Electronics – 96032

POLITECNICO DI MILANO

Boxcar Averagers

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Disclaimer

Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes

Purpose of the lesson

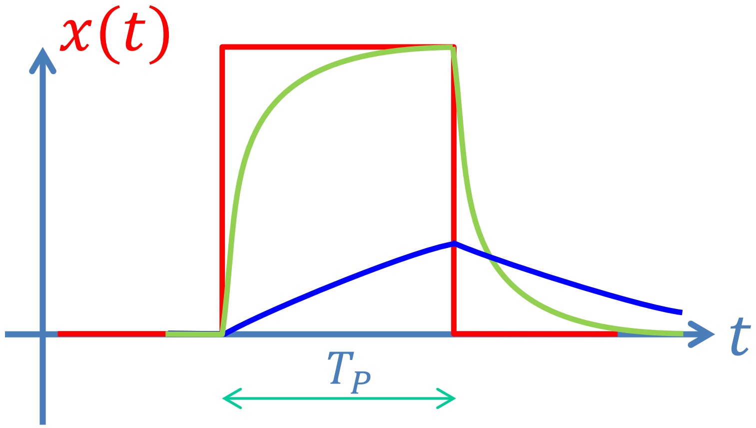
- It's time to begin a discussion on the techniques for improving S/N
- Noise-reduction techniques obviously depend on the type of signal and of noise:

		Noise	
		HF (White)	LF (flicker)
Signal	LF (constant)	previous lesson	next lessons
	HF (pulse)	this lesson	next lessons

Outline

- Boxcar averagers
- Ratemeters

Fast pulse + white noise and LPF

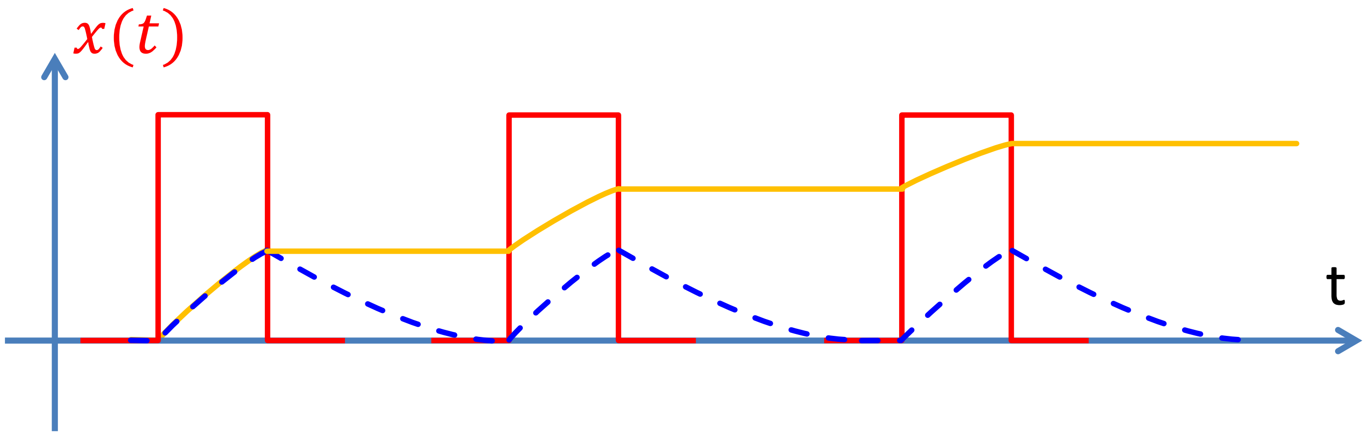


$$\left(\frac{S}{N}\right)_{LPF} = A\frac{1-e^{-T_P/T_F}}{\sqrt{\lambda/2T_F}}$$

$$\left(\frac{S}{N}\right)_{GI} = A\sqrt{\frac{T_P}{\lambda}}$$

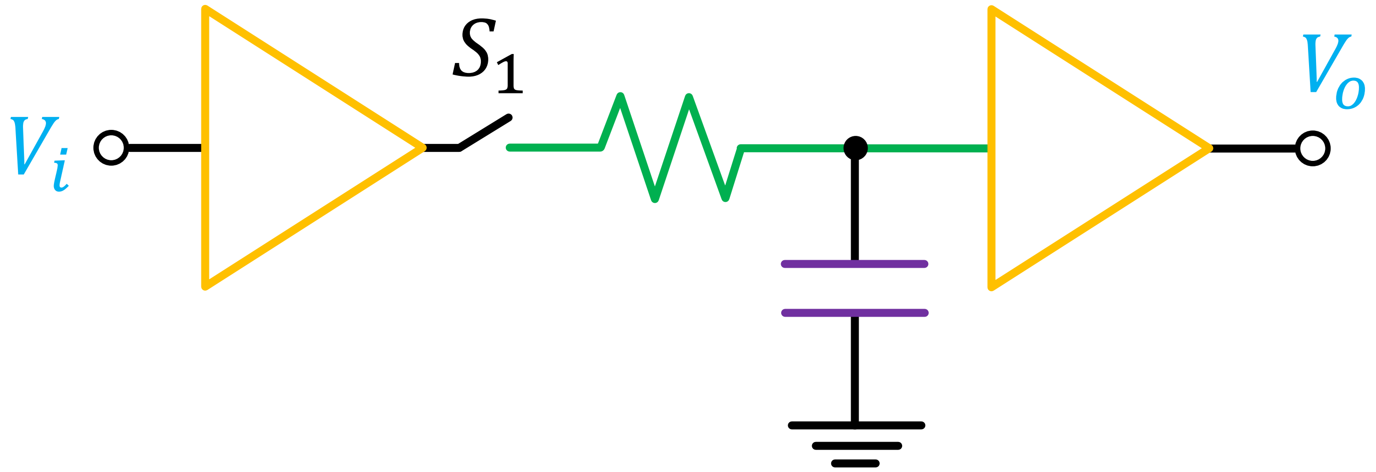
- Short $T_F \Rightarrow$ larger signal, but larger noise
- Long $T_F \Rightarrow$ smaller signal, but smaller noise
- GIs do not increase appreciably S/N if T_P is small
- Can we retain the advantage of a long T_F without sacrificing the signal?

Repetitive pulses



- If the capacitor is not discharged after each pulse, we can gain signal
- S/N improves because we are now averaging over many pulses
- Still a time-variant filter!

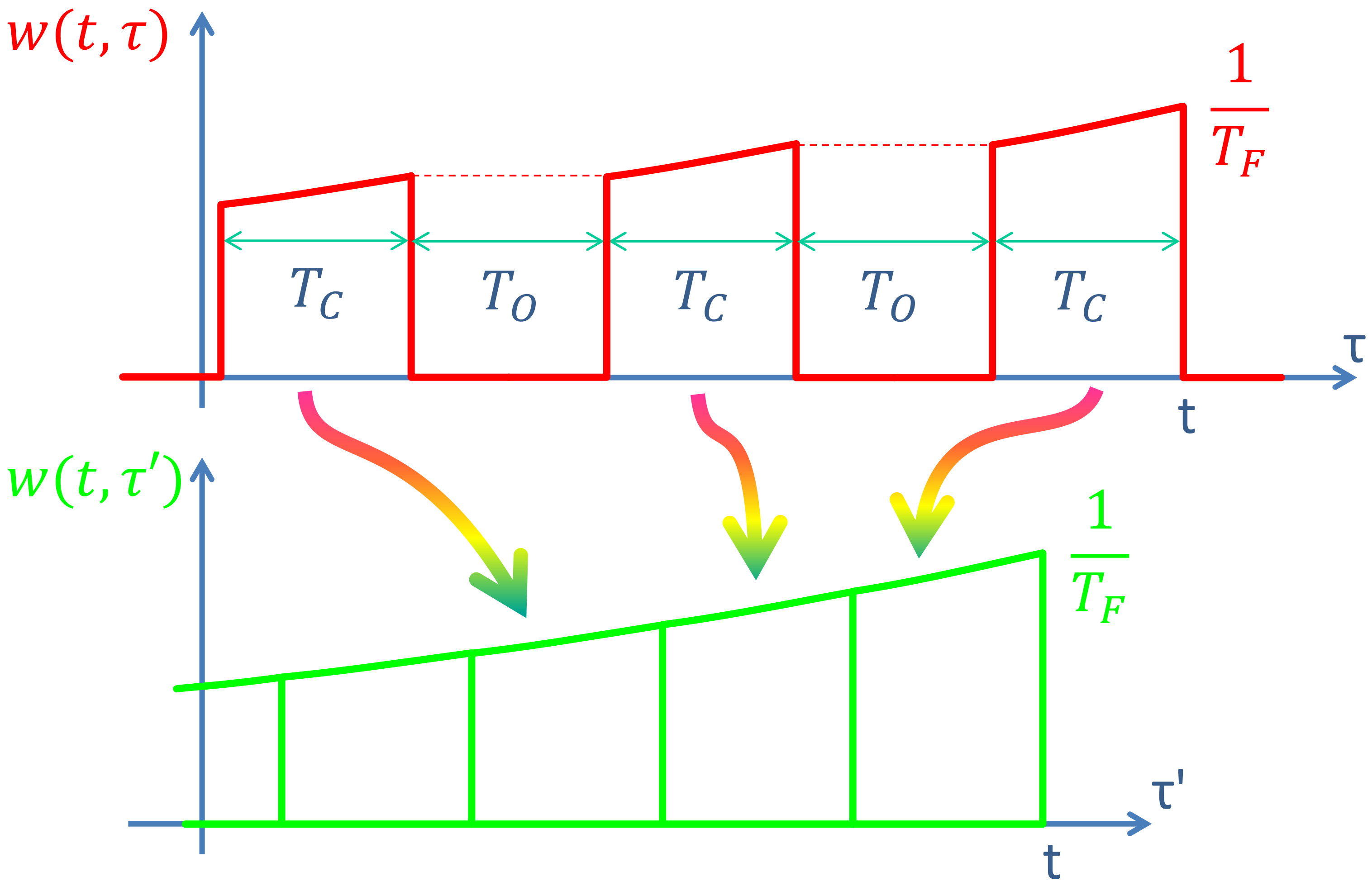
Boxcar averager



- Switch closed ⇒ LPF
- Switch open ⇒ the voltage stored on the capacitor cannot decrease

Weighting function

(actual behavior may not be periodic)



Filter behavior

- In the «equivalent time» τ' we have

$$w(t,\tau') = \frac{1}{T_F} e^{-\frac{t-\tau'}{T_F}} \mathrm{u}(t-\tau')$$

- For white input noise we have:

$$\overline{n_y^2} = \lambda \int w^2(t,\tau)d\tau = \lambda \int w^2(t,\tau')d\tau'$$

and the filter behaves exactly as an LPF and has the same S/N

Improvement of S/N

- The improvement is the same as the LPF:

$$\left(\frac{S}{N}\right)_{out} = \left(\frac{S}{N}\right)_{in} \sqrt{\frac{2T_F}{T_n}}$$

- The output depends on $x(\tau')$ only, i.e., on the input signal when the switch is closed
- The time to reach steady-state depends on T_C and T_O (i.e., on the sampling rate) but the performance does not

Single-pulse BA

- We consider $T_C \ll T_F$ so that we can neglect the discharge over the time interval T_C
- A single-pulse boxcar is then equivalent to a GI with gate time T_C and gain $K = 1/T_F$:

$$\overline{n_y^2} = \lambda \frac{T_C}{T_F^2} \qquad\qquad y = A \frac{T_C}{T_F}$$

$$\left(\frac{S}{N}\right)_{sp} = \left(\frac{S}{N}\right)_{in} \sqrt{\frac{T_C}{T_n}}$$

Equivalent number of samples

$$\left(\frac{S}{N}\right)_{BA} = \left(\frac{S}{N}\right)_{sp} \sqrt{\frac{2T_F}{T_C}} = \left(\frac{S}{N}\right)_{sp} \sqrt{N_{eq}}$$

$$N_{eq} = \frac{2T_F}{T_C}$$

N_{eq} represent the improvement in S/N due to the exponential average and is called the equivalent number of samples

General case – signal

- We address the case in which $T_C \ll T_F$ does not hold
- The weighting function of a single-pulse boxcar is now

$$w(t,\tau) = \frac{1}{T_F} e^{-\frac{t-\tau}{T_F}} \quad t - T_C \leq \tau \leq t$$

- For an input signal nearly constant over T_C we have then

$$y = A \int_{t-T_C}^{t} \frac{1}{T_F} e^{-\frac{t-\tau}{T_F}} d\tau = A \int_{-T_C}^{0} \frac{1}{T_F} e^{\frac{\gamma}{T_F}} d\gamma = A\left(1 - e^{-\frac{T_C}{T_F}}\right)$$

General case – noise

For a white stationary input noise we have

$$\overline{n_y^2} = \lambda k_{w_{tt}}(0) = \lambda \int w^2(t,\tau)d\tau = \frac{\lambda}{T_F^2}\int_{t-T_C}^{t} e^{-\frac{2(t-\tau)}{T_F}} d\tau$$

$$= \frac{\lambda}{T_F^2}\int_{-T_C}^{0} e^{\frac{2\gamma}{T_F}} d\gamma = \frac{\lambda}{2T_F}\left(1 - e^{-\frac{2T_C}{T_F}}\right)$$

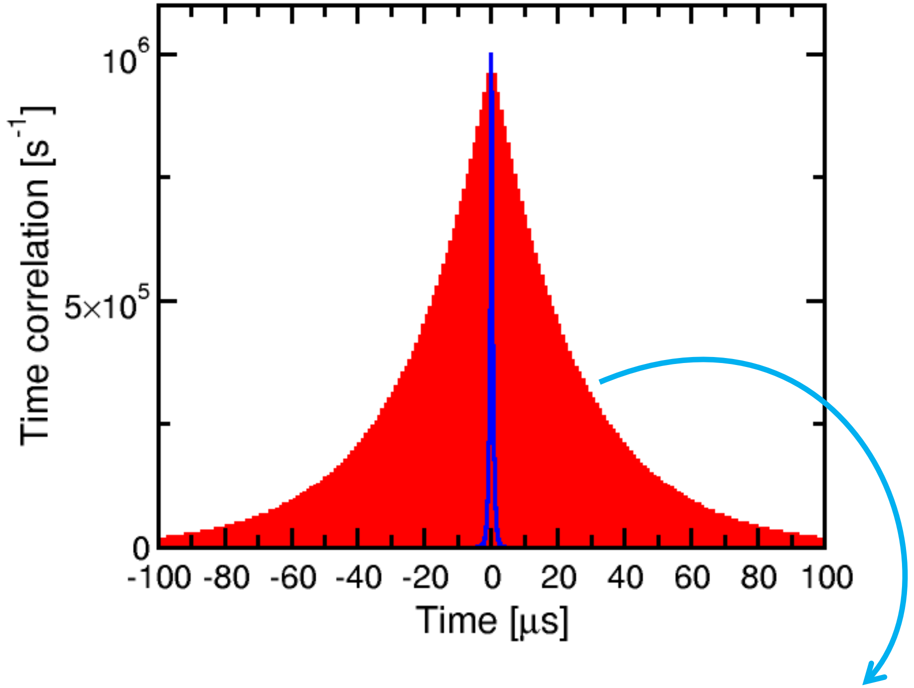
General case – S/N and N_{eq}

$$\left(\frac{S}{N}\right)_{sp} = \frac{A\left(1-e^{-\frac{T_C}{T_F}}\right)}{\sqrt{\frac{\lambda}{2T_F}\left(1-e^{-\frac{2T_C}{T_F}}\right)}} = \left(\frac{S}{N}\right)_{BA} \frac{1-e^{-\frac{T_C}{T_F}}}{\sqrt{1-e^{-\frac{2T_C}{T_F}}}}$$

$$N_{eq} = \frac{1-e^{-\frac{2T_C}{T_F}}}{\left(1-e^{-\frac{T_C}{T_F}}\right)^2} = \frac{1+e^{-\frac{T_C}{T_F}}}{1-e^{-\frac{T_C}{T_F}}} \approx \frac{2T_F}{T_C} \quad (\text{if } T_C \ll T_F)$$

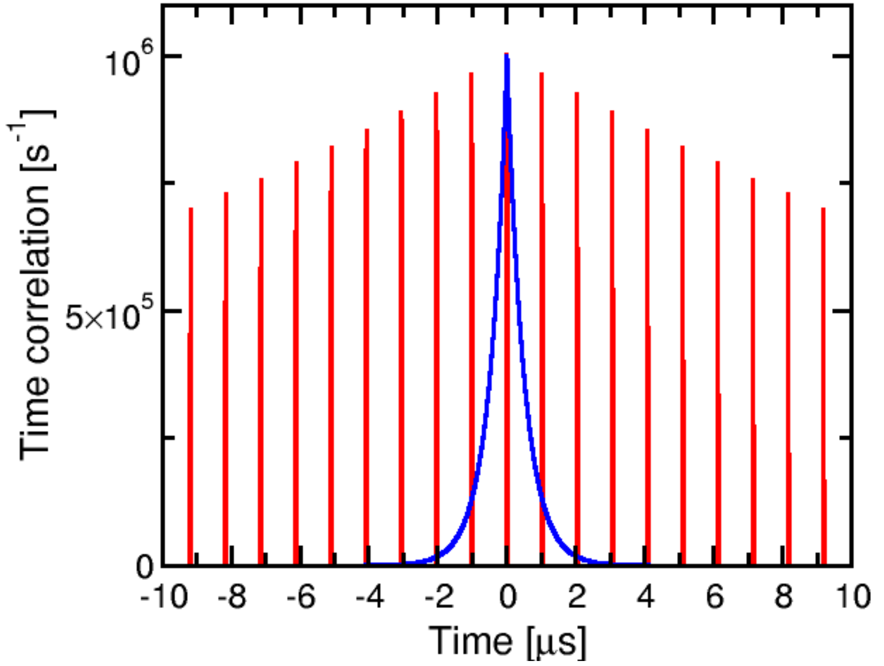
Correlation function (periodic case)

$$T_C = 20\ \text{ns},\ T_O = 1\ \mu\text{s},\ T_F = 0.5\ \mu\text{s}$$



Time constant of envelope

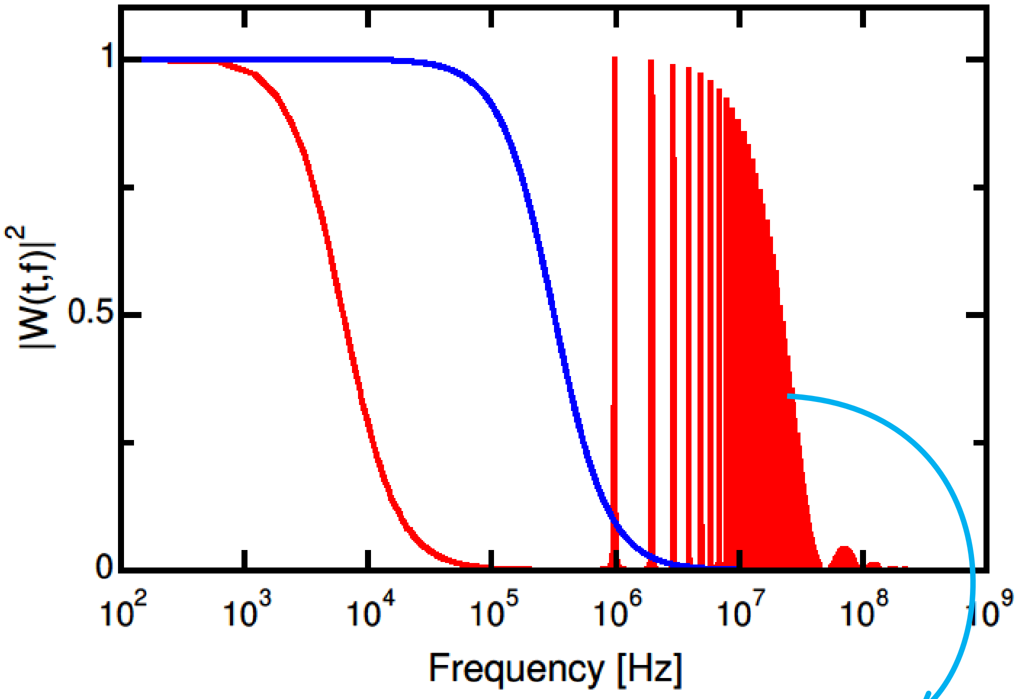
$$\approx T_F(T_C + T_O)/T_C$$

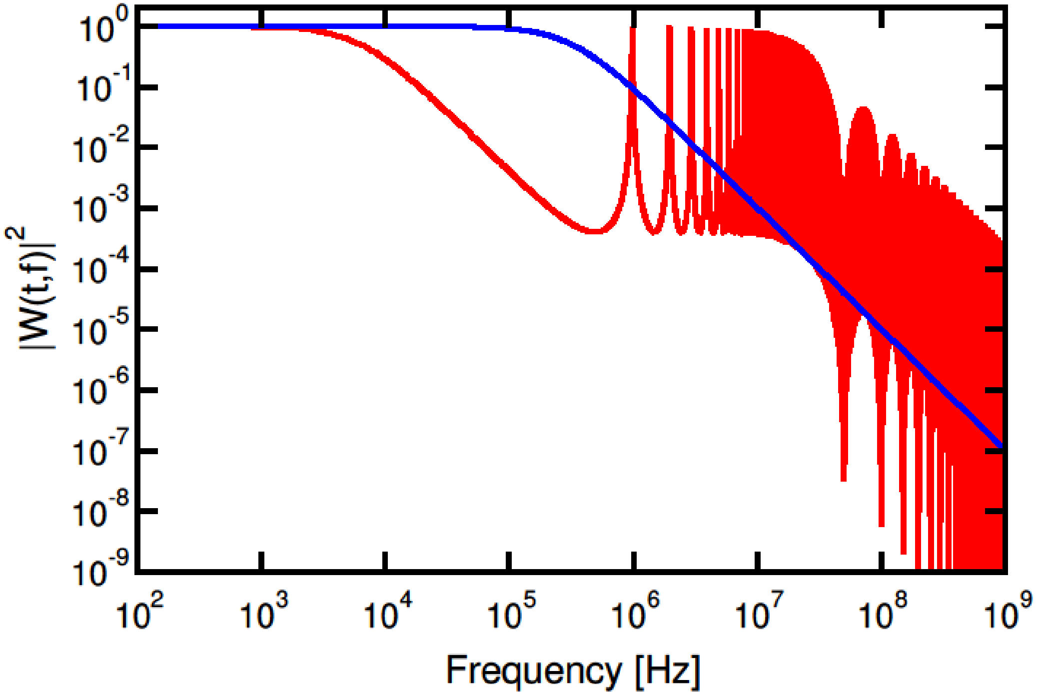


$$k_{hh}(\gamma) = \frac{1}{2T_F} e^{-|\gamma|/T_F}$$

Frequency domain

$T_C = 20\text{ ns}, T_O = 1\ \mu\text{s}, T_F = 0.5\ \mu\text{s}$

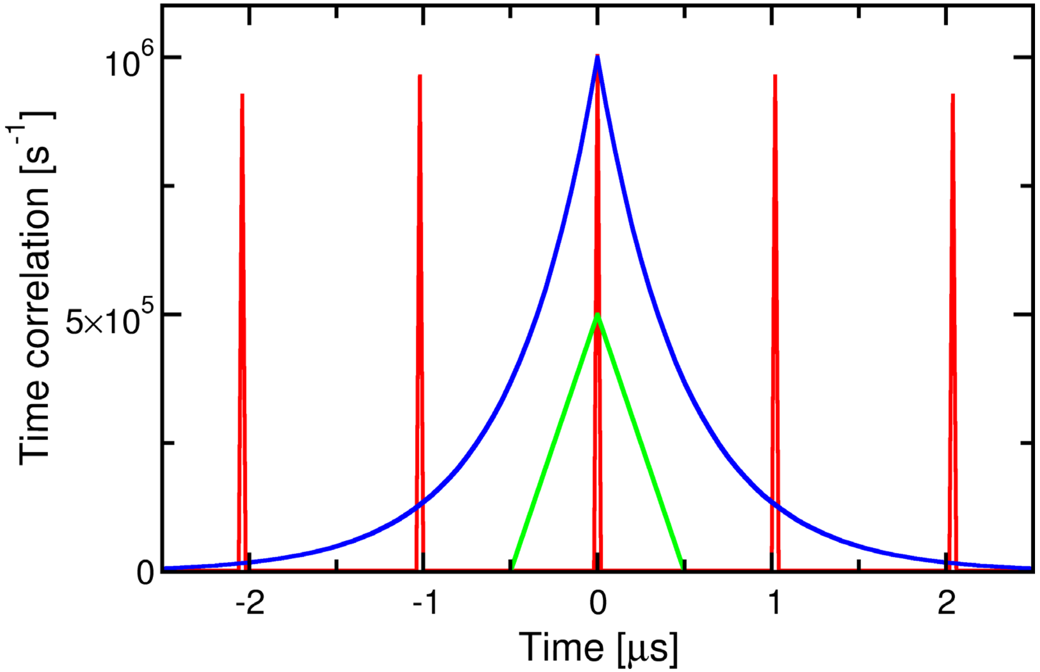


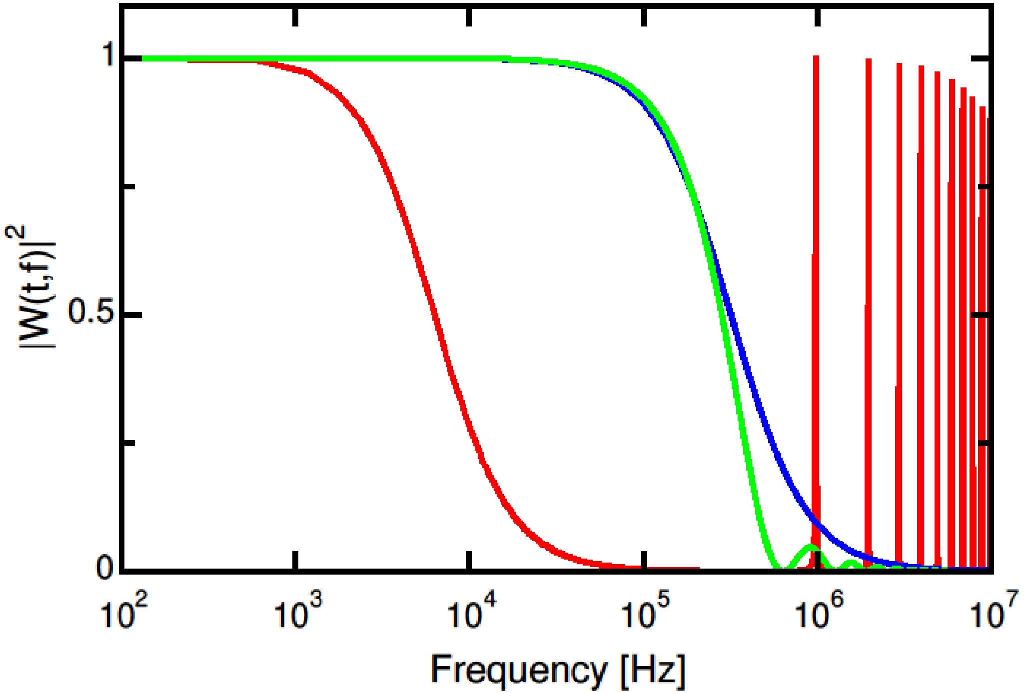


The envelope is nearly the single-pulse transform $\approx \text{sinc}^2\,(\pi f T_C)$

$$|W_{LP}|^2 = \frac{1}{1 + (2\pi f T_F)^2}$$

Effect on correlated noise





$$\overline{n_y^2(t)} = \int R_{xx}(\gamma) k_{w_{tt}}(\gamma) d\gamma = \int S_x(f) |W(t,f)|^2 df$$

Effect on correlated noise

- If the input noise is non-white and correlated over a time T_C – but not significantly so over $T_C + T_O$ – BA can provide a much better S/N than LPF
- For white noise, the two filters give the same output noise (value in $\tau = 0$ of the WF autocorrelations is the same)

Main typical parameters

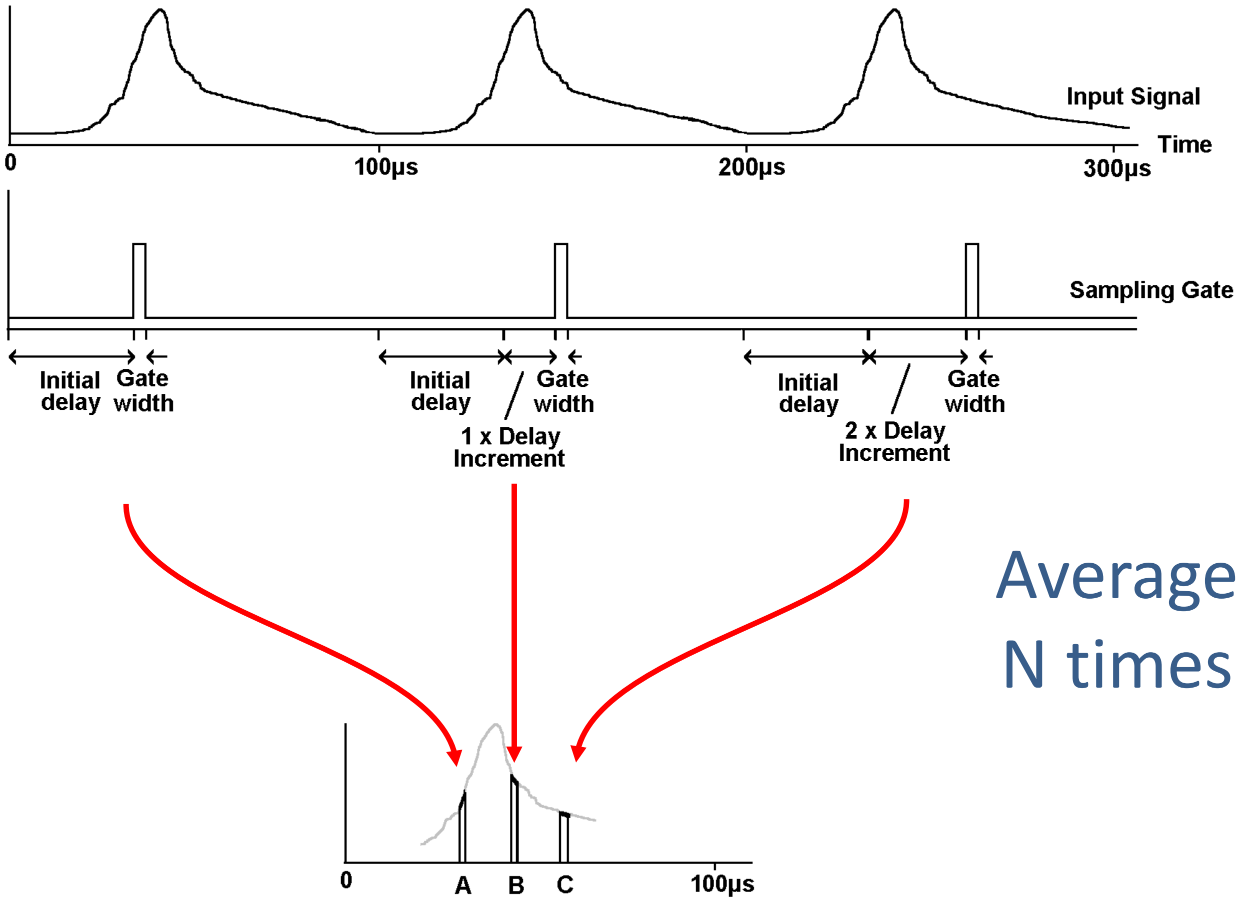
- Gate width (typ. from 1 – 2 ns to 20 – 30 µs; can be even shorter in fast samplers)
- Number of samples (from 1 to several 1000s)
- Delay (intrinsically 10 – 15 ns; then from 3 to 300 ns typ. higher values are possible by custom modification)
- Trigger rate (typ. lower than 100 kHz)

Waveform recovery mode

- Trigger delay is not fixed but rather is incremented so that it sweeps between an initial and a final value
- Each “point” is repeated N times to improve S/N and the data is sent to output
- In this way, the input waveform is reconstructed (equivalent-time sampling)

Waveform recovery mode

From [1]

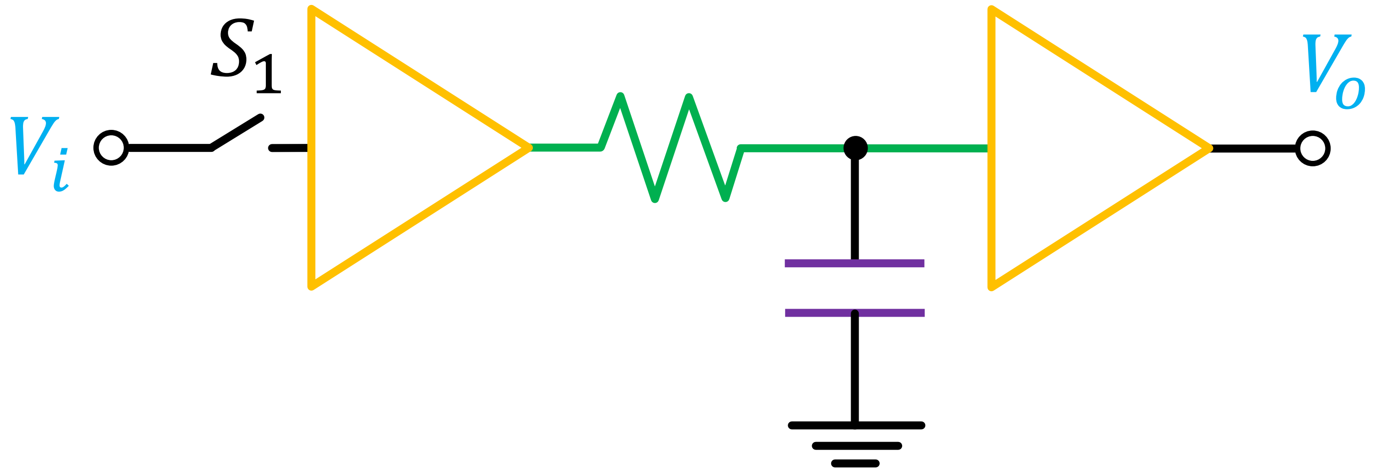


Average
N times

Outline

- Boxcar averagers
- Ratemeters

Ratemeter

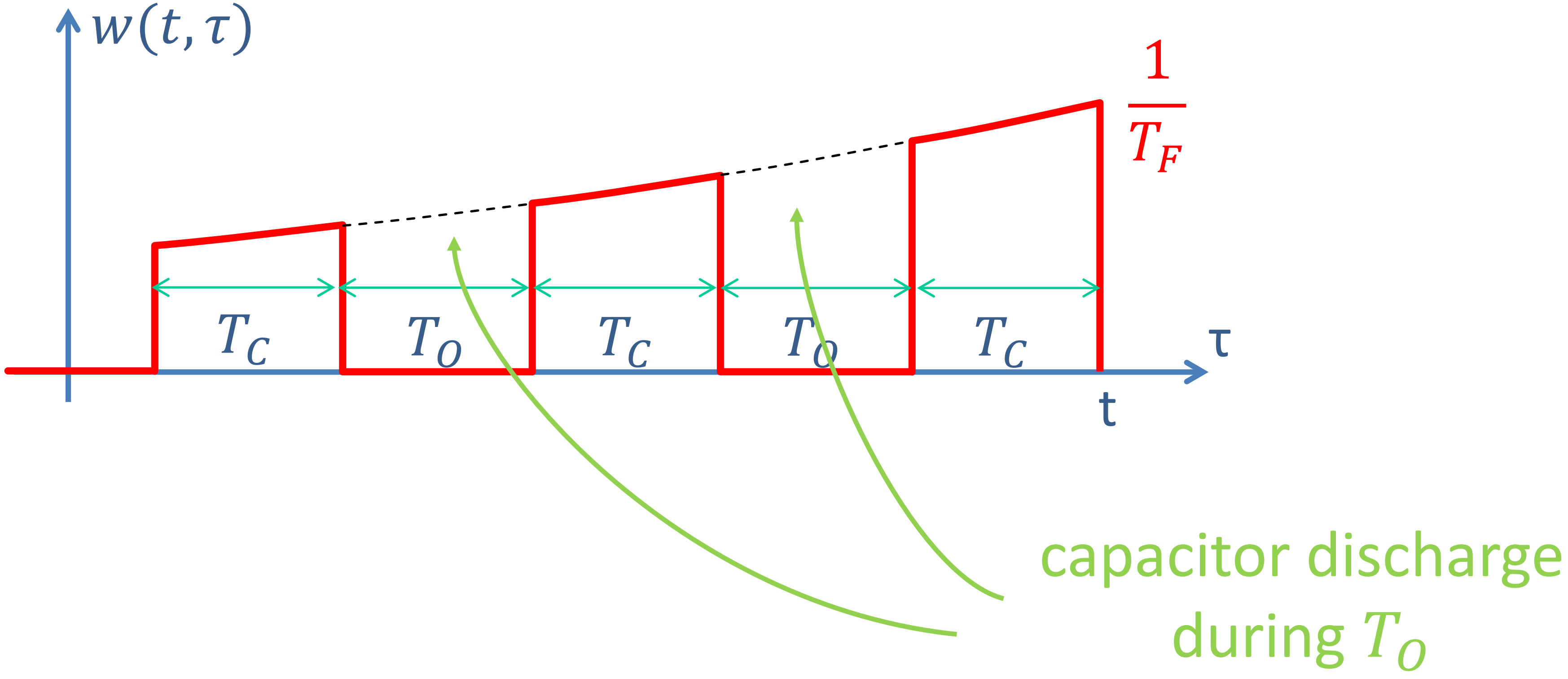


S_1 blocks the input signal but does not stop the capacitor discharge

$\Rightarrow$ this is not a boxcar

Weighting function

(actual behavior may not be periodic)



Output signal

- The contribution of the n-th pulse is $(t = 0)$

$$y^n = A\int_{-n(T_C+T_O)-T_C}^{-n(T_C+T_O)} \frac{e^{\tau/T_F}}{T_F}d\tau = A\left(1-e^{-\frac{T_C}{T_F}}\right)e^{-\frac{n(T_C+T_O)}{T_F}}$$

- The output becomes

$$y = \sum_{n=0}^{\infty} y^n = A\left(1-e^{-\frac{T_C}{T_F}}\right)\sum_{n=0}^{\infty}\left(e^{-\frac{T_C+T_O}{T_F}}\right)^n = A\frac{1-e^{-\frac{T_C}{T_F}}}{1-e^{-\frac{T_C+T_O}{T_F}}}$$

Output noise (input WN case)

$$k_{w_{tt}}^{n}(0)=\int_{-n(T_C+T_O)-T_C}^{-n(T_C+T_O)}\frac{e^{2\tau/T_F}}{T_F^2}d\tau=\frac{1-e^{-\frac{2T_C}{T_F}}}{2T_F}e^{-\frac{2n(T_C+T_O)}{T_F}}$$

$$k_{w_{tt}}(0)=\sum_{n=0}^{\infty}k_{w_{tt}}^{n}(0)=\frac{1}{2T_F}\frac{1-e^{-\frac{2T_C}{T_F}}}{1-e^{-\frac{2(T_C+T_O)}{T_F}}}$$

$$\overline{n_{out}^2}=\lambda k_{w_{tt}}(0)$$

S/N

$$\left(\frac{S}{N}\right)_{out} = A\sqrt{\frac{2T_F}{\lambda}}\,\frac{1-e^{-\frac{T_C}{T_F}}}{\sqrt{1-e^{-\frac{2T_C}{T_F}}}}\,\frac{\sqrt{1-e^{-\frac{2(T_C+T_O)}{T_F}}}}{1-e^{-\frac{T_C+T_O}{T_F}}}$$

boxcar

single-pulse boxcar

effect of the (exponential) average

Equivalent number of samples

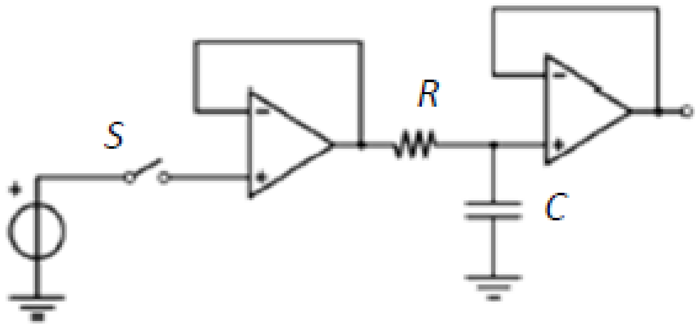
$$N_{eq} = \frac{1 + e^{-\frac{T_C + T_O}{T_F}}}{1 - e^{-\frac{T_C + T_O}{T_F}}} \approx \frac{2T_F}{T_C + T_O}$$

If $T_C + T_O \ll T_F$

The equivalent number of samples and the whole filter performance (signal and noise) depend on T_O.

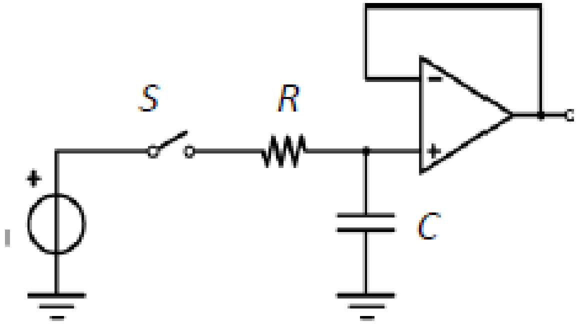
S/N is always smaller than the boxcar's

BA and RI: passive circuit comparison



RATEMETER INTEGRATOR

- Switch S acts as gate on the input source
- Switch S is decoupled from the RC passive filter by the voltage buffer
- The RC integrator is unaffected by S, it has constant parameters, it does NOT have a HOLD state
- The sample average is done on a given time, defined by the RC value



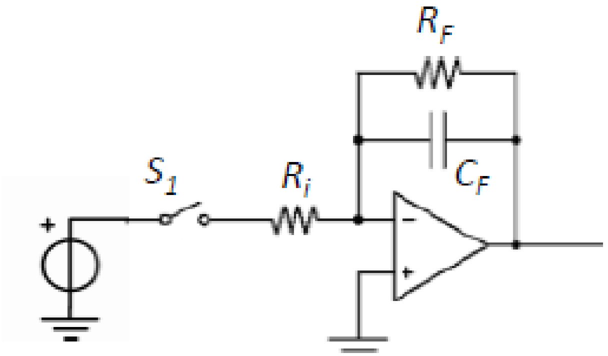
BOXCAR INTEGRATOR

- Switch S acts as gate on the input source
- Switch S acts also on the RC passive filter (changes the resistor value)
- The time constant T_F of the integrator filter is switched from finite RC (S-down) to infinite (S-up, HOLD state)
- The sample average is done on a given number of samples, defined by the T_F/T_G value

From [2]

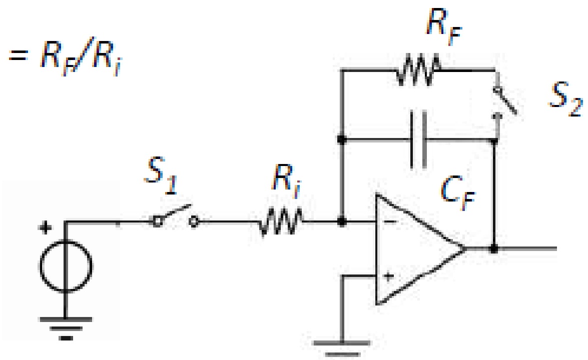
BA and RI: active circuit comparison

DC gain $G = R_F/R_i$



RATEMETER INTEGRATOR

- Switch S_1 acts as gate on the input
- Switch S_1 is decoupled from the active RC integrator by the buffer action of the OP-AMP virtual ground
- The $R_F C_F$ integrator is unaffected by S_1; it has constant parameters, it does NOT have a HOLD state
- The sample average is done on a given time, defined by the $R_F C_F$ value



BOXCAR INTEGRATOR

- Switch S_1 acts as gate on the input
- Switch S_1 is decoupled from the active RC integrator by the buffer action of the OP-AMP virtual ground
- A second switch S_2 is required for switching the time constant T_F of the integrator from finite $R_F C_F$ (S_2-down) to infinite (S_2-up, HOLD state)
- The sample average is done on a given number of samples, defined by the T_F/T_G value

From [2]

References

1. www.signalrecovery.com/download/220518-A-MNL-D.pdf
2. http://home.deib.polimi.it/cova/elet/lezioni/SSN05c_Filters_LPF3.pdf