Electronics – 96032

POLITECNICO DI MILANO





Time-variant Filters: Gated Integrators

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Disclaimer

Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes

Purpose of the lesson

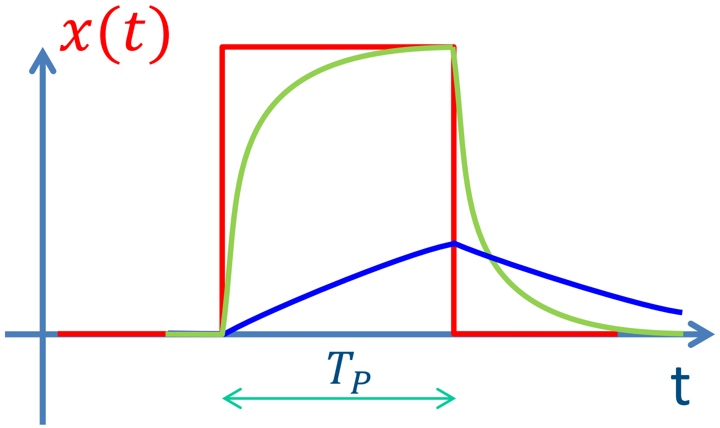
- It’s time to begin a discussion on the techniques for improving S/N
- Noise-reduction techniques obviously depend on the type of signal and of noise:

		Noise	
		HF (White)	LF (flicker)
Signal	LF (constant)	previous lesson	next lessons
	HF (pulse)	this lesson	next lessons

Outline

- Time-variant filters
- Gated integrators

Fast pulse + WN and LPF



$$\left(\frac{S}{N}\right)_{out} = A\frac{1-e^{-T_P/T_F}}{\sqrt{\lambda/2T_F}}$$

- Short $T_F \Rightarrow$ larger signal, but larger noise
- Long $T_F \Rightarrow$ smaller signal, but smaller noise
- We can try to use a time-variant filter, that operates only when the pulse is present (but we need to develop the theory first…)

Time-variant filters

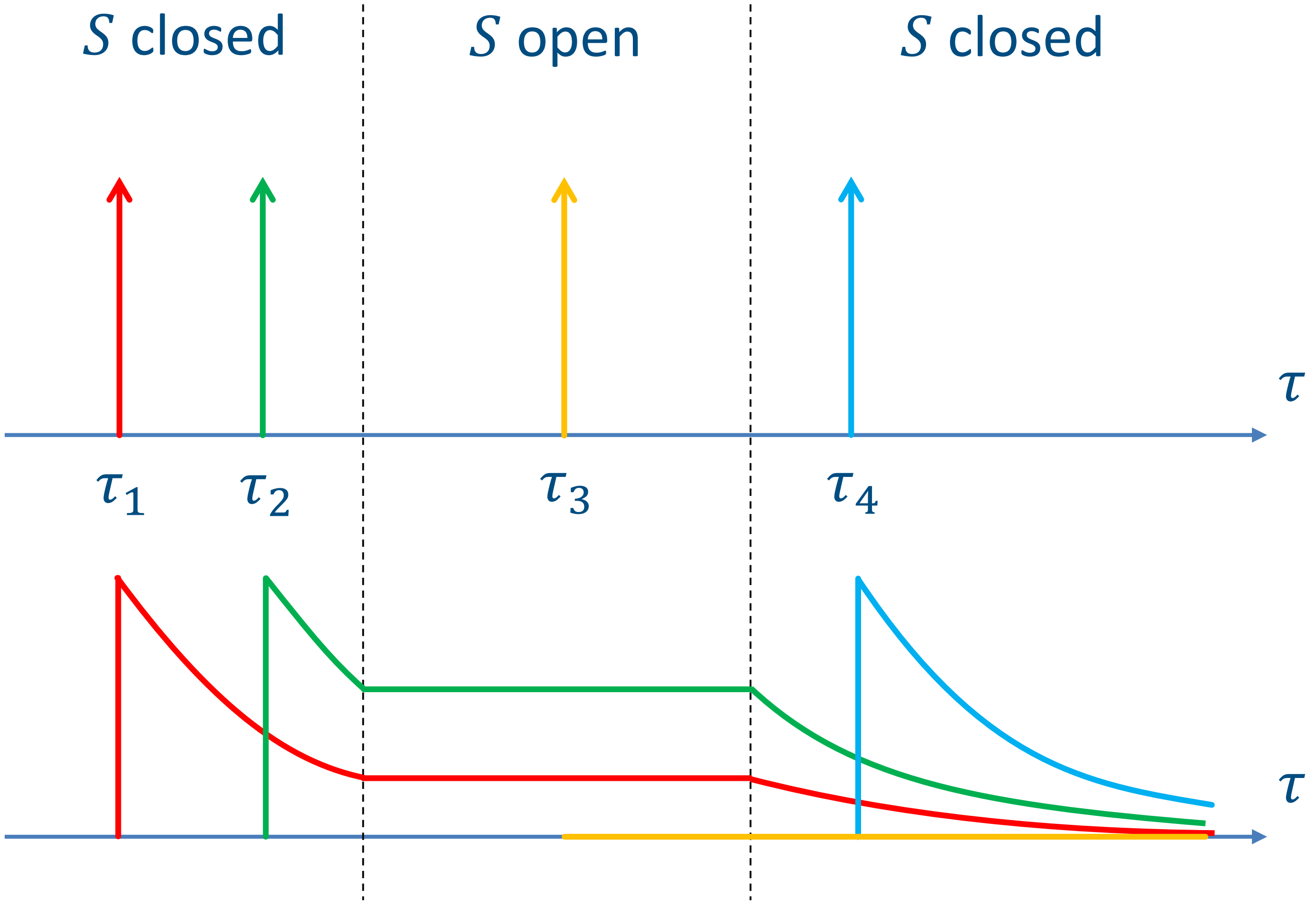
- The output depends explicitly on time
- A weighting function w can always be defined such that

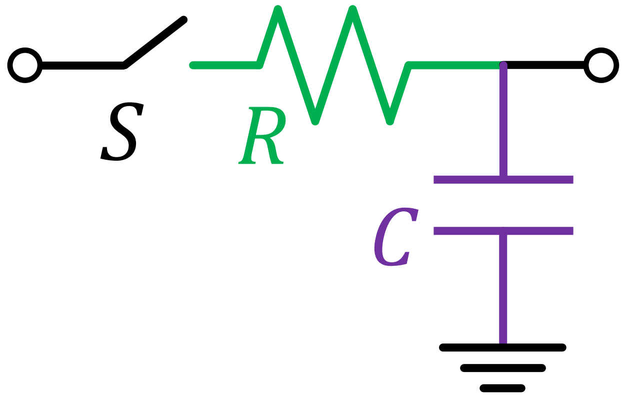
$$y(t) = \int x(\tau) w(t,\tau)\, d\tau$$

$\equiv 0 \; \forall \; t < \tau$, so integral may run from $-\infty$ to $+\infty$

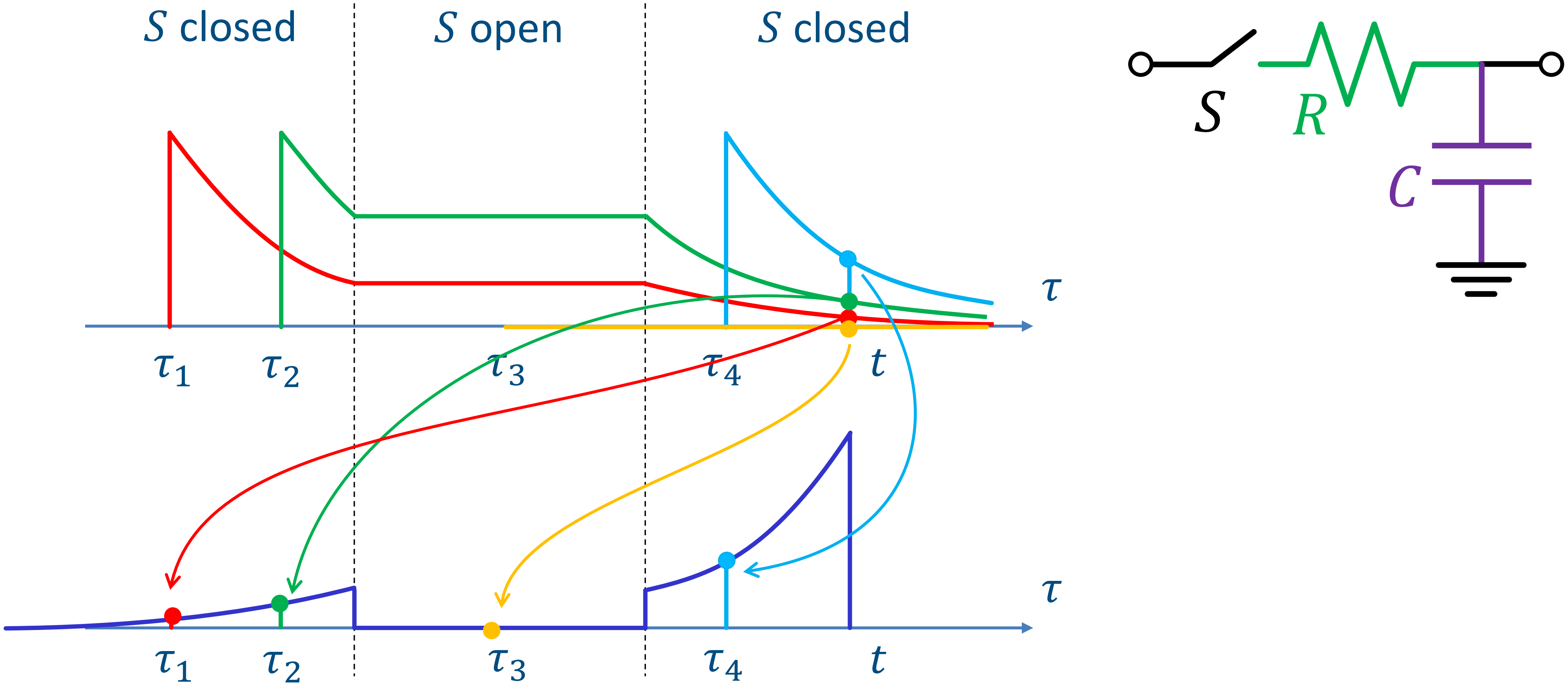
- If $x(\tau) = \delta(\tau - \tau_0), y(t) = w(t, \tau_0) \Rightarrow w(t,\tau)$ is still the system response at time t to a delta function applied in τ, but it is not the delta-function response shifted and reversed, i.e., $w(t,\tau) \neq h(t-\tau)$

Example 1: δ-function response

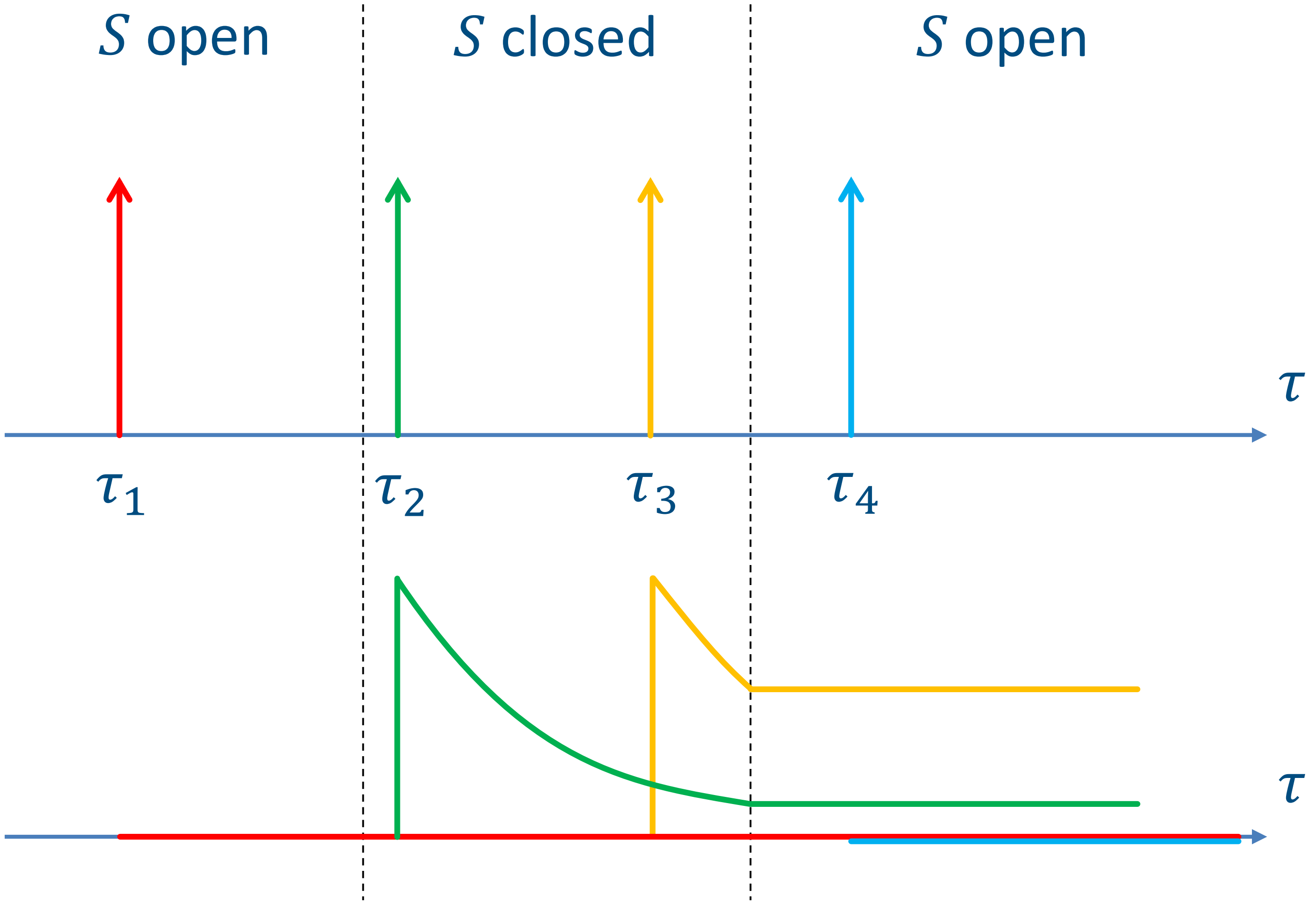


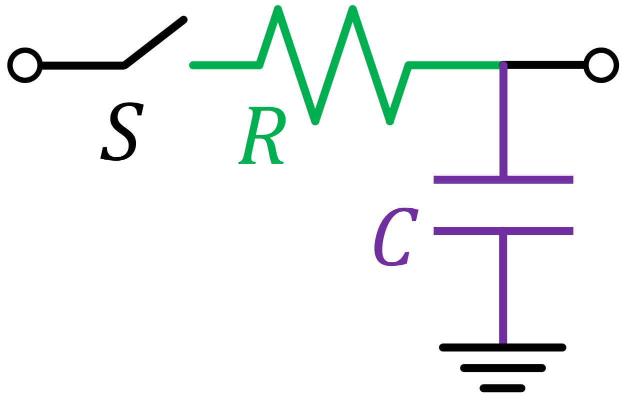


Example 1: weighting function

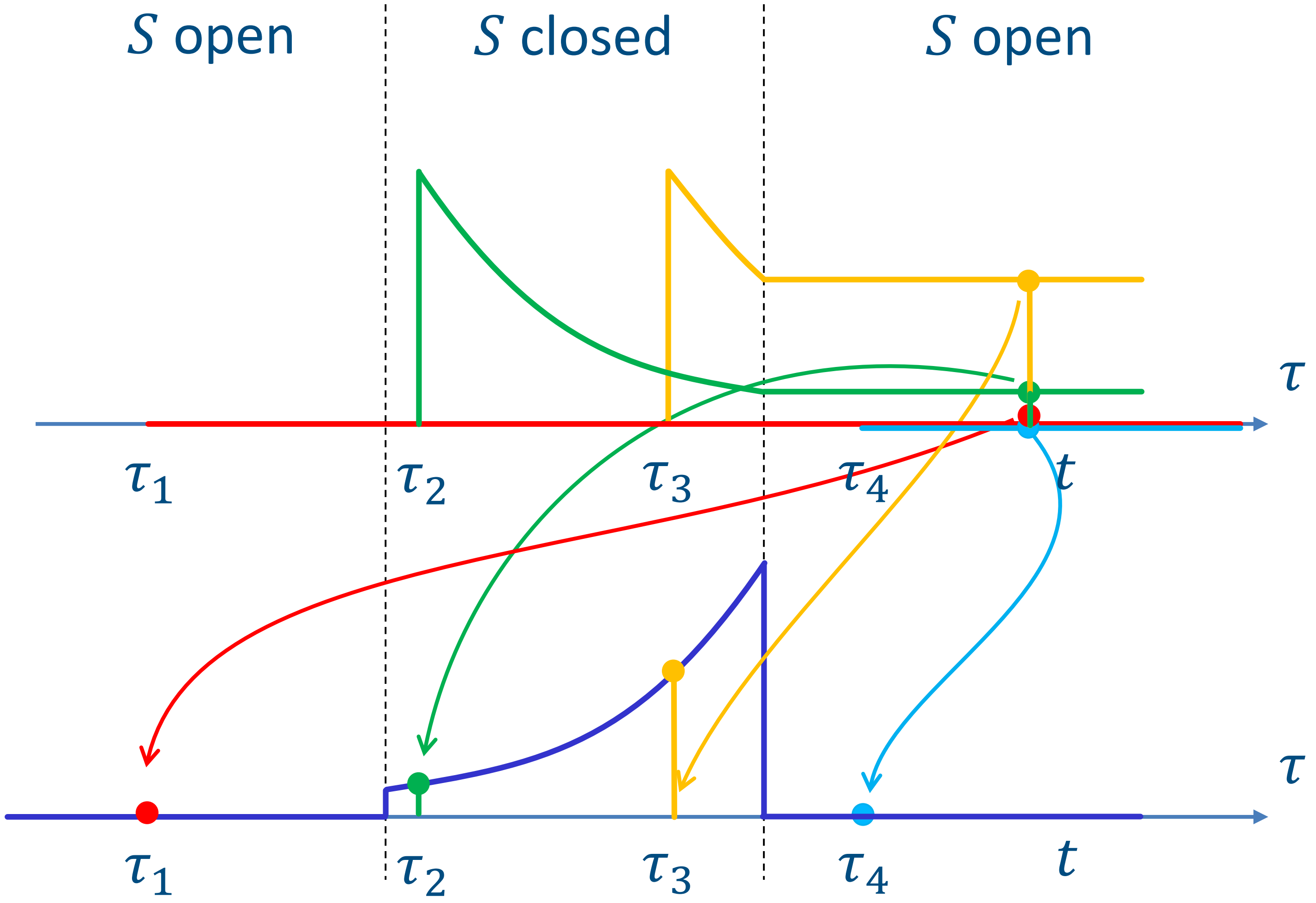


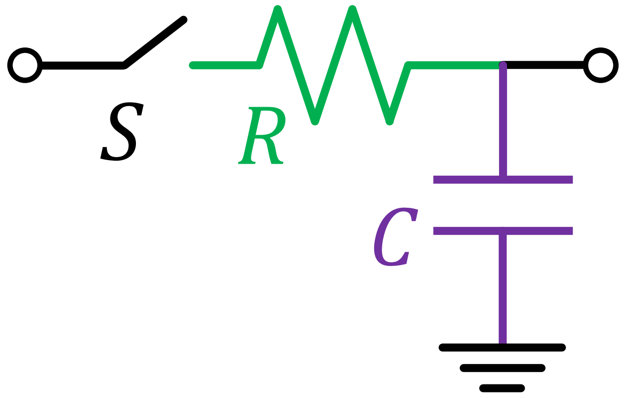
Example 2: δ-function response





Example 2: weighting function





Response to noise

$$R_{yy}(t_1,t_2) = \overline{y(t_1)y(t_2)} = \overline{\iint x(\alpha)w(t_1,\alpha)x(\beta)w(t_2,\beta)d\alpha d\beta}$$

$$= \iint R_{xx}(\alpha,\beta)w(t_1,\alpha)w(t_2,\beta)d\alpha d\beta$$

$$\overline{n_y^2(t_1)} = R_{yy}(t_1,t_1) = \iint R_{xx}(\alpha,\beta)w(t_1,\alpha)w(t_1,\beta)d\alpha d\beta$$

Stationary input noise

$$R_{yy}(t_1,t_2)=\overline{y(t_1)y(t_2)}=\iint R_{xx}(\beta-\alpha)w(t_1,\alpha)w(t_2,\beta)d\alpha d\beta$$

$$=\iint R_{xx}(\gamma)w(t_1,\alpha)w(t_2,\alpha+\gamma)d\alpha d\gamma$$

$$=\int R_{xx}(\gamma)d\gamma\int w(t_1,\alpha)w(t_2,\alpha+\gamma)d\alpha=\int R_{xx}(\gamma)k_{w_{12}}(\gamma)d\gamma$$

Output noise rms value

- For stationary input noise

$$\overline{n_y^2(t)} = R_{yy}(t,t) = \int R_{xx}(\gamma) k_{w_{tt}}(\gamma) d\gamma$$

$$k_{w_{tt}}(\gamma) = \int w(t,\alpha) w(t,\alpha+\gamma) d\alpha$$

- The output is noise is non-stationary, due to the time-variant nature of the filter

Frequency domain - signal

- From Parseval theorem

$$y(t) = \int x(\tau)w(t,\tau)\,d\tau = \int X(f)W^*(t,f)df$$

where

$$W(t,f) = \int w(t,\tau)e^{-j2\pi f\tau}d\tau$$

- No simple definition of $Y(f)$ can be provided in the general case

Frequency domain – noise

For stationary input noise

$$\overline{n_y^2(t)} = \int R_{xx}(\gamma) k_{w_{tt}}(\gamma) d\gamma = \int S_x(f) |W(t,f)|^2 df$$

The case of white stationary noise

$$R_{xx}(\gamma) = \lambda\delta(\gamma)$$

$$\overline{n_y^2(t)} = \int R_{xx}(\gamma)k_{w_{tt}}(\gamma)d\gamma = \lambda\int \delta(\gamma)k_{w_{tt}}(\gamma)d\gamma = \lambda k_{w_{tt}}(0)$$

$$= \lambda\int w^2(t,\alpha)d\alpha$$

Parseval theorem

$$S_x(f) = \lambda$$

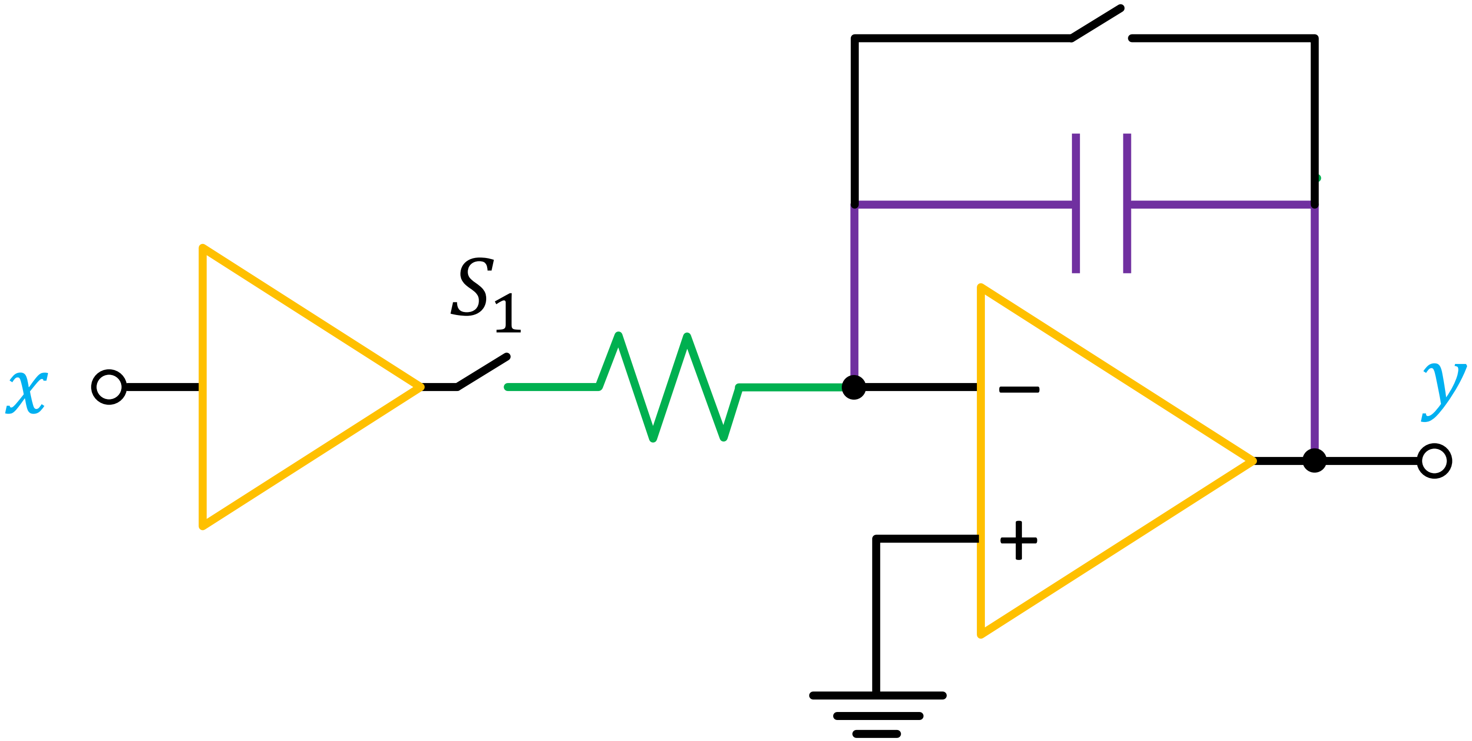
$$\overline{n_y^2(t)} = \int S_x(f)|W(t,f)|^2 df = \lambda\int |W(t,f)|^2 df$$

Outline

- Time-variant filters
- Gated integrators

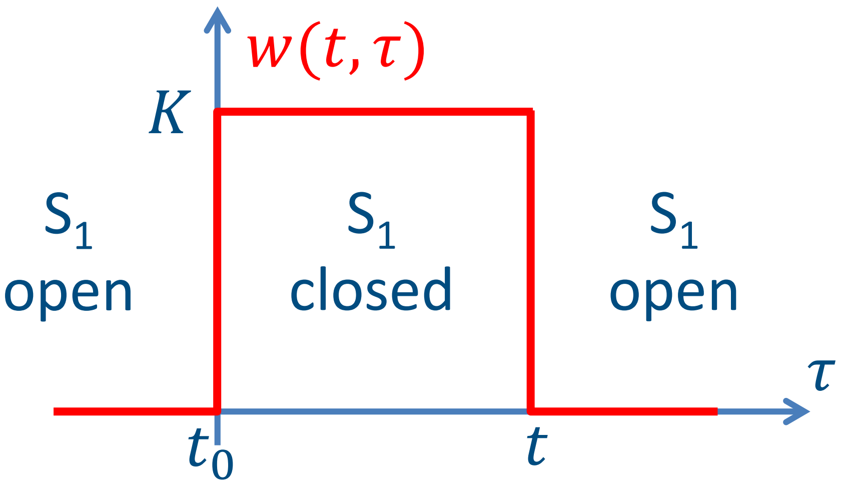
Gated integrator

Time-varying filter which integrates the input signal for a finite interval of time



Weighting function (time domain)

- Remember that $w(t,\tau)$ is the system response at time t to a delta function applied in τ

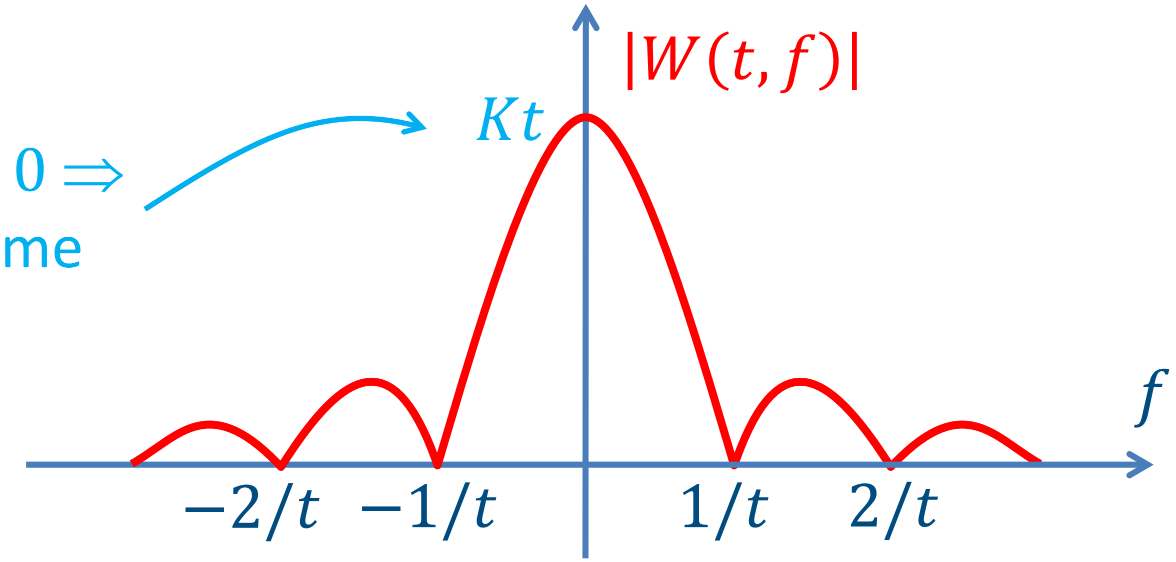


- Setting $t_0 = 0$ for simplicity we have

$$w(t,\tau) = K(\mathrm{u}(\tau) - \mathrm{u}(\tau - t))$$

Weighting function (frequency domain)

Value in $f = 0 \Rightarrow$ area in the time domain



Time shift of a factor $t/2$

$$W(t,f) = \mathcal{F}[w(t,\tau)] = Kt\,\mathrm{sinc}(\pi f t)\, e^{-j\pi f t}$$

(transform is carried out with respect to the variable τ)

Signal response

$$y(t) = \int x(\tau)w(t,\tau)d\tau = K\int_0^t x(\tau)d\tau = Kt\langle x\rangle$$

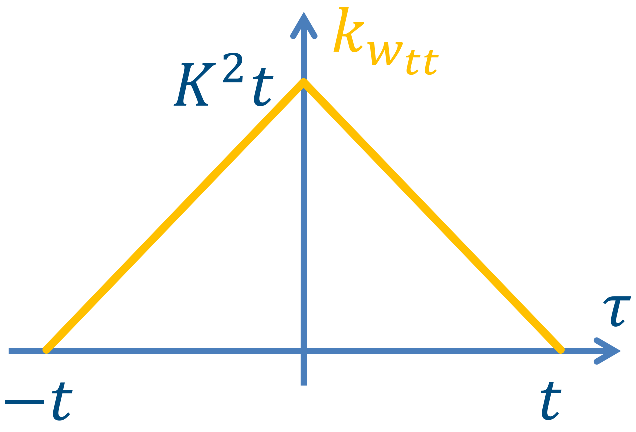
$$y(t) = Kt\int X(f)\,\mathrm{sinc}(\pi f t)\,e^{j\pi f t}df$$

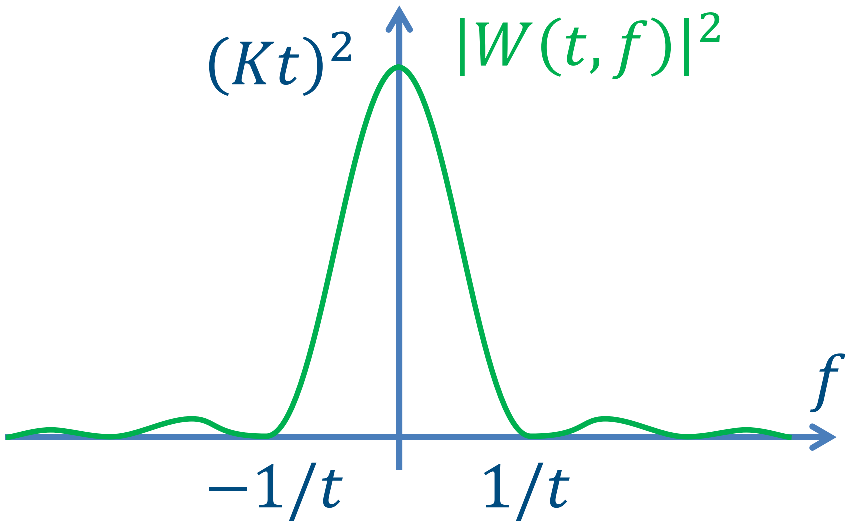
Results for LTV filters are usually easier in the time domain

Weighting function autocorrelation

$$k_{w_{tt}}(\tau) = \int w(t,\alpha)w(t,\alpha+\tau)d\alpha = K^2 t\,\mathrm{tri}(t)$$

$$|W(t,f)|^2 = K^2 t^2\,\mathrm{sinc}^2(\pi f t)$$





Output noise

- Time domain

$$\overline{n_y^2} = \lambda k_{w_{tt}}(0) = \lambda t K^2$$

- Frequency domain:

$$\overline{n_y^2} = \lambda \int |W(t,f)|^2 df = \lambda K^2 t^2 \int \text{sinc}^2(\pi f t)\, df = \lambda t K^2$$

$= 1/t$

- The noise equivalent BW of the gated integrator is $BW_n = \frac{1}{2t}$

Input and output S/N

- Constant pulse over the integration time and quasi-white noise with equivalent bandwidth $f_n = 1/2T_n$ at the input:

$$\left(\frac{S}{N}\right)_x = \frac{V_i}{\sqrt{\overline{n_x^2}}} = \frac{V_i}{\sqrt{2\lambda f_n}} = V_i\sqrt{\frac{T_n}{\lambda}}$$

- At the output we have:

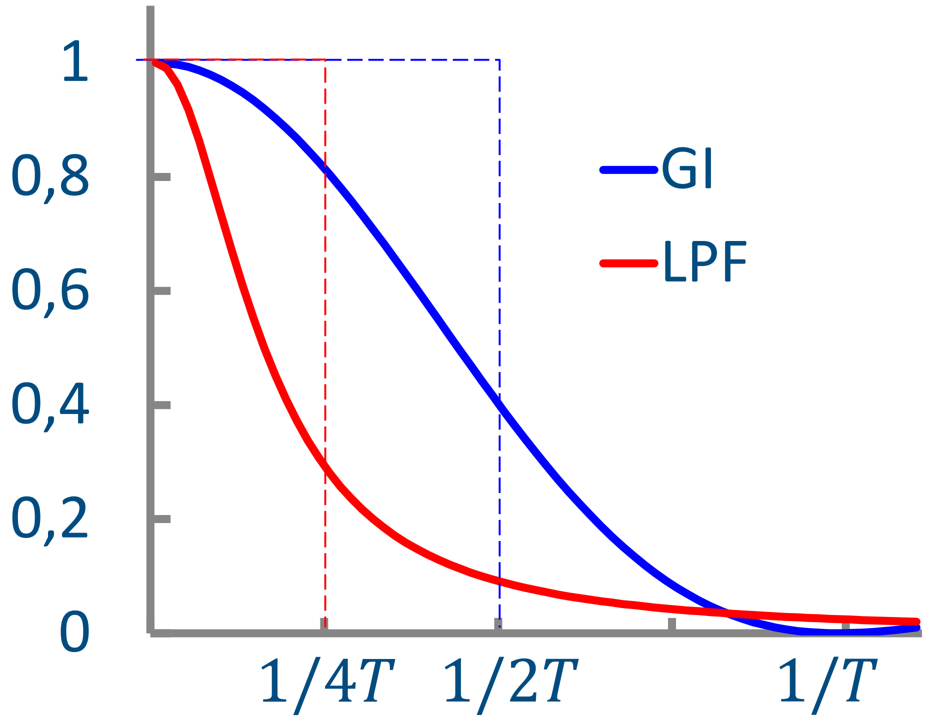
$$\left(\frac{S}{N}\right)_y = \frac{V_y}{\sqrt{\overline{n_y^2}}} = \frac{V_i K t}{\sqrt{\lambda t K^2}} = V_i\sqrt{\frac{t}{\lambda}}$$

Improvement of S/N

$$\left(\frac{S}{N}\right)_y = \left(\frac{S}{N}\right)_x \sqrt{\frac{t}{T_n}} = \left(\frac{S}{N}\right)_x \sqrt{\frac{f_n}{BW_n}}$$

Comparison against LP filter

We want to compare the noise bandwidths of the filters $\Rightarrow$ same gain at $f = 0$



$$BW_n(GI) = \frac{1}{2T_G}$$

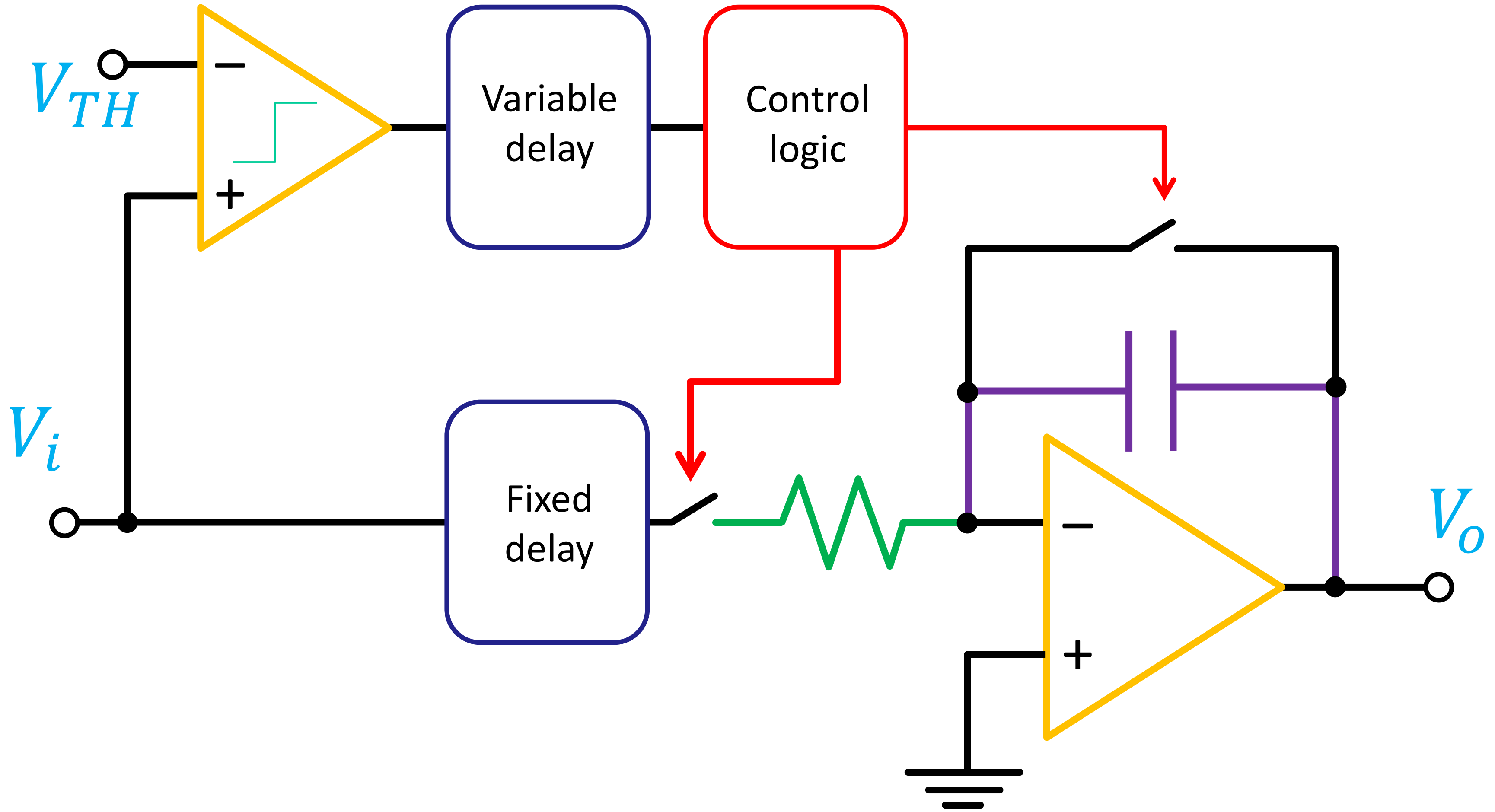
$$BW_n(LPF) = \frac{1}{4T_F}$$

To achieve the same S/N, we must set $T_F = T_G/2$

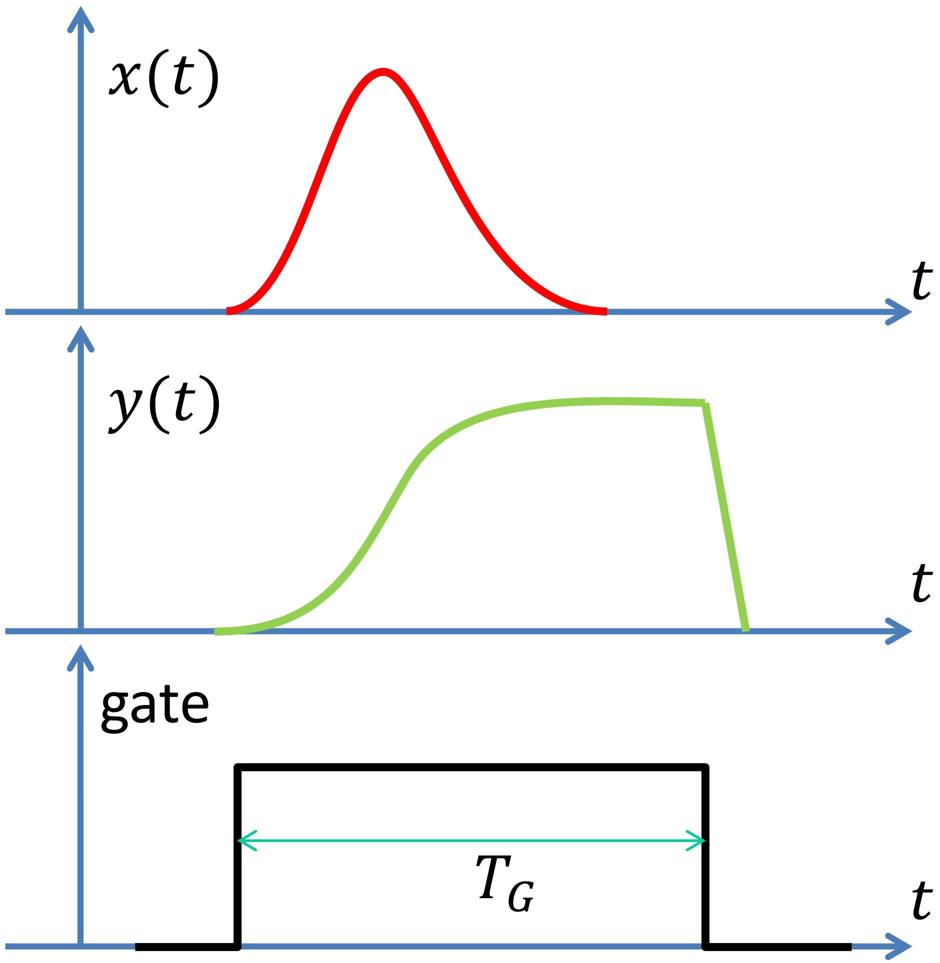
Comments

- An LPF is always present in an acquisition system to cut unwanted high-frequency noise components
- GIs are useful for fast signals (pulses), although they can also be employed for DC signals
- GIs effectively suppress frequency components $f_n = n/t \Rightarrow$ useful to reject power supply disturbs or interferences

Simplified scheme



I/O waveforms



T_G value is chosen so as to optimize S/N

Typical parameters

- Gate width (typ. from 1-2 ns to several µs – can go down to about 100 ps for fast samplers)
- Gain (typ. from 1 to 1000)
- Dead time (a few µs typ.) or Trigger Rate (typ. < 50 kHz)
- Linearity, offset,…

From [1]

References

1. https://www.thinksrs.com/products/sr255.html