Electronics – 96032

POLITECNICO DI MILANO

    

LF Noise Filtering: HPF and Baseline Restorers

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Disclaimer

Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes

Purpose of the lesson

- It is time to begin a discussion on the techniques for improving S/N
- Noise-reduction techniques obviously depend on the type of signal and of noise:

		Noise	
		HF (White)	LF (flicker)
Signal	LF (constant)	done	next lessons
Signal	HF (pulse)	done	this lesson

Outline

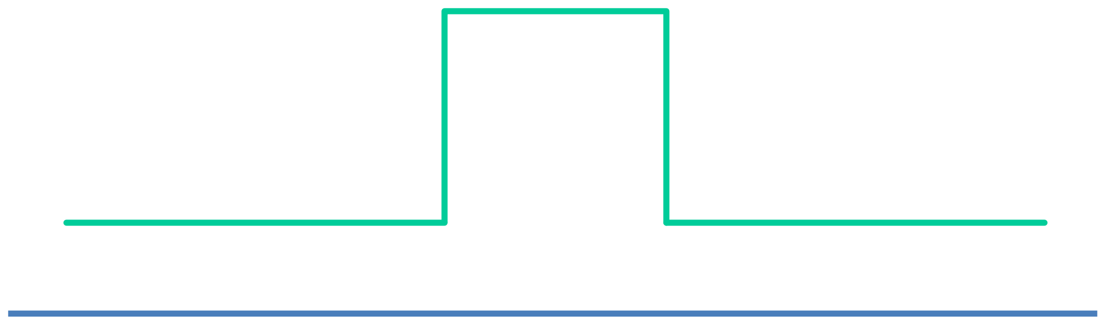
- HP filters
- Baseline restorers
- Appendix: FN LP and BP filtering

The problem

- Up to now, we have mainly considered white- or large-bandwidth noise, but LF noise (e.g., flicker) can also become an issue
- Previous filters work on HF noise components and are not effective in this case
- In the time domain, LF noise has long correlation time $\Rightarrow$ it cannot be effectively reduced by averaging

LF noise filtering: concept

- Consider a pulsed signal plus LF noise (or an offset)

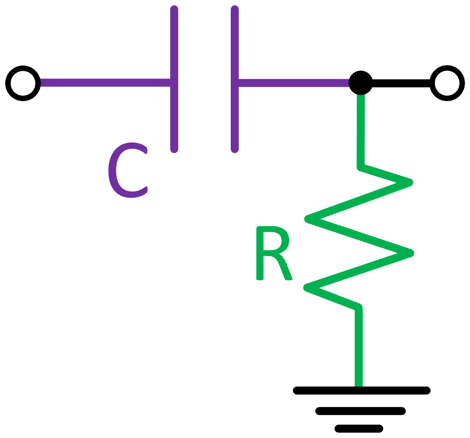


- If the noise correlation time is longer than the pulse time, averaging does not work
- We should instead «measure» the noise and subtract it from the pulsed signal (+ noise)

HP filter

- Delta-function response

$$h(t) = \delta(t) - \frac{1}{T_F} e^{-\frac{t}{T_F}} \mathrm{u}(t)$$



(think of the step response if not obvious)

- Transfer function

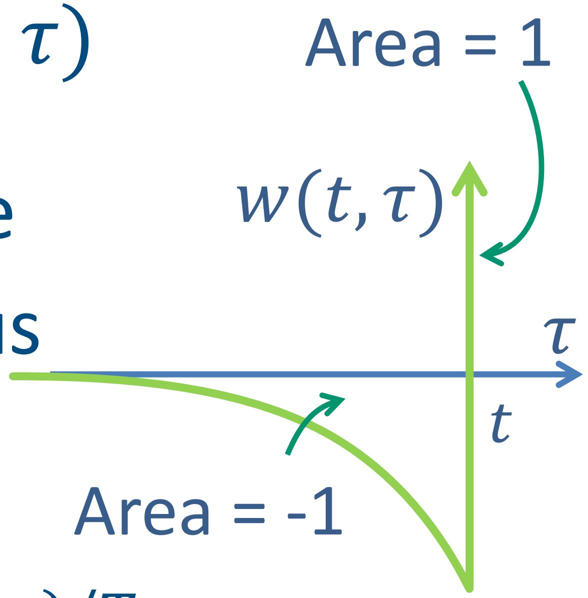
$$H(s) = \frac{sT_F}{1 + sT_F} = 1 - \frac{1}{1 + sT_F}$$

Weighting function

- The weighting function becomes

$$w(t,\tau) = h(t-\tau) = \delta(t-\tau) - \frac{1}{T_F} e^{-\frac{t-\tau}{T_F}} \mathrm{u}(t-\tau)$$

- The filter actually takes the difference between the actual input and an exponential average of previous values (stored on the capacitor):

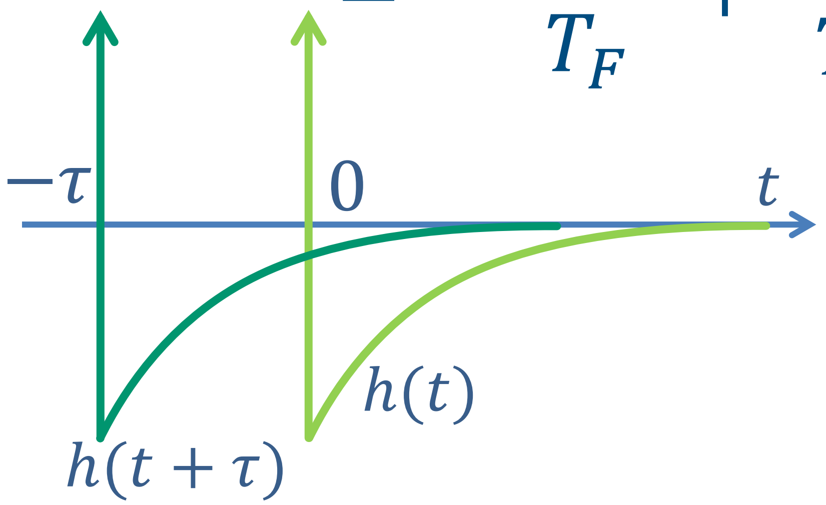


$$y(t) = \int x(\tau) w(t,\tau) d\tau = x(t) - \frac{1}{T_F} \int x(\tau) e^{-(t-\tau)/T_F} d\tau$$

Weighting function time correlation

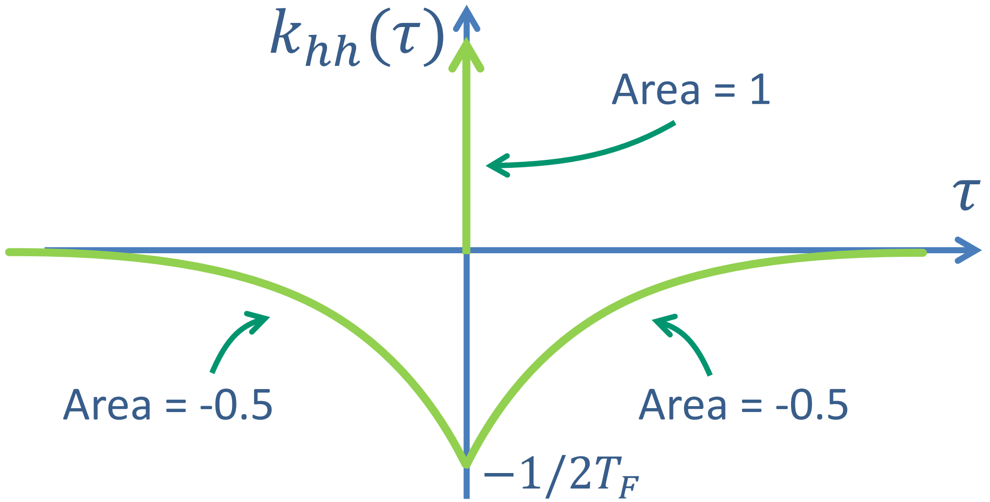
$$k_{hh}(\tau) = \int h(t)h(t+\tau)dt = -\int_0^\infty \left(\delta(t) - \frac{1}{T_F} e^{-\frac{t}{T_F}}\right) \frac{1}{T_F} e^{-\frac{t+\tau}{T_F}} dt$$

$$= -\frac{e^{-\frac{\tau}{T_F}}}{T_F} + \frac{e^{-\frac{\tau}{T_F}}}{T_F^2} \int_0^\infty e^{-\frac{2t}{T_F}} dt = -\frac{e^{-\frac{\tau}{T_F}}}{2T_F} \Rightarrow -\frac{e^{-\frac{|\tau|}{T_F}}}{2T_F}$$



$$k_{hh}(\tau) = \delta(\tau) - \frac{e^{-\frac{|\tau|}{T_F}}}{2T_F}$$

Output rms noise



$$\overline{n_y^2} = \int R_{xx}(\tau)k_{hh}(\tau)d\tau = R_{xx}(0) - \frac{1}{2T_F}\int R_{xx}(\tau)e^{-\frac{|\tau|}{T_F}}d\tau$$

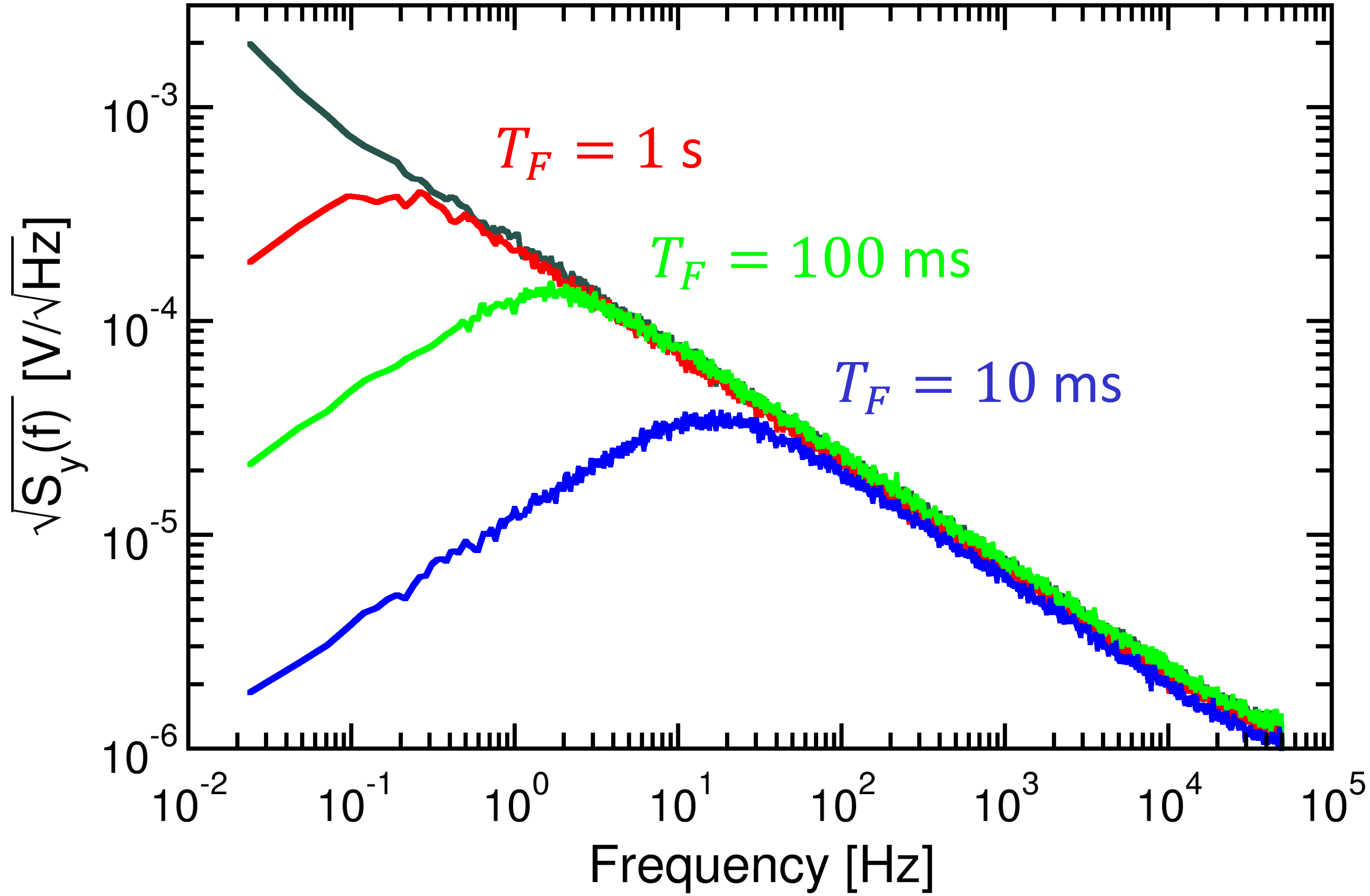
Effect on noise

- If we consider a rectangular R_{xx} we obtain

$$\overline{n_y^2} = \overline{n_x^2} - \frac{1}{T_F}\int_0^{T_n} \overline{n_x^2} e^{-\frac{\tau}{T_F}} d\tau = \overline{n_x^2} e^{-\frac{T_n}{T_F}}$$

- For white or non-correlated noise, T_n is small and $\ll T_F \Rightarrow$ the filter has little effect
- For LF noise ($T_n \gg T_F$) the filter is effective

Example: flicker noise HP filtering

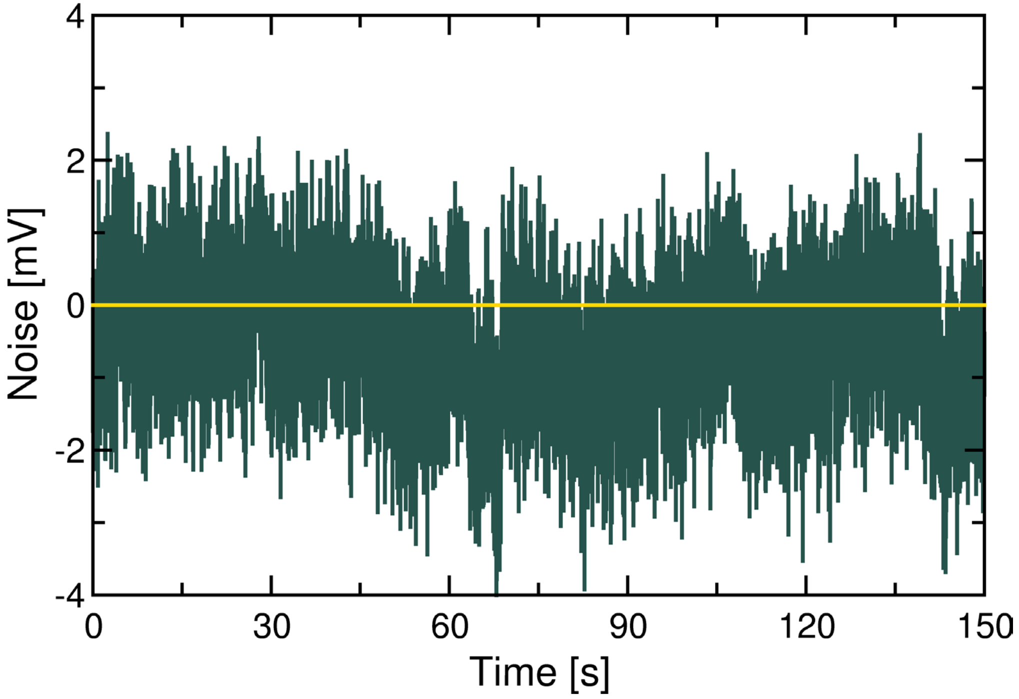


$$S_y = \frac{K}{f} \frac{(2\pi f T_F)^2}{1 + (2\pi f T_F)^2}$$

(the filter rejects the components below $1/2\pi T_F$)

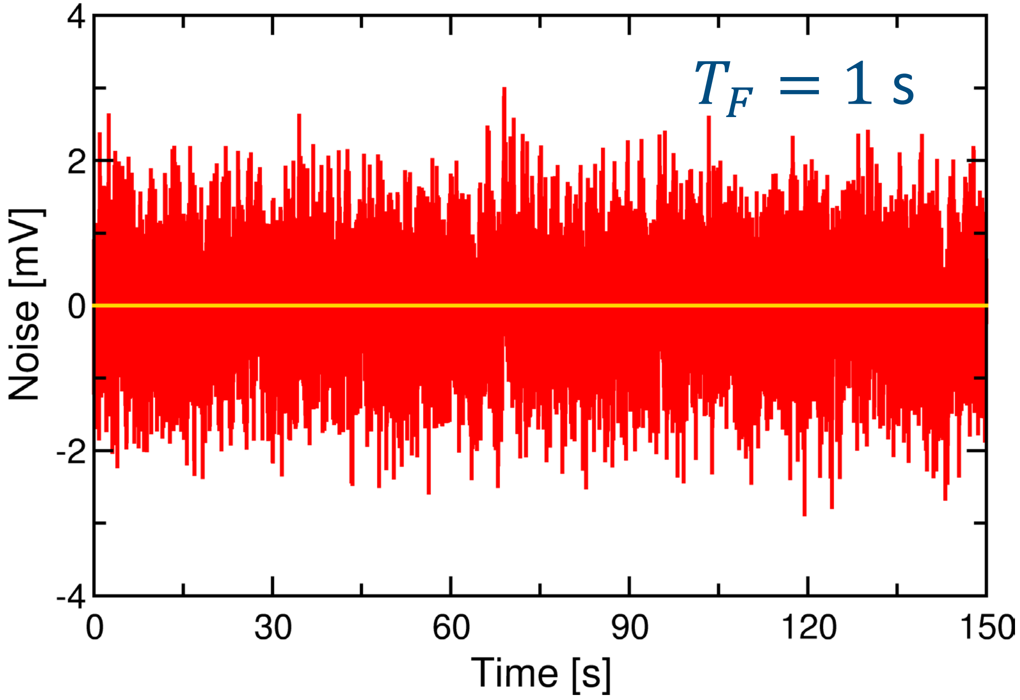
Noise in the time domain

Flicker noise



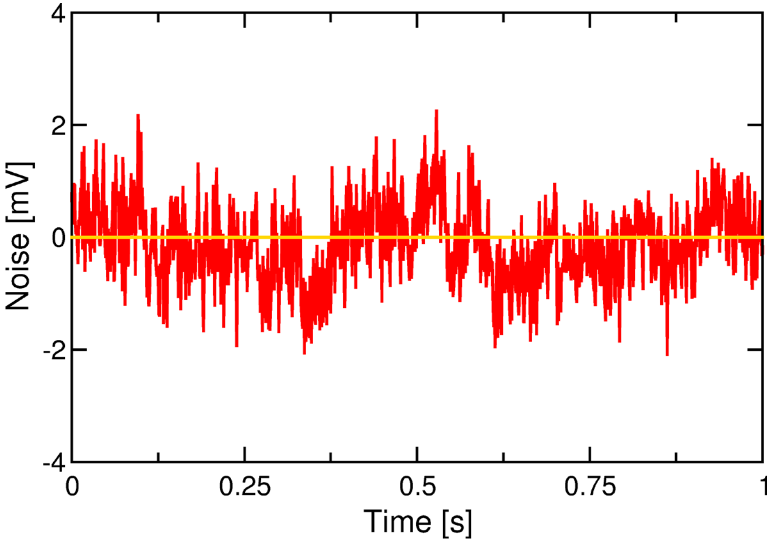
Long-time correlation

HP-filtered FN

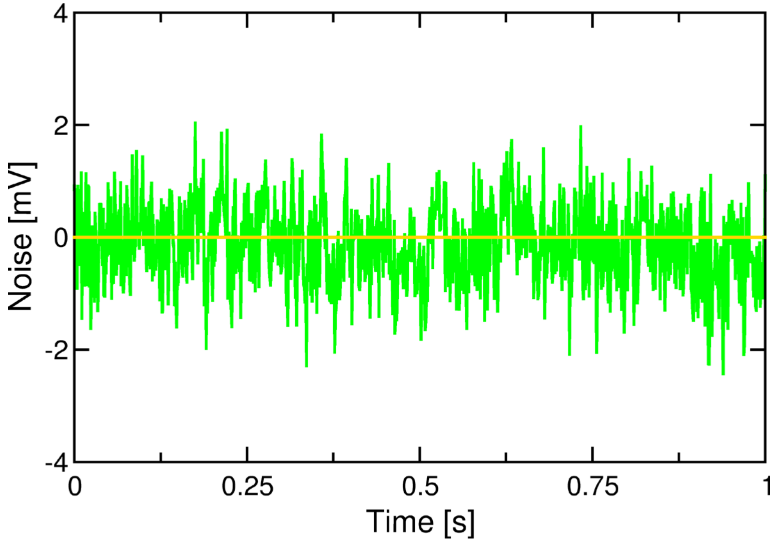


No long-time correlation, however…

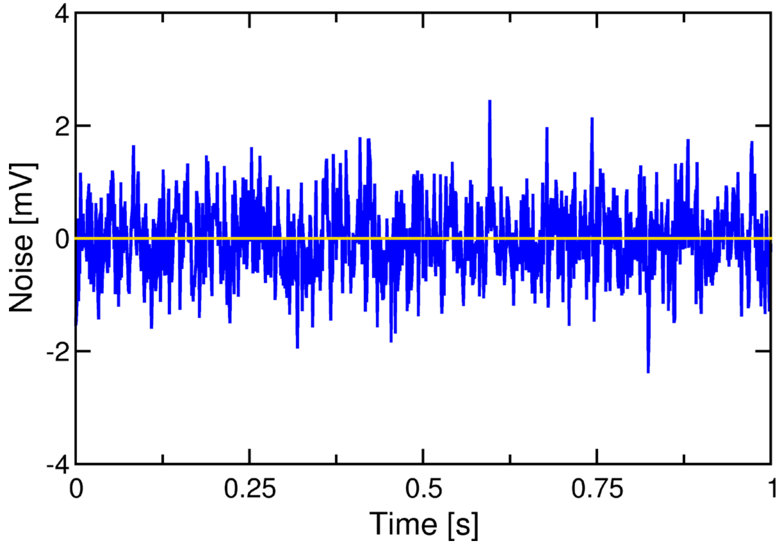
Shorter timescale…



$T_F = 1\ \mathrm{s}$



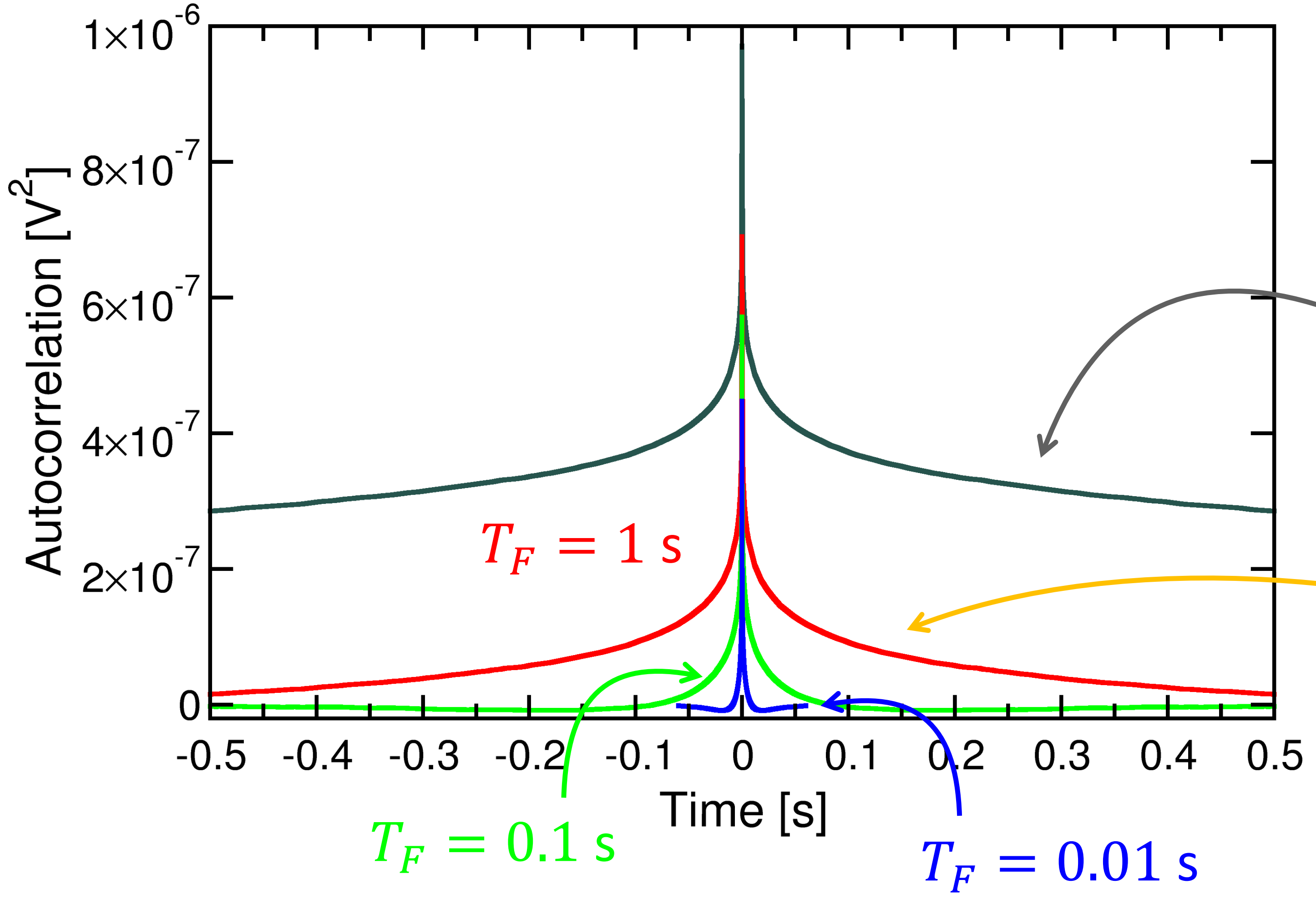
$T_F = 0.1\ \mathrm{s}$



$T_F = 0.01\ \mathrm{s}$

Filtered noise is not white and still shows some correlation over times comparable to T_F

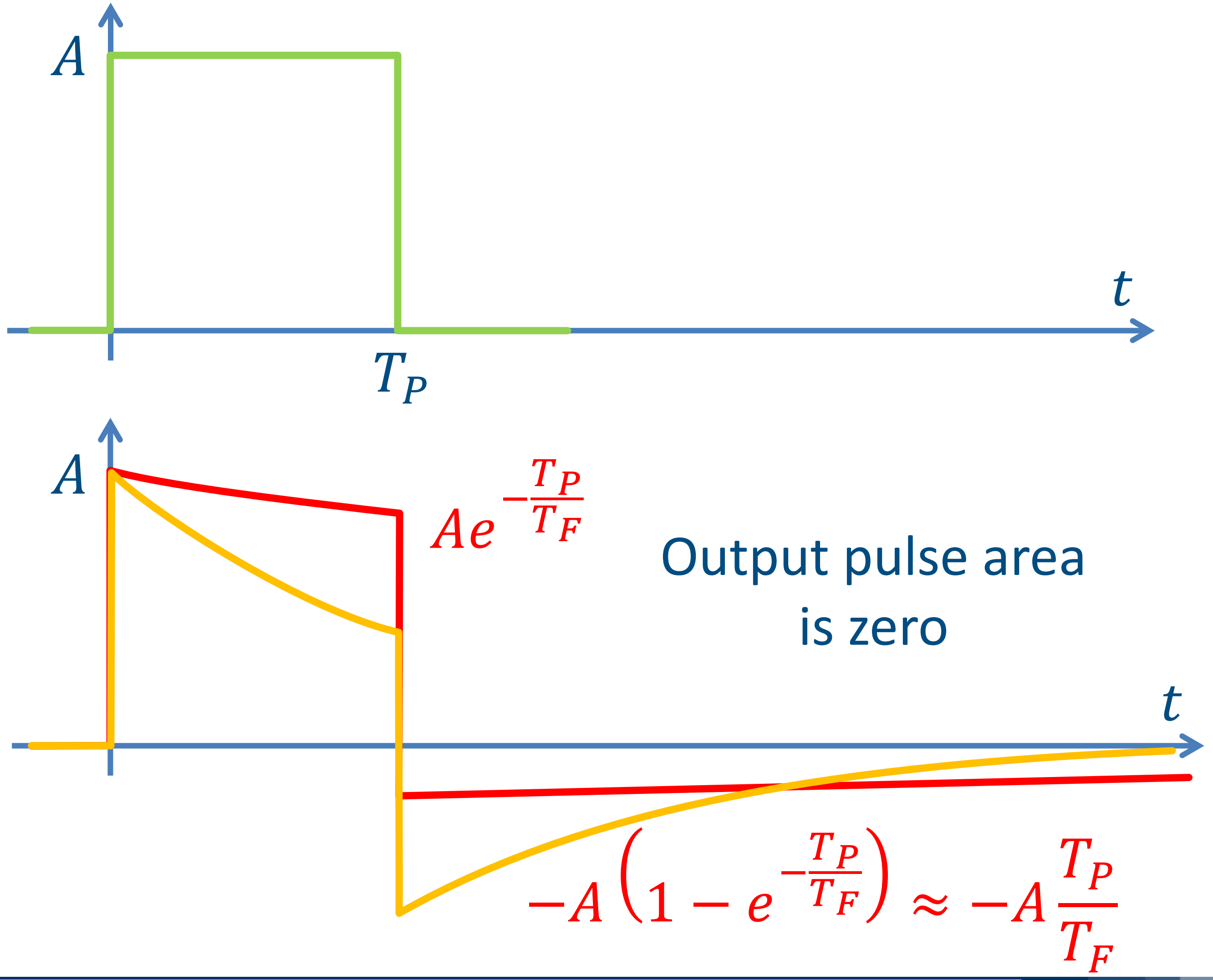
Noise autocorrelation



Limited by the observation time (remember that flicker noise is not stationary)

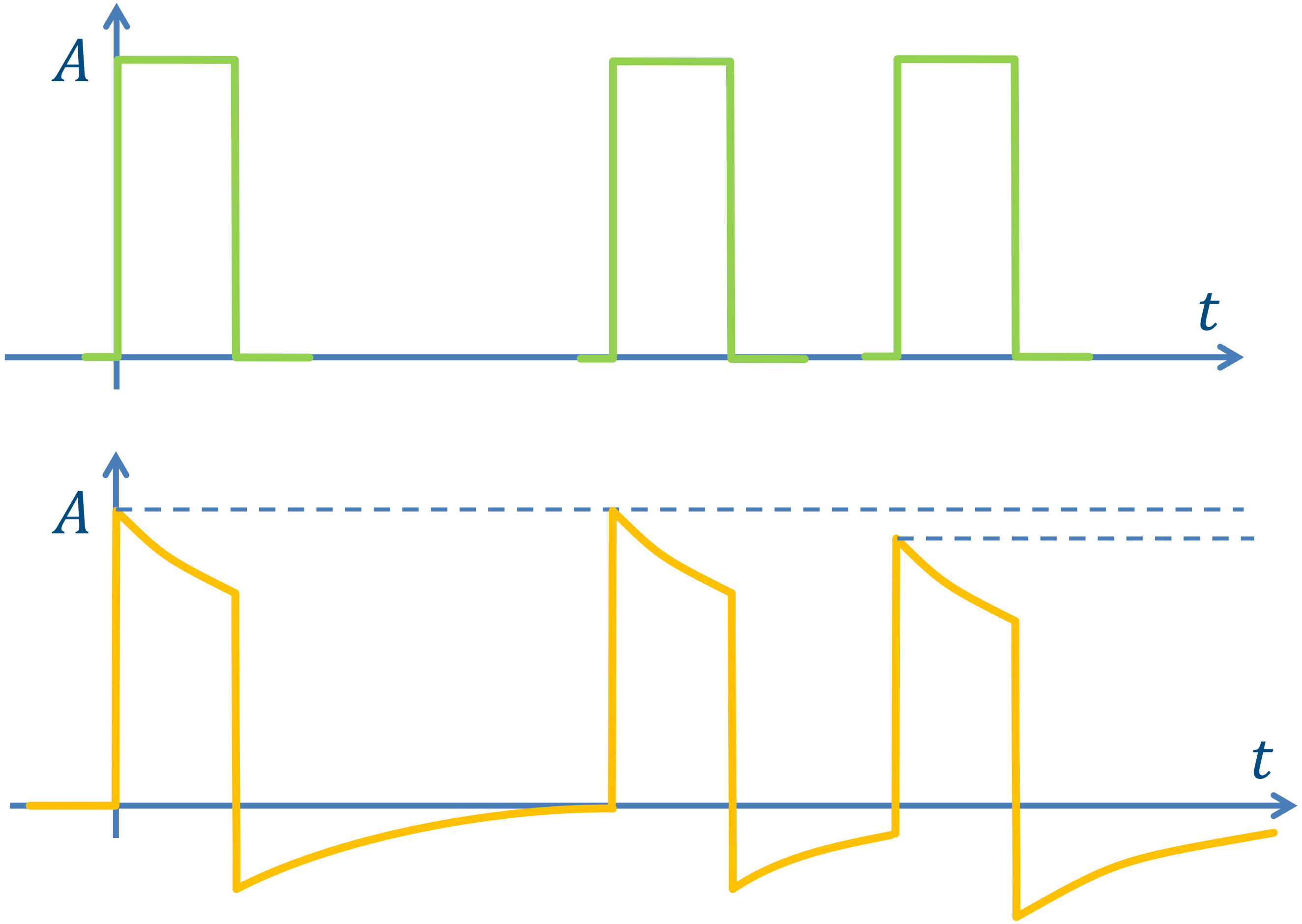
Correlation time now comparable with T_F

Effect on pulsed signal



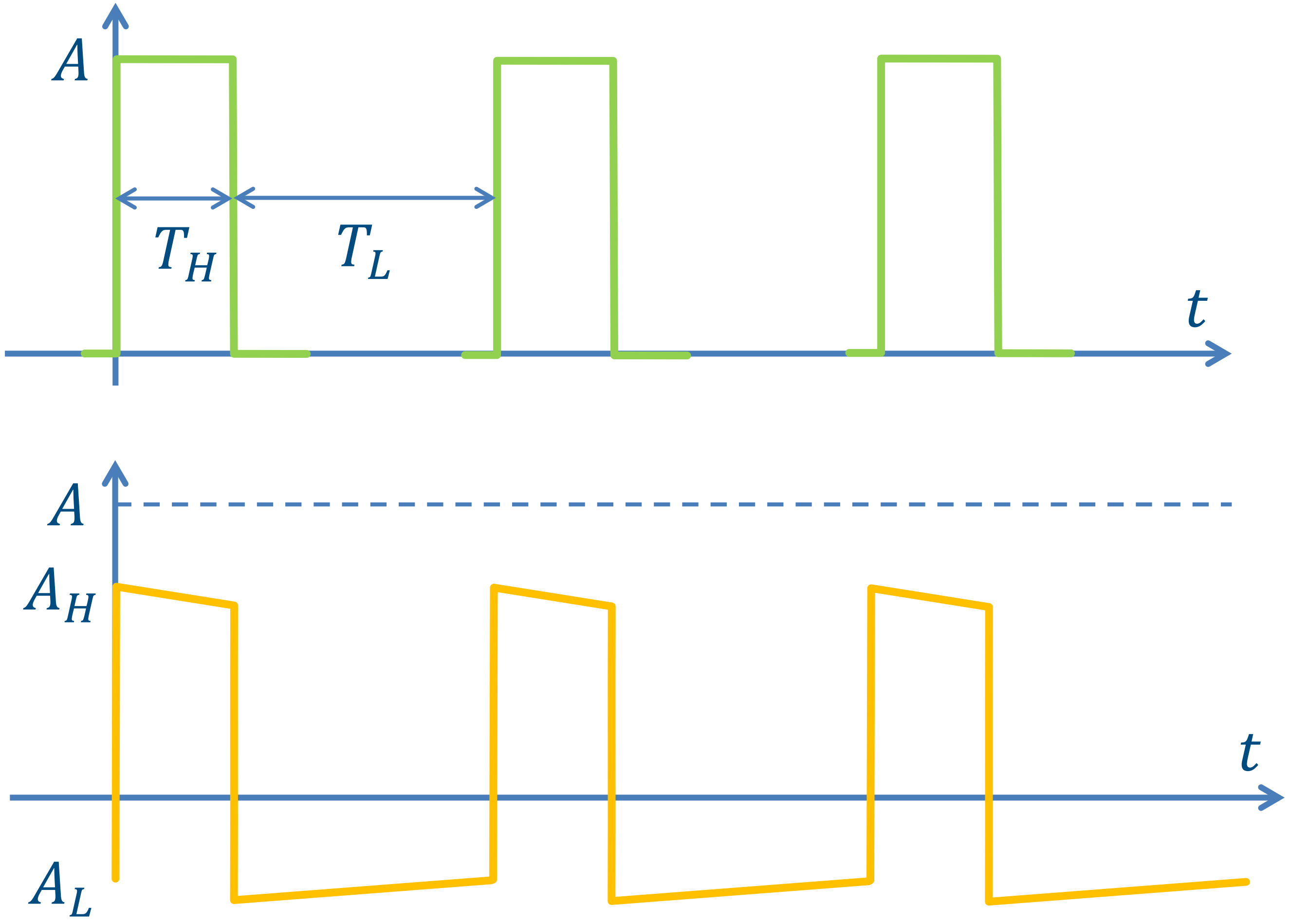
- The pulse results in an undershoot plus a tail
- There is a trade-off on T_F:
  - Long T_F: small pulse distortion and undershoot but long tail
  - Short T_F: strong pulse distorsion and undershoot but short tail

Effect on pulse sequence



- If the distance between successive pulses is shorter than $5T_F$, the pulse amplitude is incorrectly measured
- The error depends on the pulse repetition rate

The periodic case



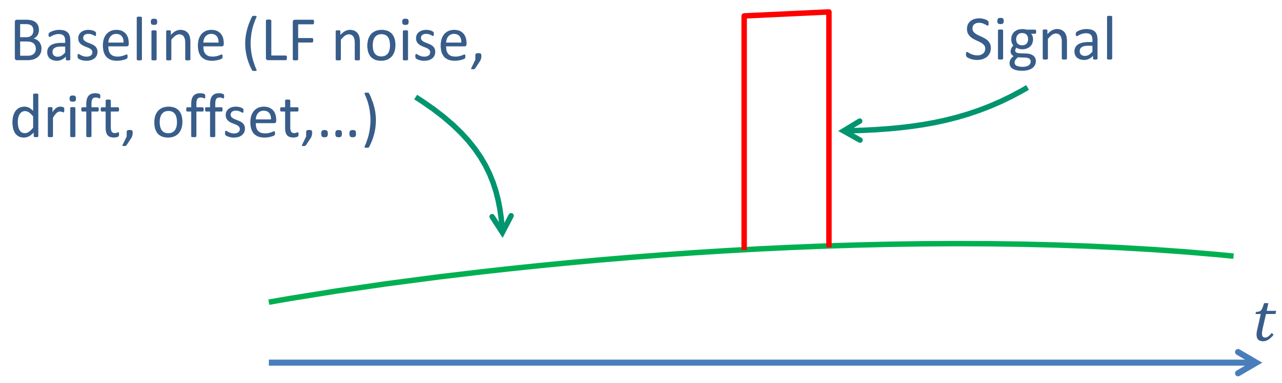
- TF has a zero in the origin $\Rightarrow$ average value of the output signal must be zero:

$$A_H T_H = (A - A_H) T_L$$

$$A_H = A \frac{T_L}{T_H + T_L}$$

- The error depends on the duty cycle

A better choice



1) Store the value of the baseline (without the signal)
2) Subtract the baseline from the signal

Obviously, a time-variant filter!

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- Baseline restorers
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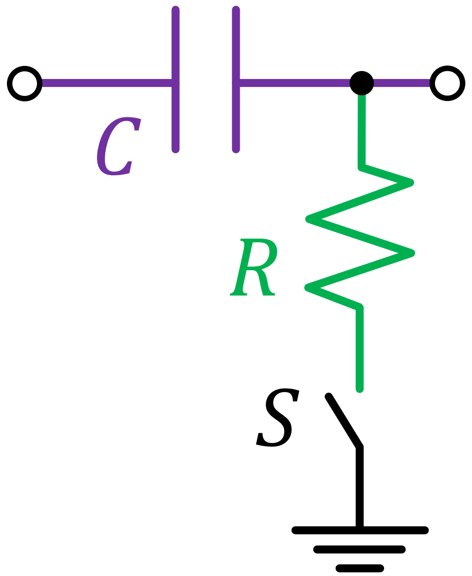
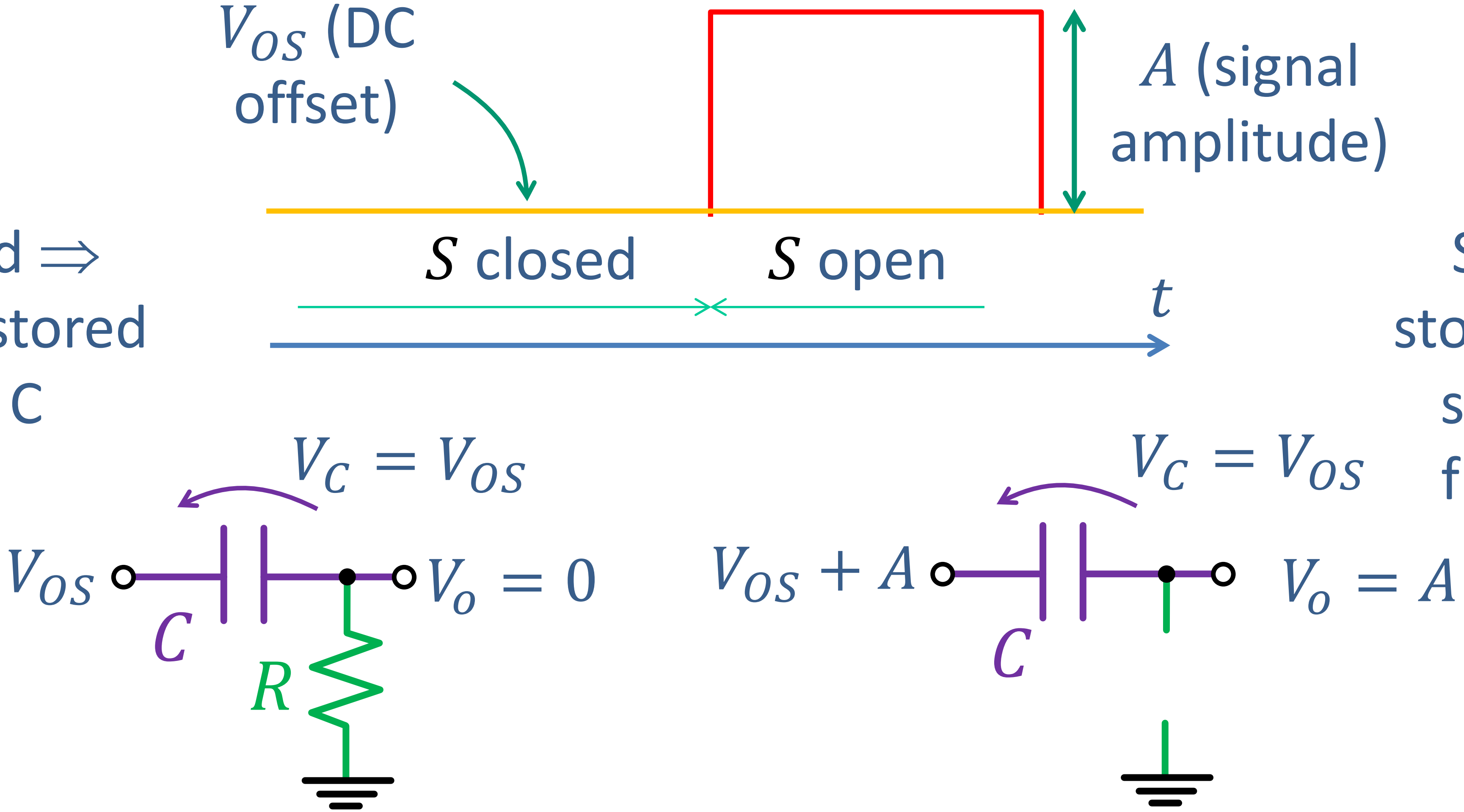
Baseline restorer



1) Baseline measurement ⇒ switch closed
2) BL subtraction from signal ⇒ switch open

Example: constant DC offset

S closed ⇒ offset is stored onto C

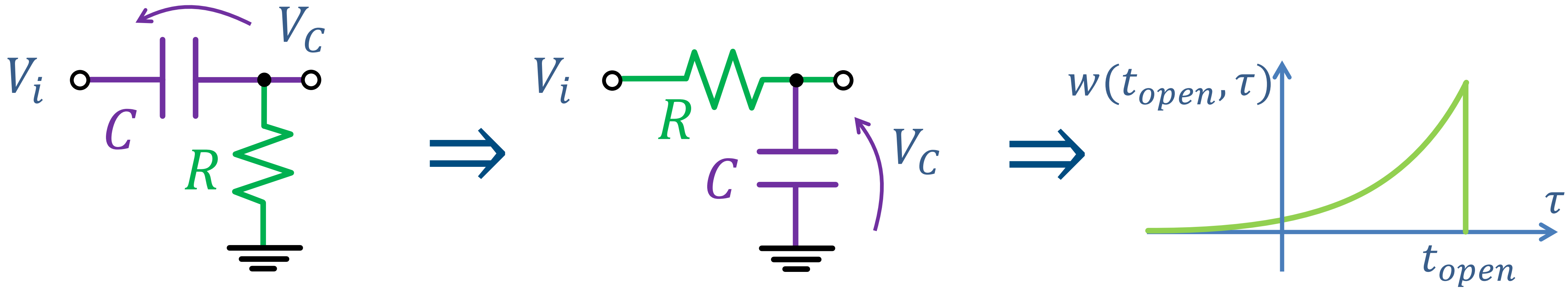


S open ⇒ stored value is subtracted from input

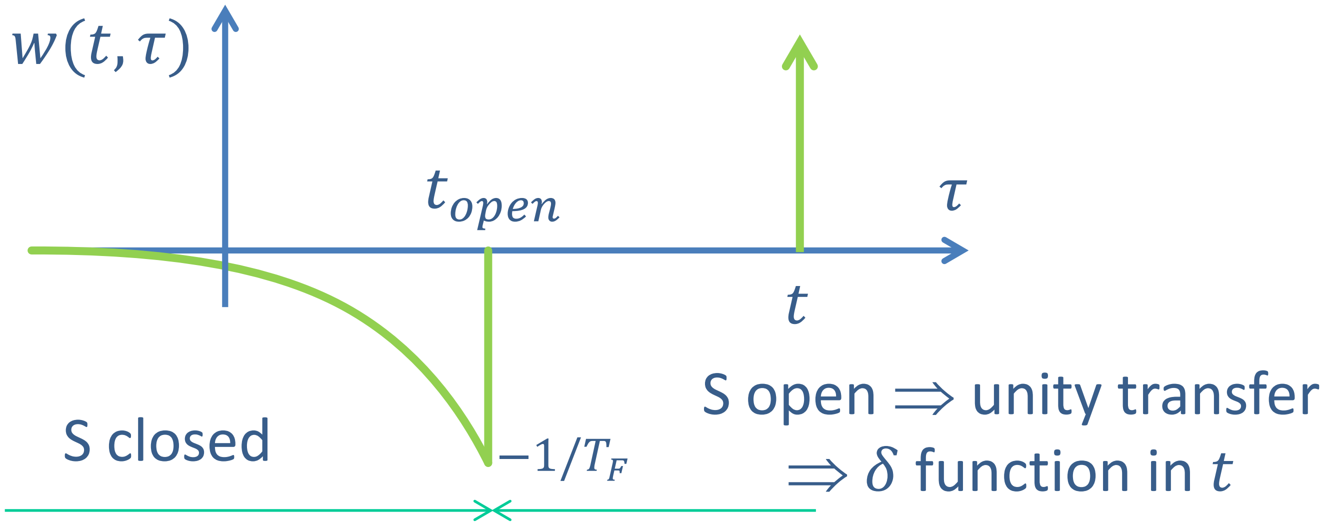
Weighting function construction

$$v_o(t) = v_i(t) - v_C(t) = v_i(t) - v_C(t_{open})$$

- The first term $v_o(t) = v_i(t)$ means $w(t,\tau) = \delta(\tau - t)$
- $v_C(t_{open})$ is built when the switch is closed. The circuit is

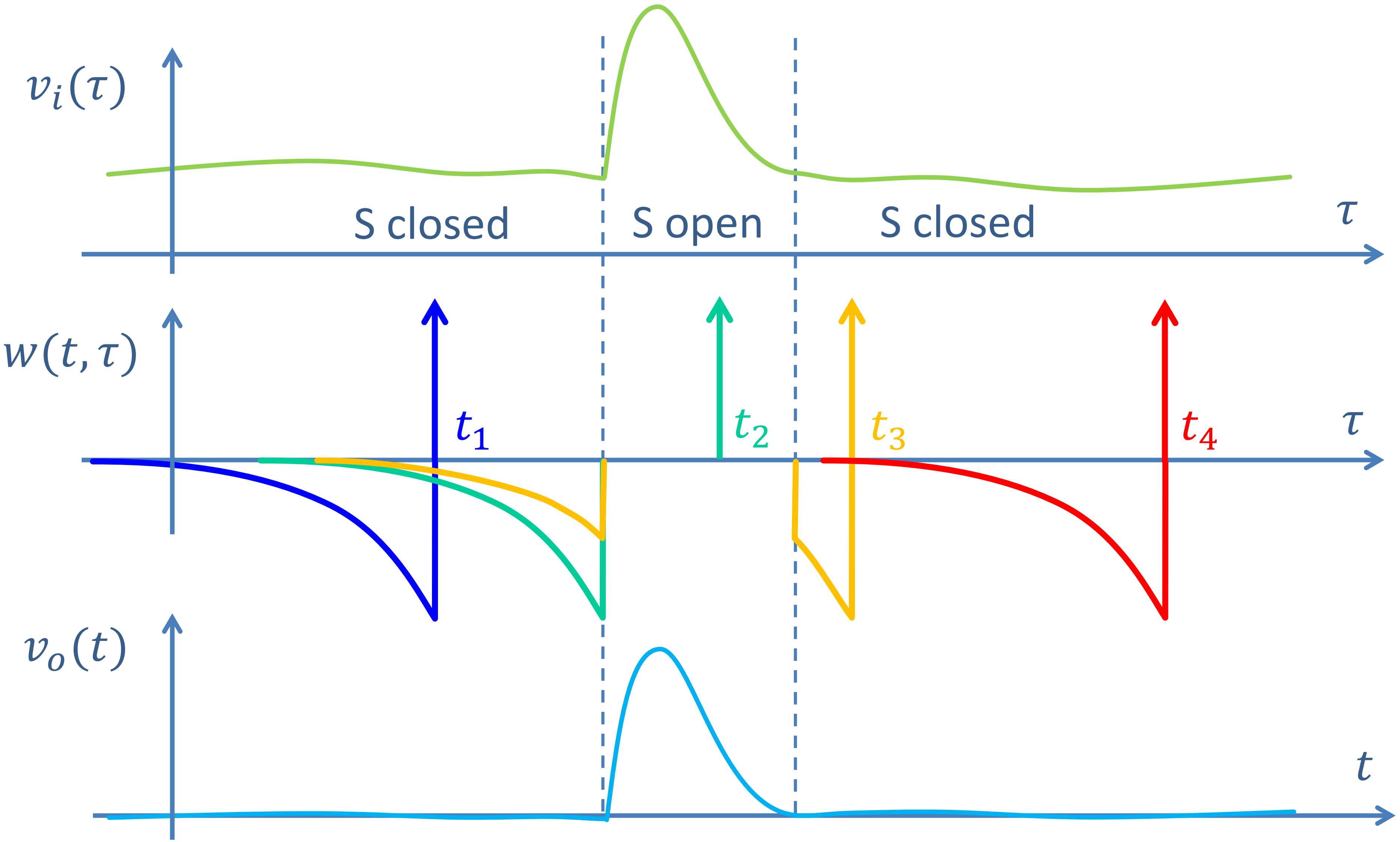


Weighting function

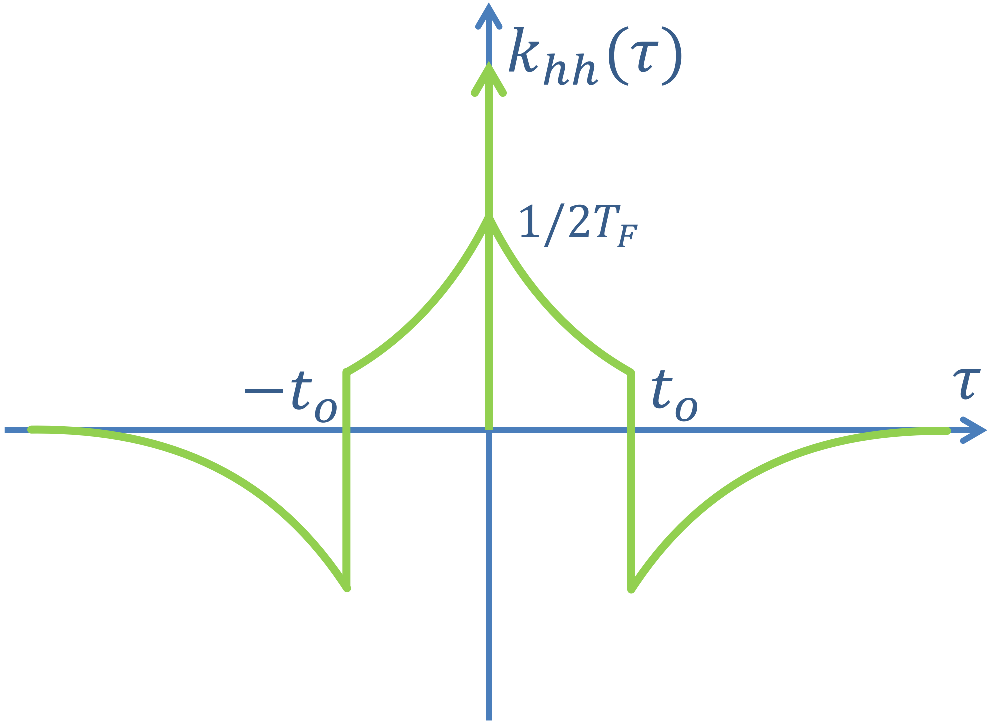


Remember that $w(t,\tau)$ is the system response at time t to a delta-function applied in τ

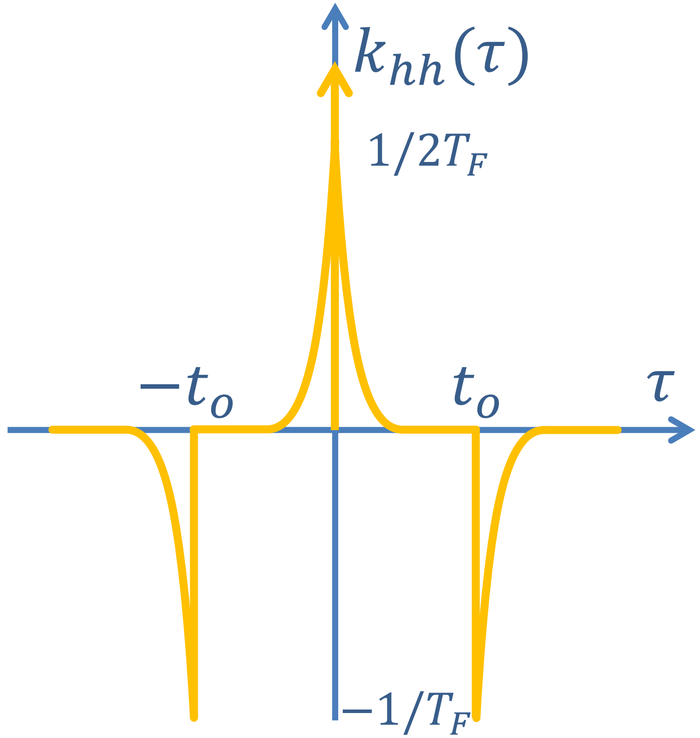
Example



Weighting function time correlation



$T_F \gg t_o$



$T_F \ll t_o$

Effect on noise

$$\overline{n_y^2} = \int R_{xx}(\tau) k_{hh}(\tau) d\tau$$

- For HF noise $\overline{n_y^2}$ is even larger than $\overline{n_x^2}$, as we subtract uncorrelated samples
- For LF noise ($T_n \gg t_o$) the negative areas of k_{hh} reduce $\overline{n_y^2}$ and the filter is effective

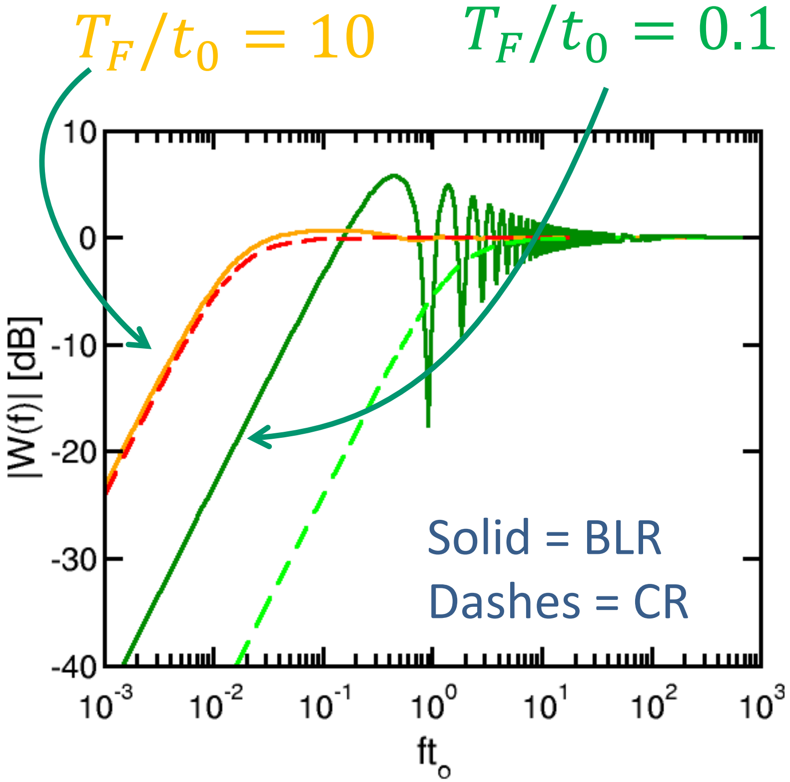
Frequency domain

- We set $t = 0$ for simplicity and we recall the time-reversal property, obtaining:

$$W(f) = 1 - \frac{e^{j2\pi f t_o}}{1 - j2\pi f T_F}$$

- At LF $e^{j2\pi f t_o} \approx 1 + j2\pi f t_o$

$$W(f) \approx -j2\pi f (T_F + t_o)$$



Intrinsic HP filtering

From [2]

- In all real cases, even with DC coupled electronics:

  weighting is inherently NOT extended down to zero frequency,

  because an intrinsic high-pass filtering is present in any real operation.

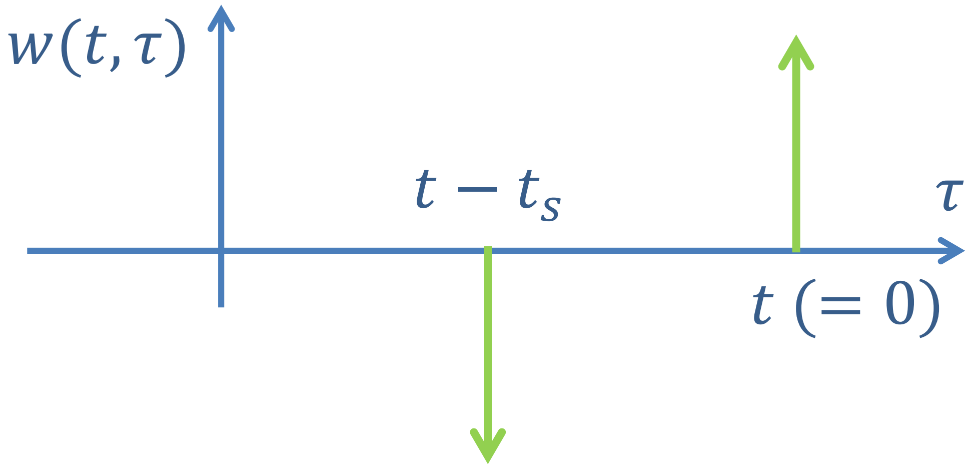
- The intrinsic filtering action arises because:

- a) operation is **started at some time before** the acquisition of the measure

  b) operation is **started from zero** value

- EXAMPLE: measurement of amplitude of the output signal of a DC amplifier.
  Zero-setting is mandatory: the baseline voltage is preliminarly adjusted to zero, or it is measured, recorded and then subtracted from the measured signal.
  It may be done a long time before the signal measurements (e.g. when the amplifier is switched on) or repeated before each measurement; it may be done manually or automated, but it must be done anyway.
  Zero-setting produces a high-pass filtering: let us analyze why and how

Correlated double sampling



$$w(0,\tau) = \delta(\tau) - \delta(\tau + t_s) \Leftrightarrow W(f) = 1 - e^{j2\pi f t_s}$$

$$|W(f)| = \sqrt{2(1 - \cos(2\pi f t_s))}$$

- At LF, $|W(f)| \approx 2\pi f t_s \Rightarrow |W(f)|$ behaves like an HPF
- Further calculations can be found in [2]

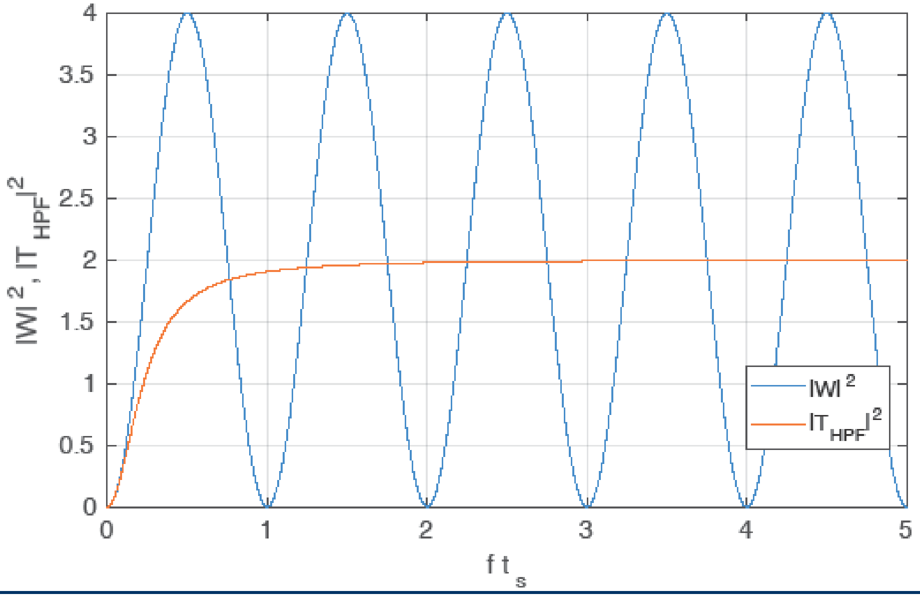
HPF approximation

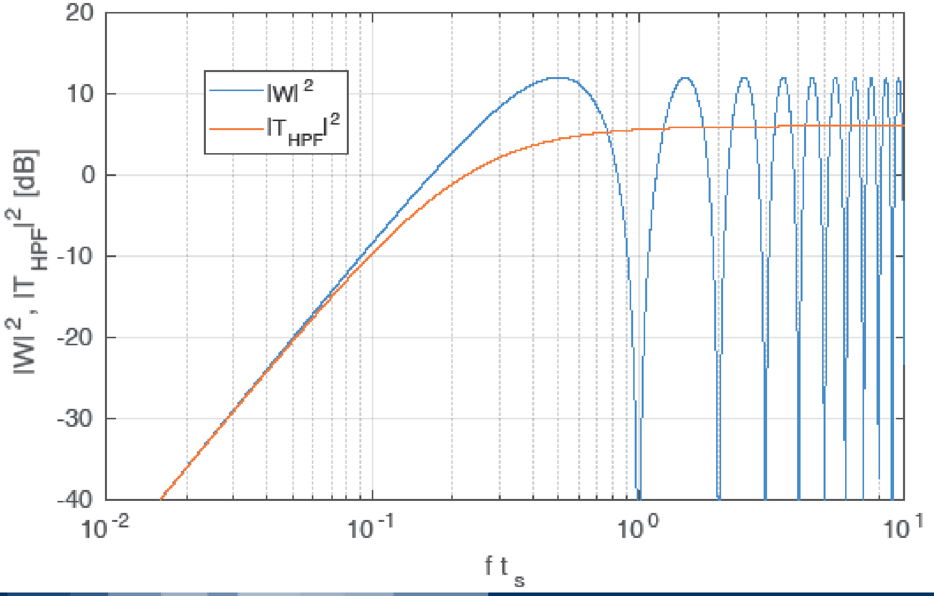
- To find an HPF approximation, we can write

$$|W(f)|^2 = 2(1 - \cos \omega t_s) \approx (\omega t_s)^2 \text{ at LF}$$

$$|T_{HPF}(f)|^2 = 2\frac{(\omega\tau)^2}{1+(\omega\tau)^2} \approx 2(\omega\tau)^2 \text{ at LF}$$

$$\Rightarrow \tau = \frac{t_s}{\sqrt{2}}$$





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Output rms noise (LP filter)

$$H(s) = \frac{1}{1 + s\tau_p}, \qquad S_x(f) = \frac{K}{f} \qquad \text{(unilateral)}$$

$$\overline{V_o^2} = \int_{f_L}^{\infty} \frac{K}{f} \frac{1}{\left(1 + (2\pi f \tau_p)^2\right)} df = K \int_{x_L}^{\infty} \frac{1}{x(1 + x^2)} dx$$

$$= K \left[-\ln \sqrt{1 + \frac{1}{x^2}} \right]_{x_L}^{\infty} = K \ln \sqrt{1 + \frac{1}{x_L^2}} = K \ln \sqrt{1 + \left(\frac{f_p}{f_L}\right)^2} \approx K \ln \left(\frac{f_p}{f_L}\right)$$

Output rms noise (BP filter)

$$H(s) = \frac{s\tau_L}{(1+s\tau_L)(1+s\tau_H)}, \qquad S_x(f) = \frac{K}{f} \quad \text{(unilateral)}$$

$$\overline{V_o^2} = \int_0^\infty \frac{2\pi K}{\omega} \frac{(\omega\tau_L)^2}{(1+\omega^2\tau_L^2)(1+\omega^2\tau_H^2)} \frac{d\omega}{2\pi}$$

$$= \frac{K}{2} \frac{\tau_L^2}{\tau_L^2 - \tau_H^2} \int_0^\infty \left(\frac{2\tau_L^2\omega}{1+\omega^2\tau_L^2} - \frac{2\tau_H^2\omega}{1+\omega^2\tau_H^2} \right) d\omega = K \frac{\tau_L^2}{\tau_L^2 - \tau_H^2} \left[\ln \sqrt{\frac{1+\omega^2\tau_L^2}{1+\omega^2\tau_H^2}} \right]_0^\infty$$

$$= K \frac{\tau_L^2}{\tau_L^2 - \tau_H^2} \ln\left(\frac{\tau_L}{\tau_H}\right) \approx K \ln\left(\frac{f_H}{f_L}\right)$$

References

1. http://home.deib.polimi.it/cova/elet/lezioni/SSN07b_Filters-HPF2.pdf
2. http://home.deib.polimi.it/cova/elet/lezioni/SSN07a_1vf_noise-HPF1.pdf