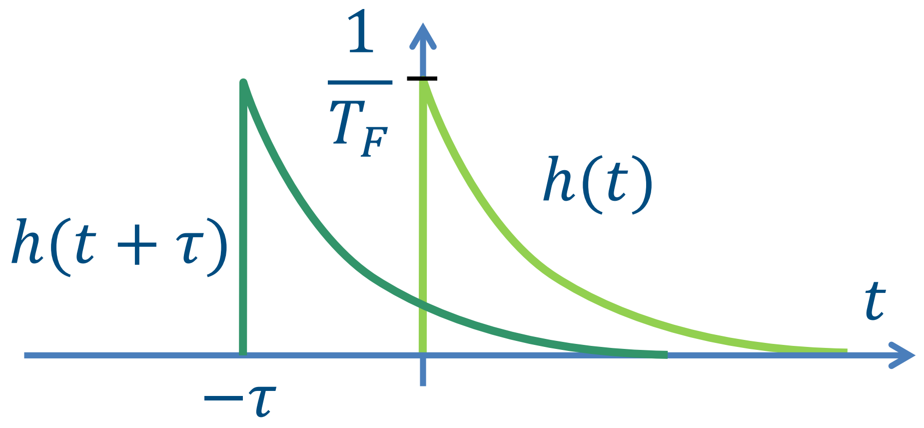


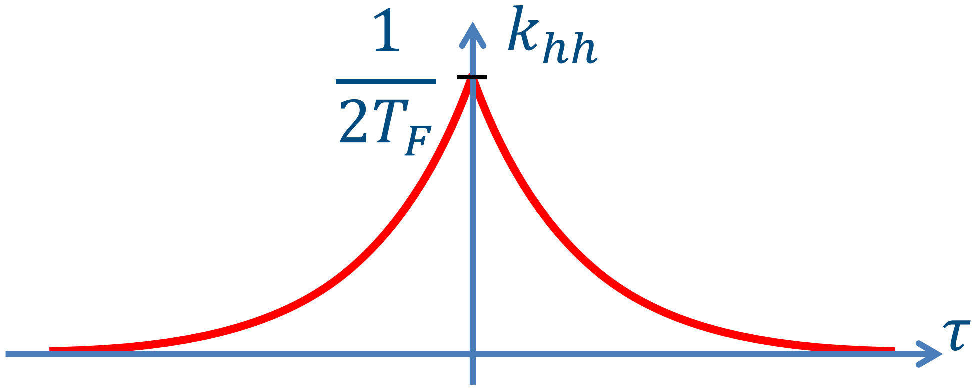
Weighting function time correlation

$$k_{hh}(\tau) = \int h(t)h(t+\tau)dt =$$

$$\frac{1}{T_F^2}\int e^{-\frac{t}{T_F}}\,\mathrm{u}(t)e^{-\frac{t+\tau}{T_F}}\,\mathrm{u}(t+\tau)dt =$$

$$\frac{e^{-\frac{\tau}{T_F}}}{T_F^2}\int_0^{\infty} e^{-\frac{2t}{T_F}}dt = \frac{e^{-\frac{\tau}{T_F}}}{2T_F} \Rightarrow \frac{e^{-\frac{|\tau|}{T_F}}}{2T_F}$$





Noise (time domain)

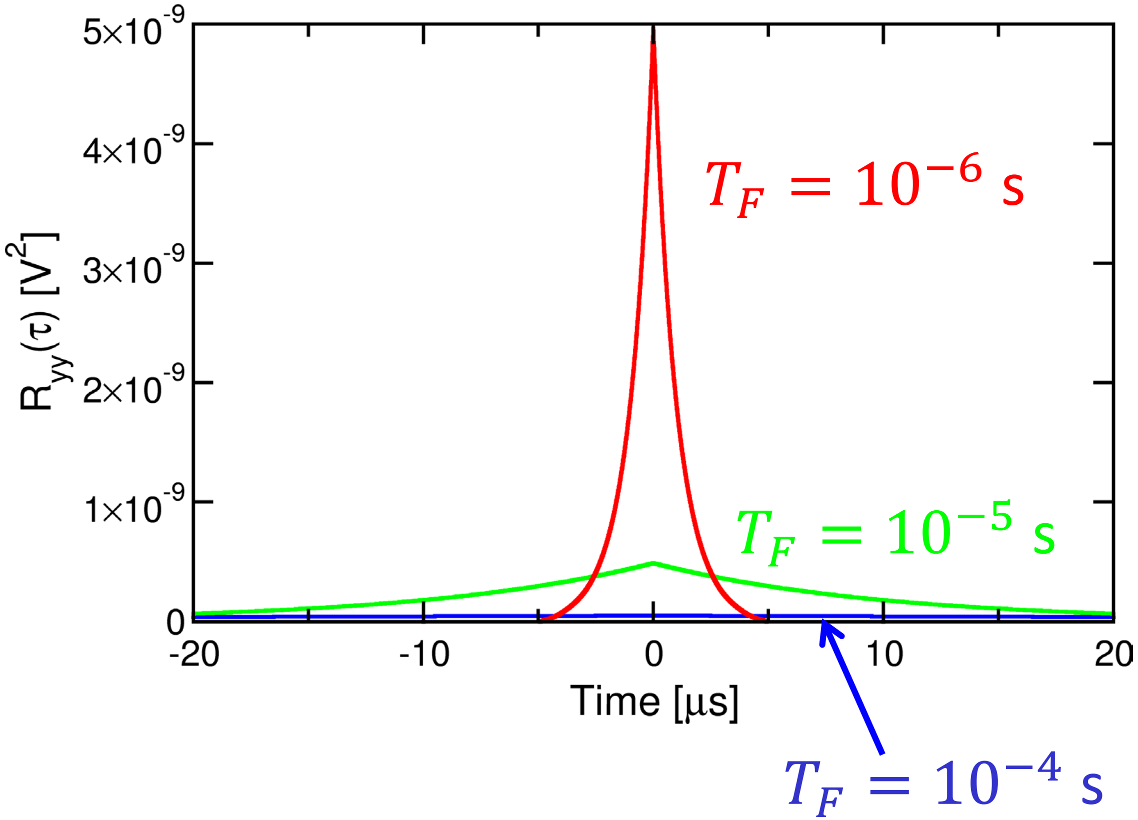
$$R_{yy}(\tau) = R_{xx}(\tau) * k_{hh}(\tau) = \lambda k_{hh}(\tau) = \frac{\lambda}{2T_F} e^{-\frac{|\tau|}{T_F}}$$

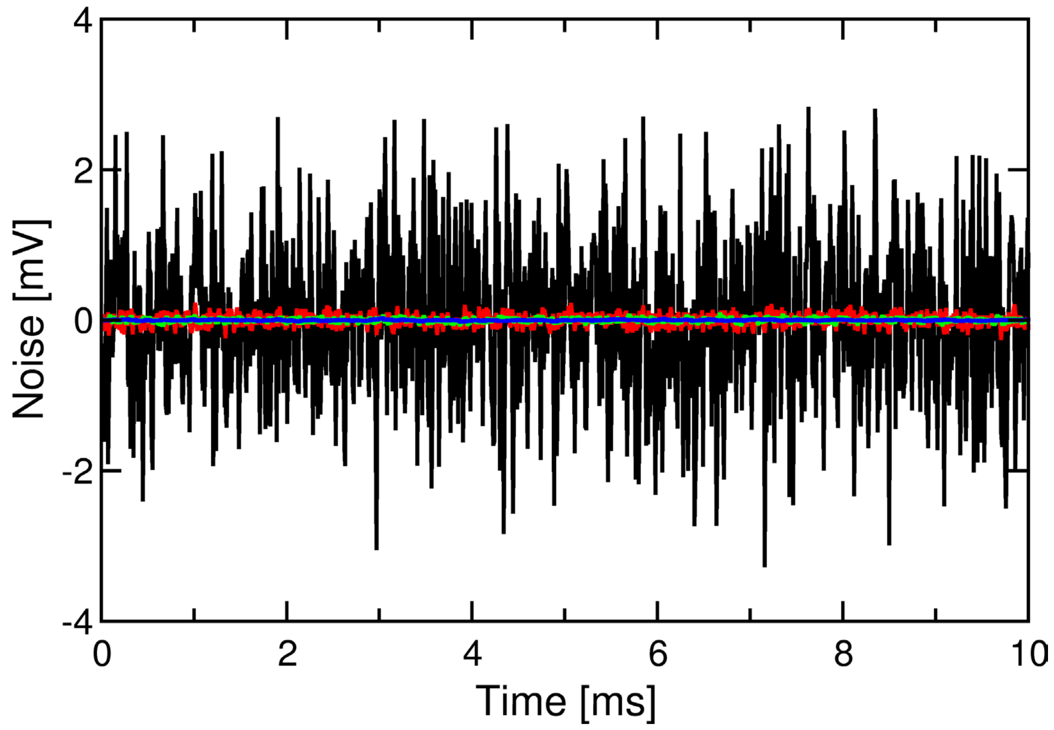
$$\overline{n_y^2} = \lambda k_{hh}(0) = \frac{\lambda}{2T_F}$$

Noise power is reduced when T_F is increased

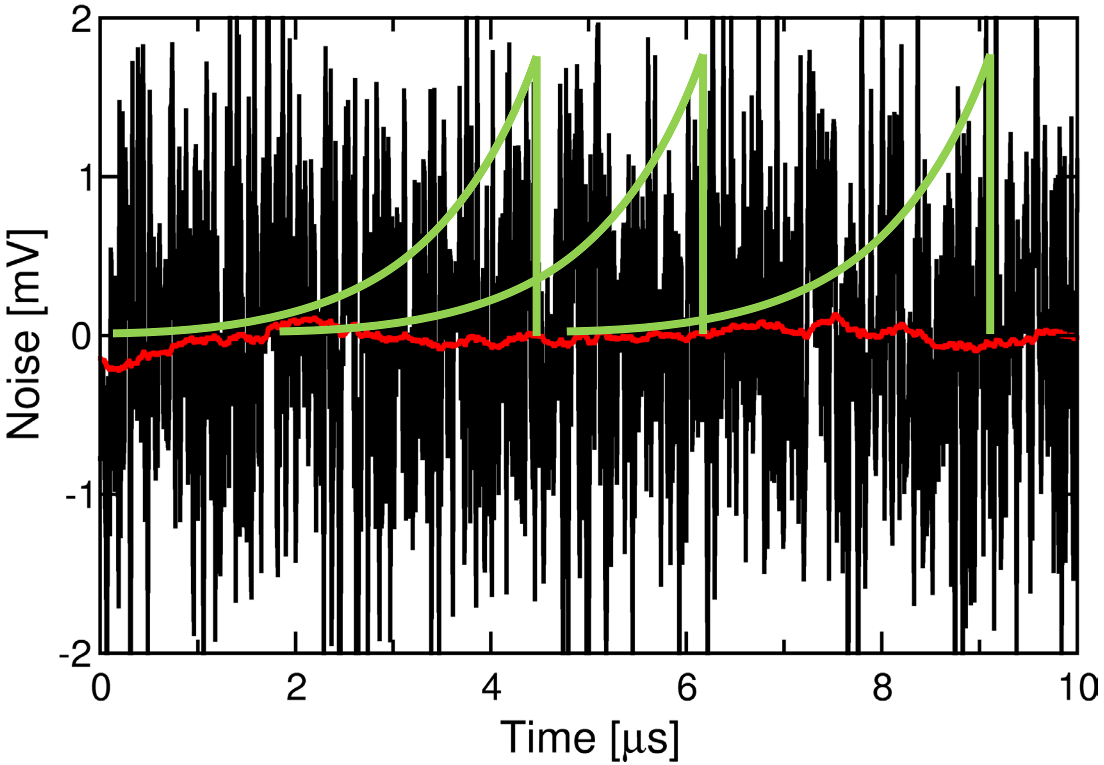
(noise is averaged over a longer interval)

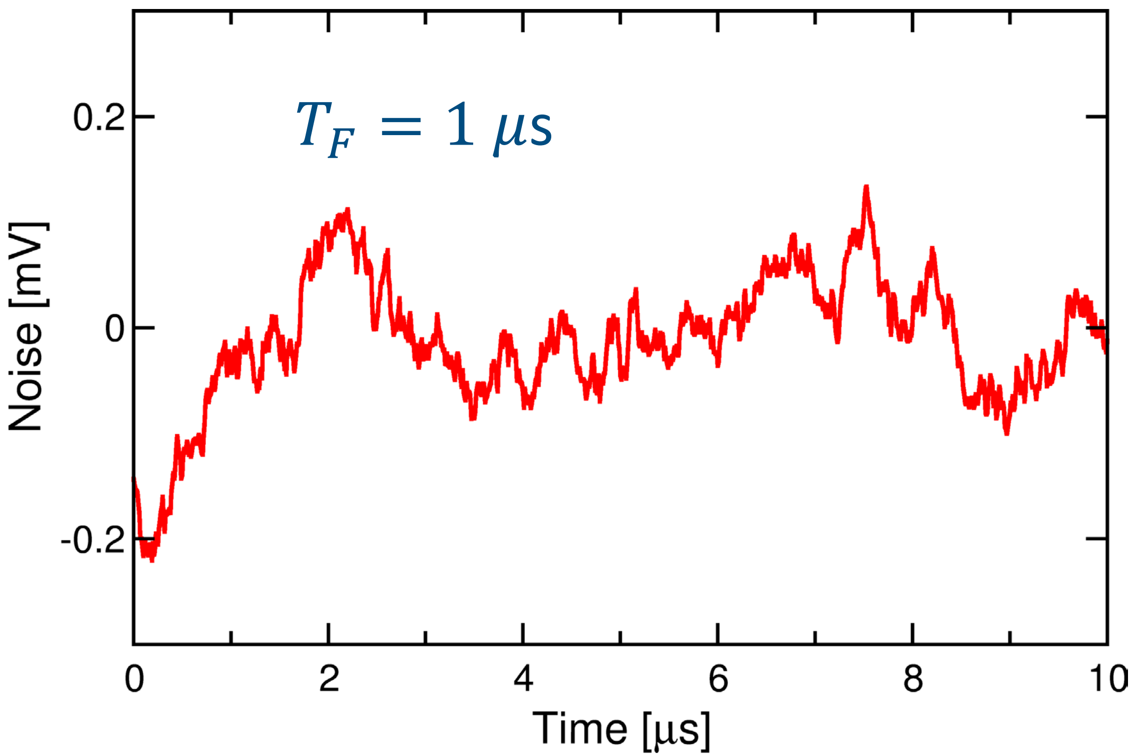
Noise reduction (time domain)





Non-white (correlated) output noise





Output noise is an average of the input WN: its values are correlated (i.e., «similar») over the extent of the WF

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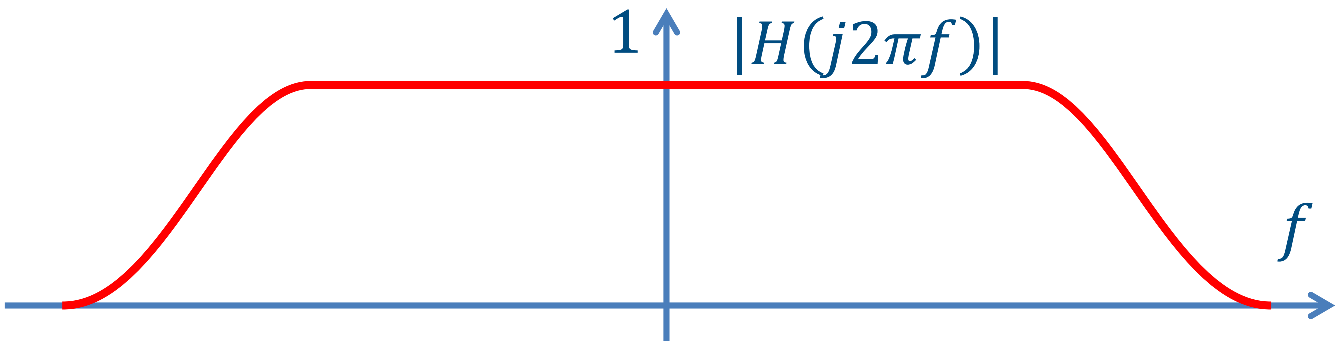
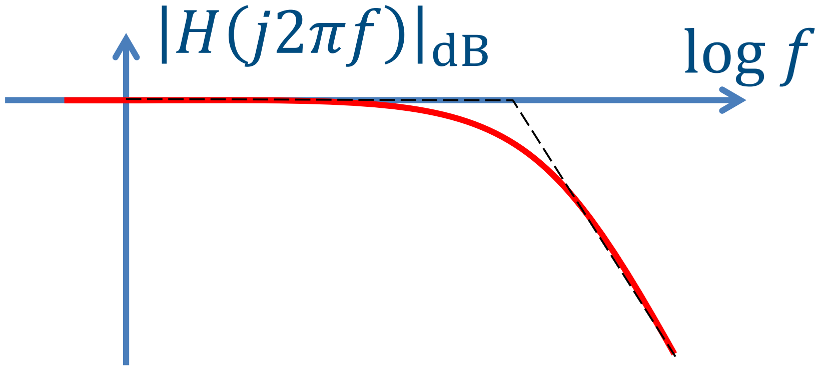
Signal (frequency domain)

Step response:

$$Y(s) = H(s)X(s) = \frac{1}{1+sT_F}\frac{A}{s} = A\left(\frac{1}{s} - \frac{T_F}{1+sT_F}\right)$$

$$\Rightarrow y(t) = A\left(1 - e^{-\frac{t}{T_F}}\right)u(t)$$

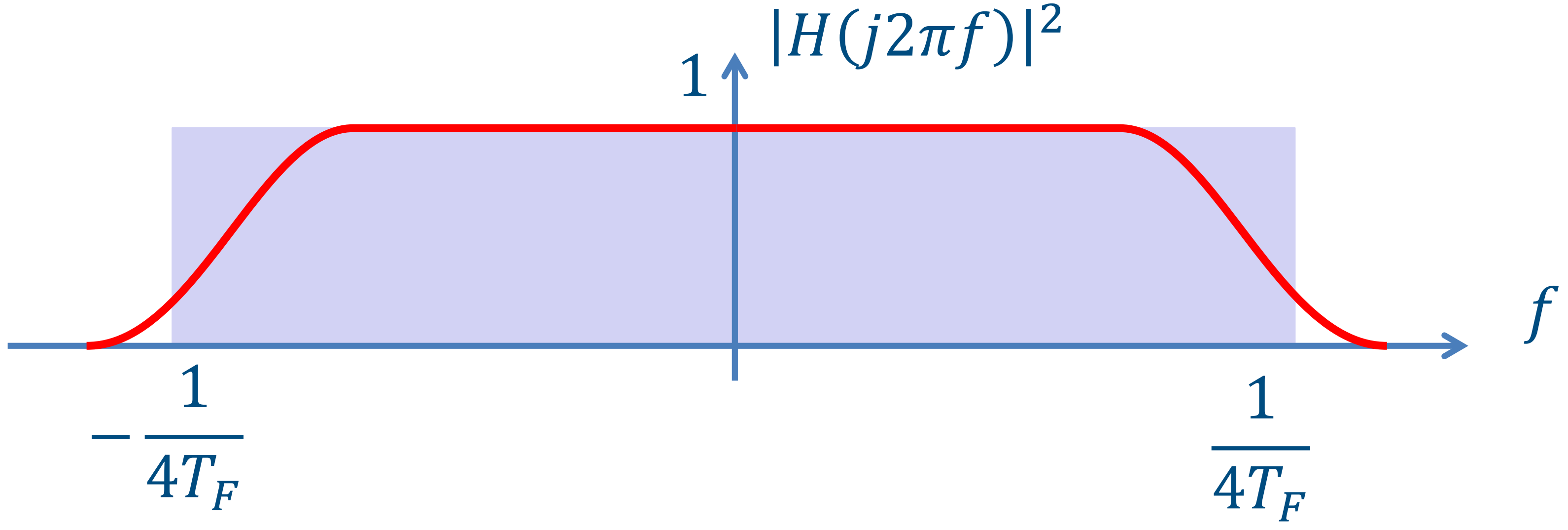




Noise (frequency domain)

$$S_y(\omega) = S_x(\omega)|H(\omega)|^2 = \lambda|H(\omega)|^2 = \frac{\lambda}{1+(\omega T_F)^2}$$

$$\overline{n_y^2} = \int_{-\infty}^{\infty} S_y(\omega)\frac{d\omega}{2\pi} = \frac{\lambda}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{1+(\omega T_F)^2} = \frac{\lambda}{2\pi T_F}[\arctan(\omega T_F)]_{-\infty}^{+\infty} = \frac{\lambda}{2T_F}$$

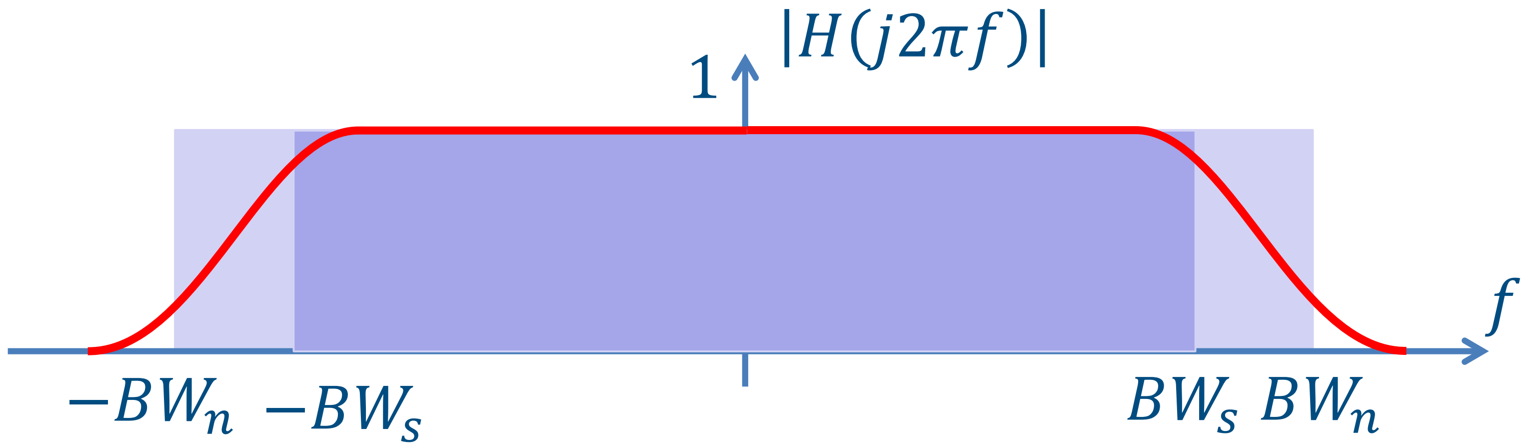


Signal and noise bandwidths

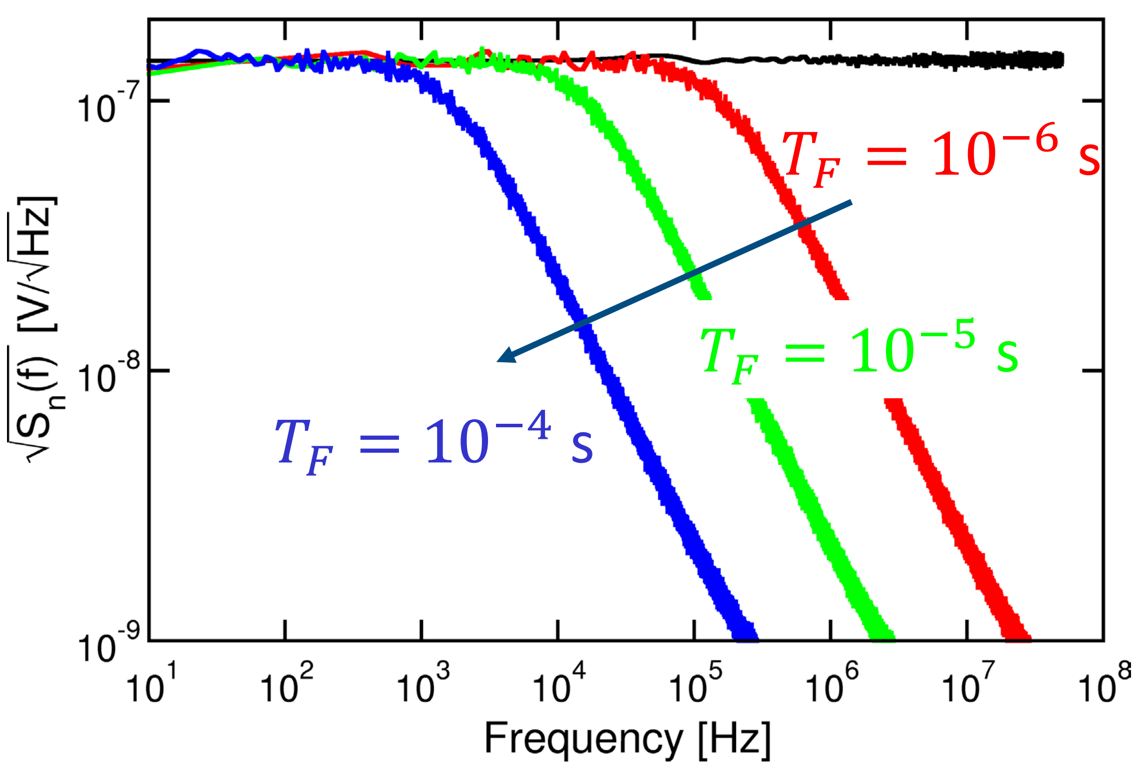
A single-pole LP filter has:

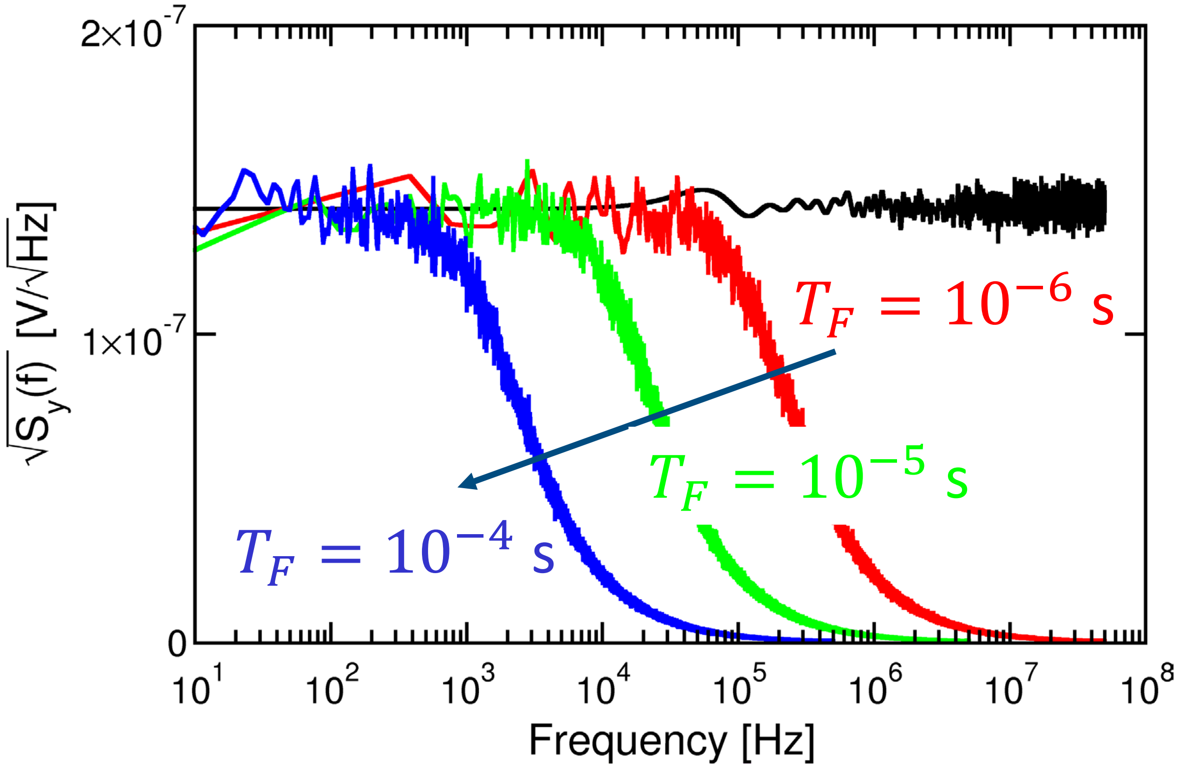
a signal bandwidth $BW_s = f_p = \frac{1}{2\pi T_F}$

a (white) noise bandwidth $BW_n = \frac{1}{4T_F} = \frac{\pi}{2} BW_s$



Noise reduction (frequency domain)





- LPF rejects high-frequency noise components, reducing $\overline{n_y^2}$
- Noise power is reduced when BW_n is lowered

Input and output S/N

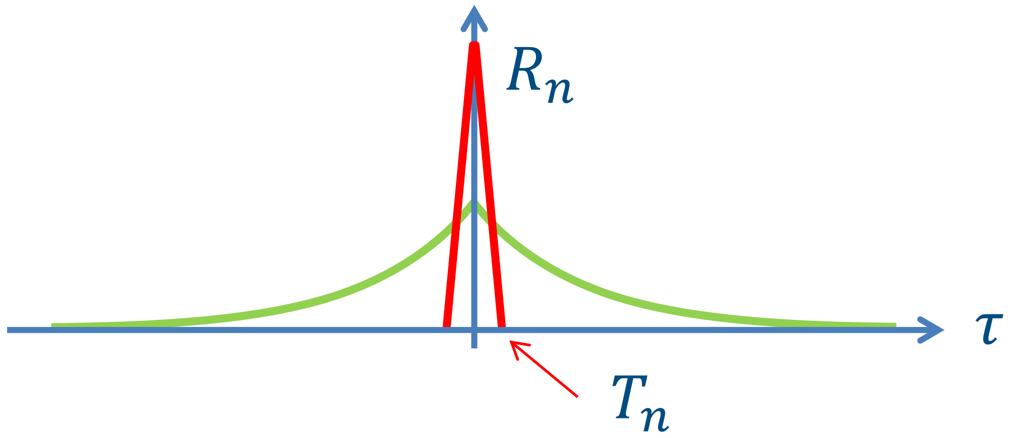
- We consider a quasi-white noise at the input, with equivalent bandwidth $f_n = 1/2T_n$

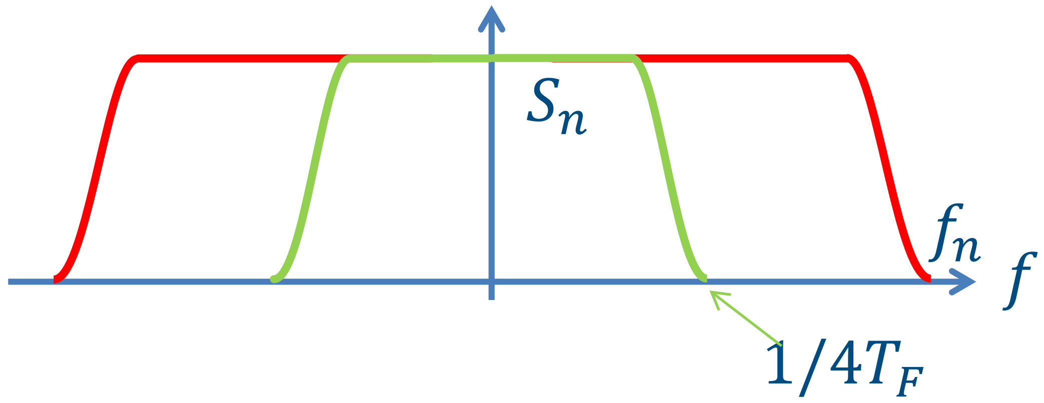
$$\left(\frac{S}{N}\right)_x = \frac{V_i}{\sqrt{\overline{n_x^2}}} = \frac{V_i}{\sqrt{2\lambda f_n}} = V_i\sqrt{\frac{T_n}{\lambda}}$$

- At the output we have

$$\left(\frac{S}{N}\right)_y = \frac{V_i}{\sqrt{\overline{n_y^2}}} = \frac{V_i}{\sqrt{2\lambda BW_n}} = V_i\sqrt{\frac{2T_F}{\lambda}}$$

Improvement of S/N





$$\left(\frac{S}{N}\right)_y = \left(\frac{S}{N}\right)_x \sqrt{\frac{2T_F}{T_n}} = \left(\frac{S}{N}\right)_x \sqrt{\frac{f_n}{BW_n}}$$