Electronics – 96032

POLITECNICO DI MILANO

Amplitude Modulation and Synchronous Detection

Alessandro Spinelli

Phone: (02 2399) 4001

alessandro.spinelli@polimi.it spinelli.faculty.polimi.it

Disclaimer

Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes

Purpose of the lesson

- It's time to begin a discussion on the techniques for improving S/N
- Noise-reduction techniques obviously depend on the type of signal and of noise:

		Noise	
		HF (White)	LF (flicker)
Signal	LF (constant)	done	this lesson
	HF (pulse)	done	done

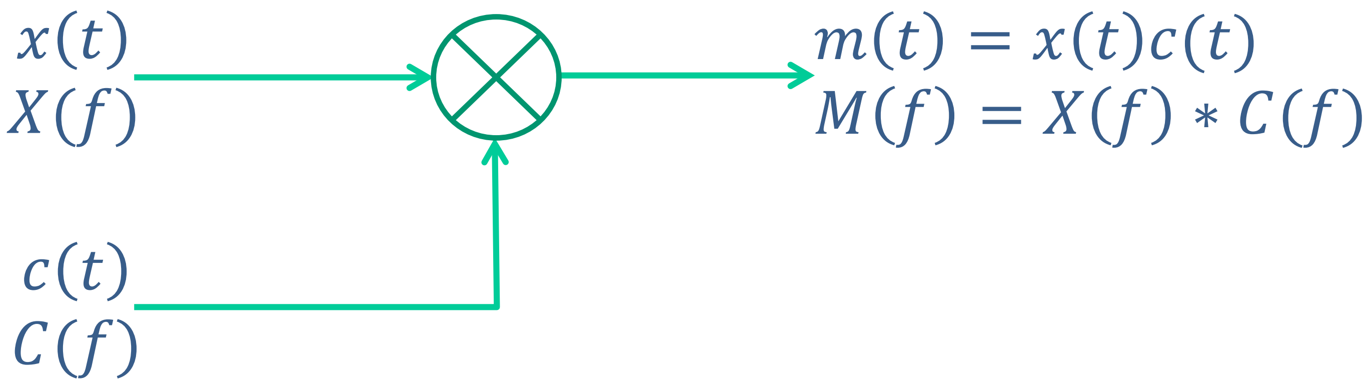
Outline

- Amplitude modulation and demodulation
- The weighting function

The problem

- If the signal is at DC level and buried in LF noise, HP or BP filters become useless
- If we could move the signal spectrum to higher frequencies, we would improve S/N
- A high-Q BP filter could then be used to recover the signal

Amplitude modulation (AM)



- The amplitude of a carrier wave $c(t)$ is modified by a modulating signal $x(t)$
- Originally developed for telephone and radio communication

Time and frequency domains

- We consider a sinusoidal carrier

$$c(t) = A\cos(\omega_c t + \phi_c) \Leftrightarrow$$

$$C(f) = \frac{A}{2}\left(e^{j\phi_c}\delta(f - f_c) + e^{-j\phi_c}\delta(f + f_c)\right)$$

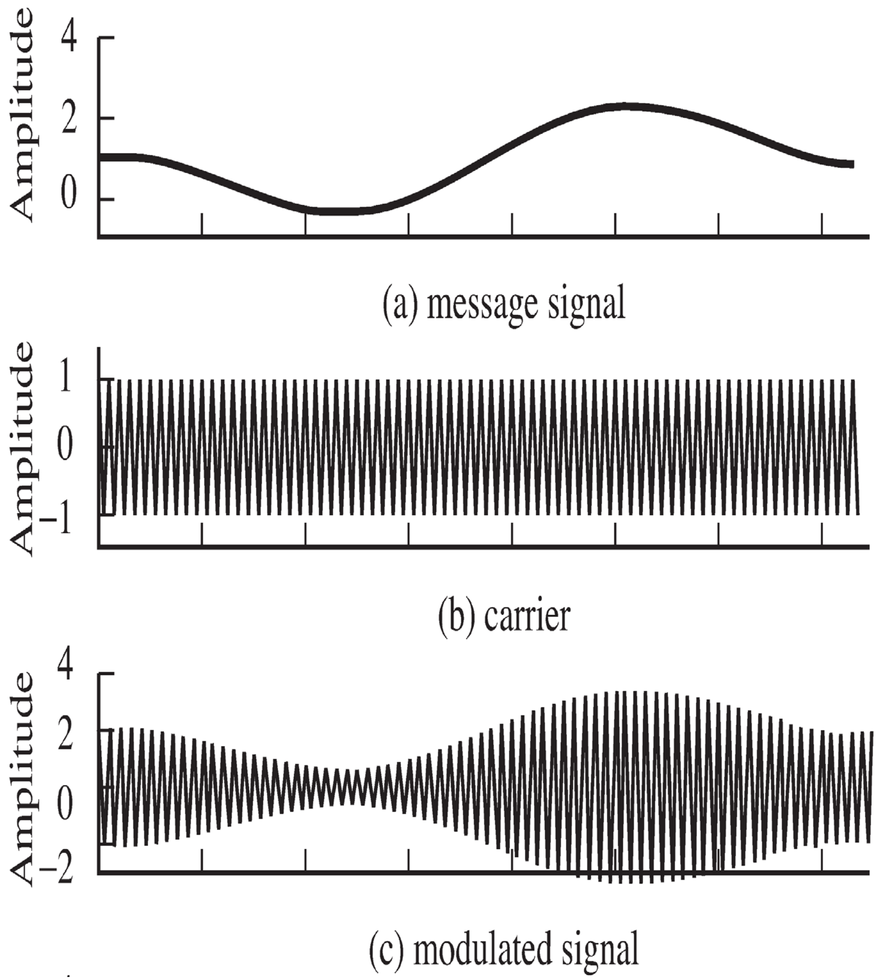
- The modulated signal is then

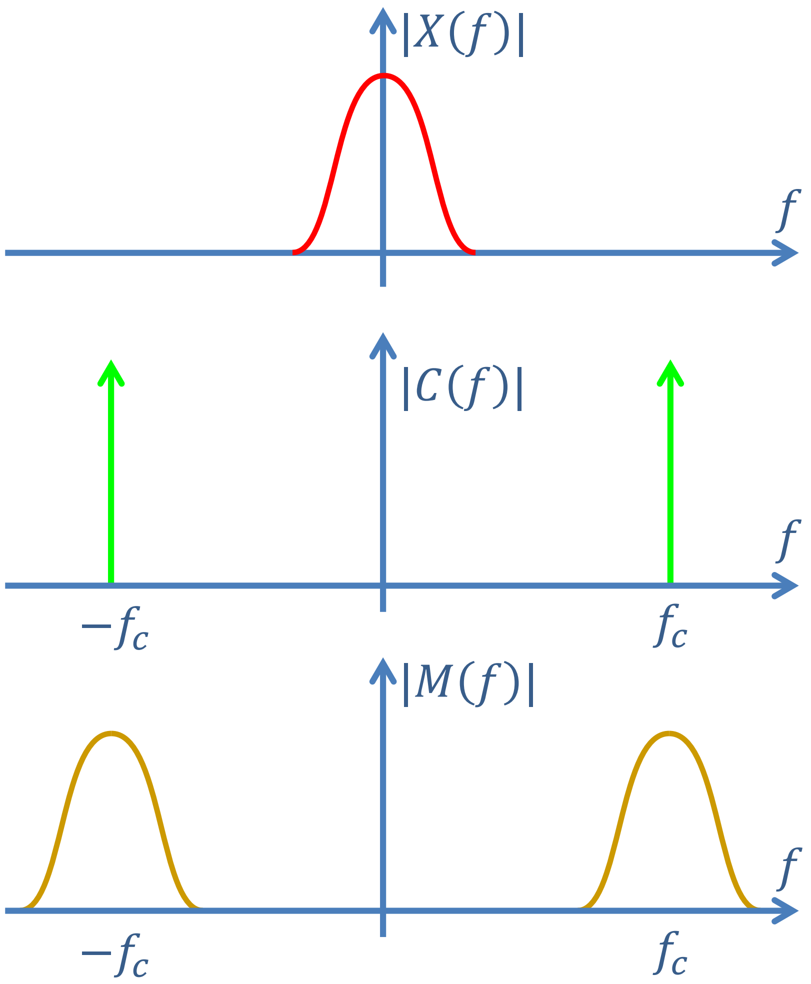
$$m(t) = x(t)c(t) \Leftrightarrow$$

$$M(f) = X(f) * C(f) = \frac{A}{2}\left(e^{j\phi_c}X(f - f_c) + e^{-j\phi_c}X(f + f_c)\right)$$

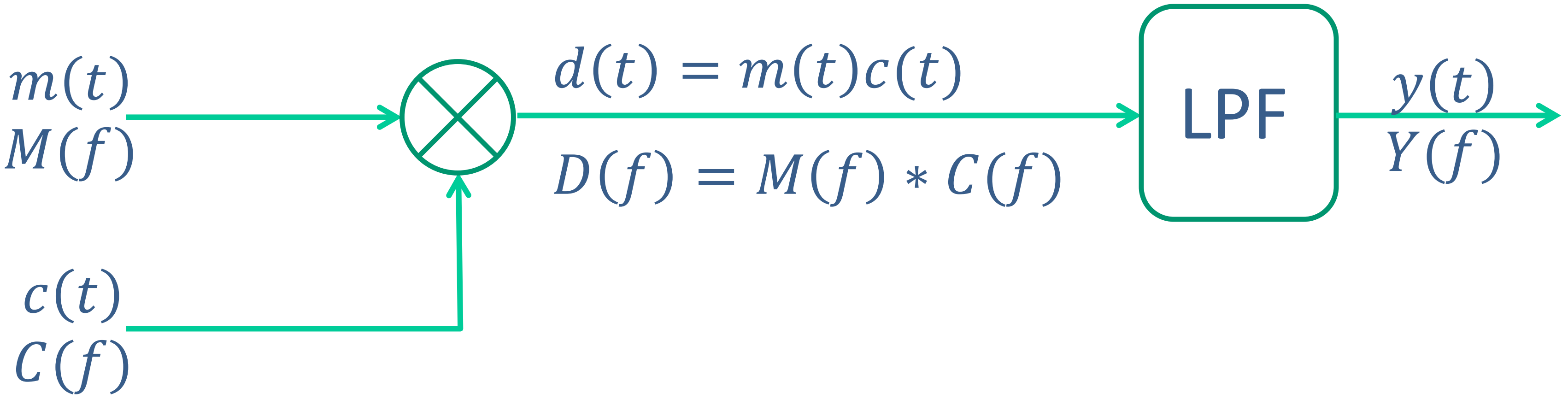
Waveforms and spectra

From [1]





Demodulation



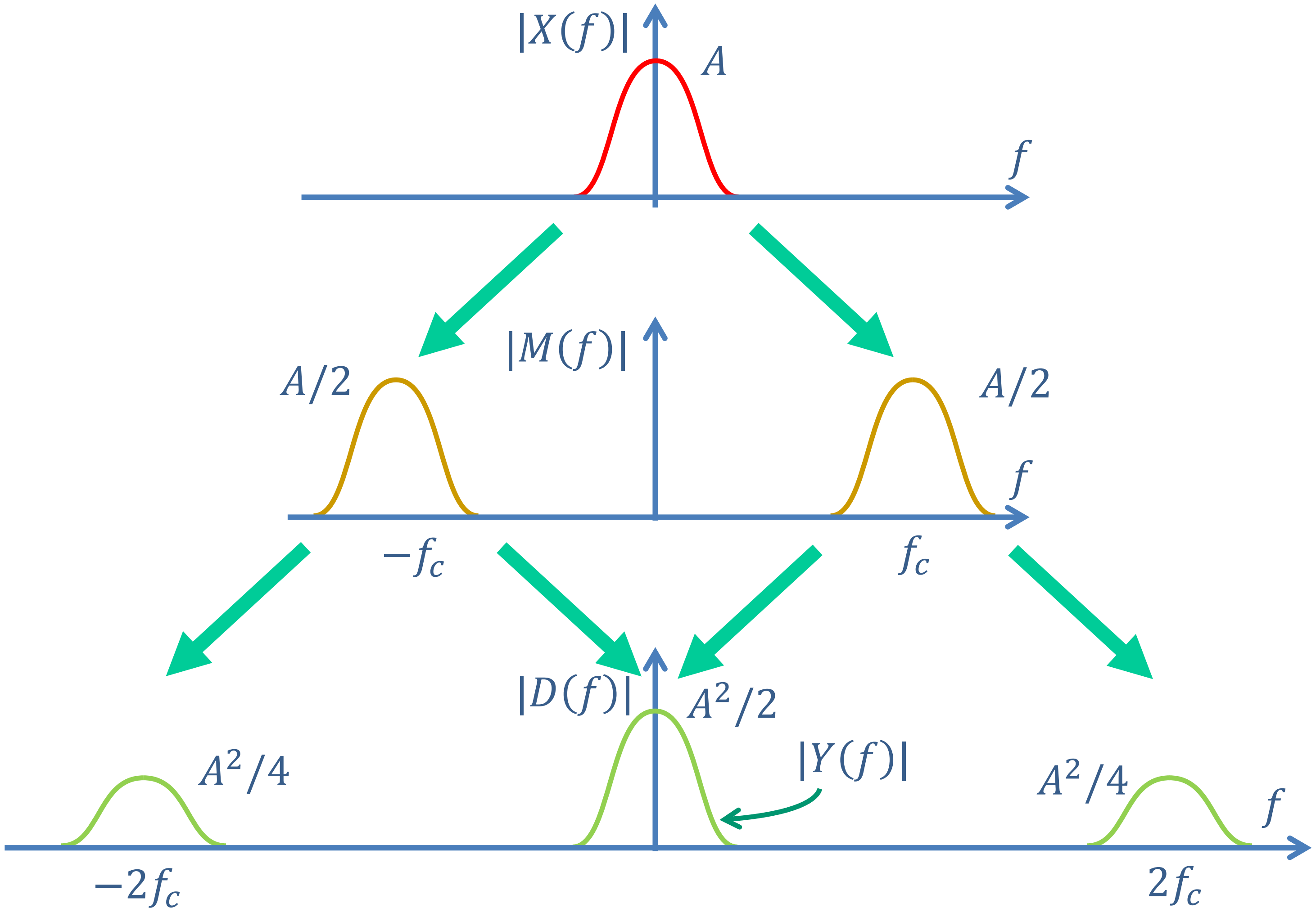
$$d(t) = m(t)c(t) = x(t)c^2(t) = x(t)A^2\cos^2(\omega_c t + \phi_c)$$

$$= \frac{A^2}{2}x(t)(1 + \cos(2\omega_c t + 2\phi_c))$$

$Y(f)$

$$D(f) = \frac{A^2}{2}X(f) + \frac{A^2}{4}\left(e^{j2\phi_c}X(f - 2f_c) + e^{-j2\phi_c}X(f + 2f_c)\right)$$

Frequency-domain view



Frequency and phase errors

- We consider demodulating with

$$w_R(t) = B\cos\big((\omega_c+\Delta\omega)t + \phi_c + \Delta\phi\big)$$

- The low-frequency term (small $\Delta\omega$) is

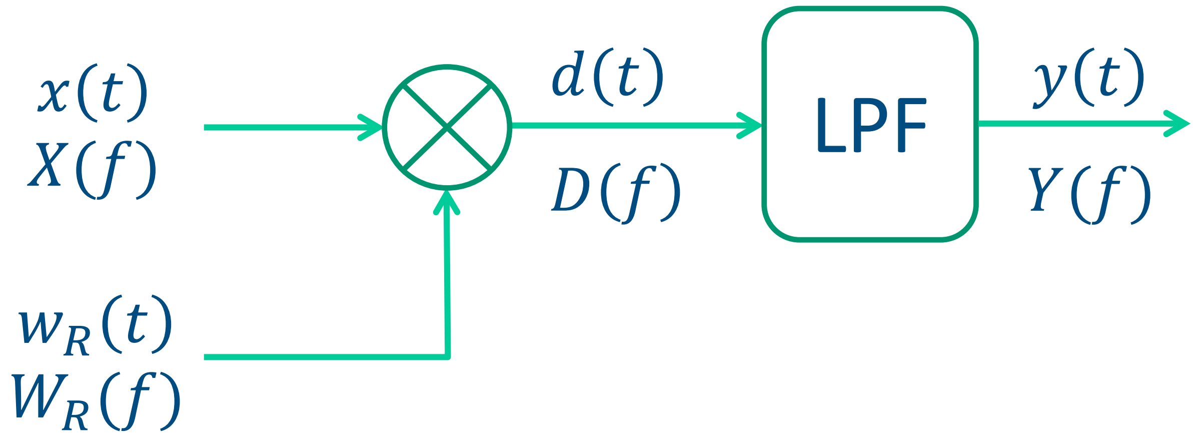
$$y(t) = ABx(t)\cos(\Delta\omega t + \Delta\phi)/2$$

  - Phase error $\Rightarrow$ signal reduction
  - Frequency error $\Rightarrow$ oscillating behavior
- The reference must be locked in frequency and phase to the carrier $\Rightarrow$ synchronous detection
- The demodulator is also called phase-sensitive detector (PSD)

Outline

- Amplitude modulation and demodulation
- The weighting function

PSD weighting function

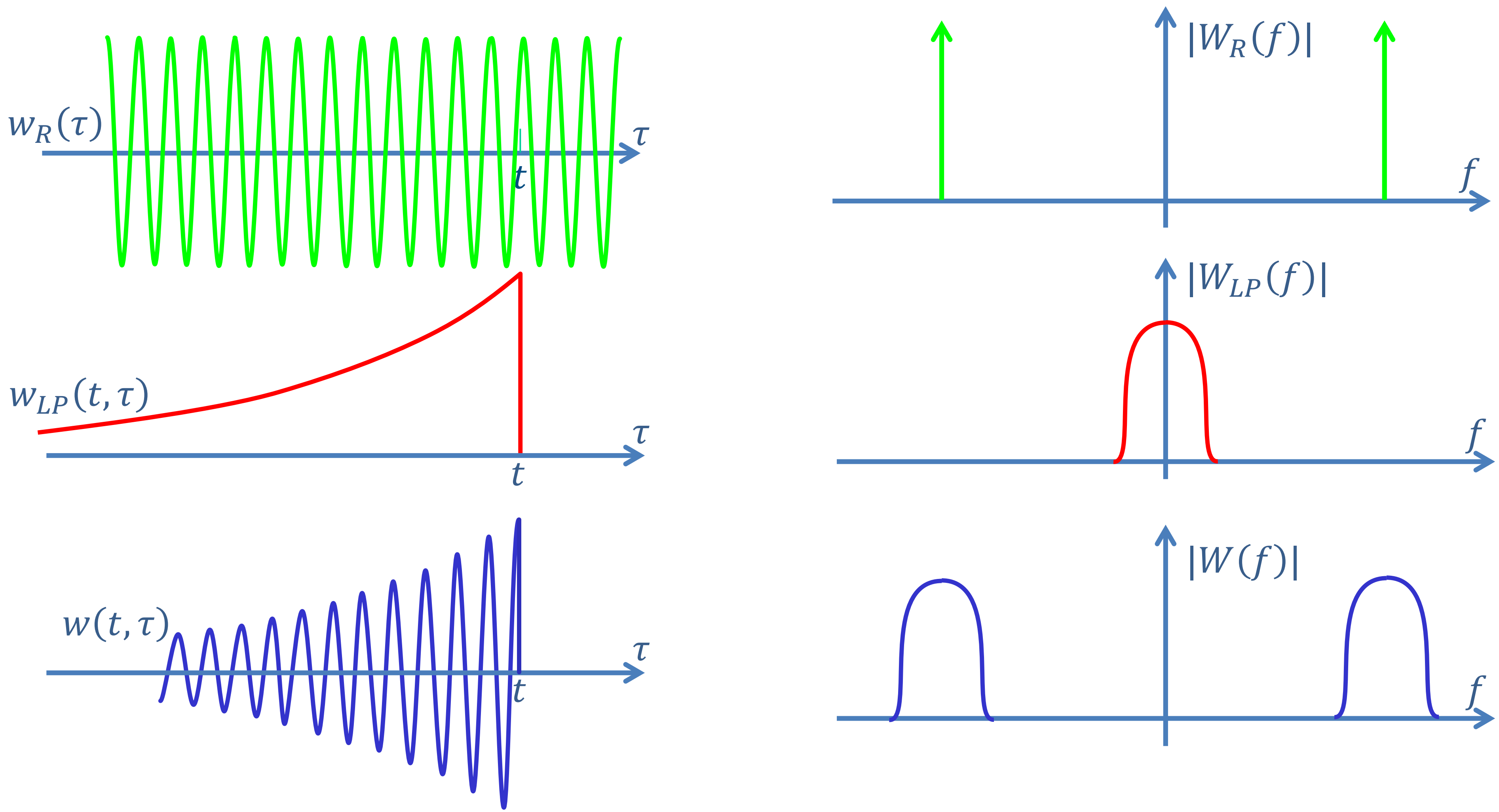


$$y(t) = \int d(\tau) w_{LP}(t,\tau) d\tau = \int x(\tau) w_R(\tau) w_{LP}(t,\tau) d\tau$$

$$w(t,\tau) = w_R(\tau) w_{LP}(t,\tau)$$

$$W(t,f) = W_R(f) * W_{LP}(t,f)$$

Time/frequency domains



Notes

- The filter is time-variant even if LPF is LTI $\Rightarrow$ remember that

$$y(t) = \int x(\tau)w(t,\tau)d\tau = \int X(f)W^*(t,f)df$$

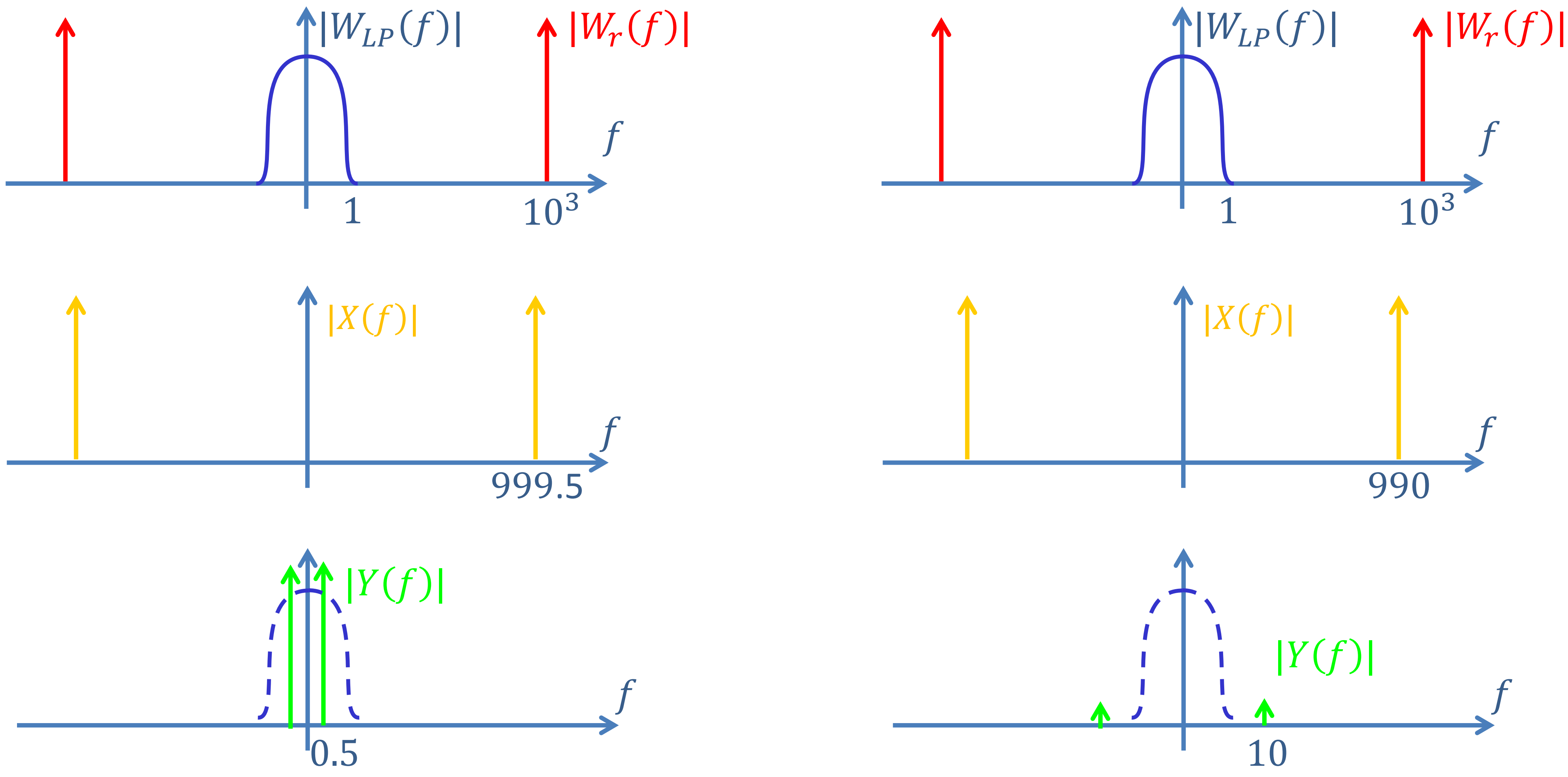
$\Rightarrow Y(f)$ is NOT equal to $X(f)W(t,f)$

- For the case of a PSD with LTI LPF, we have instead

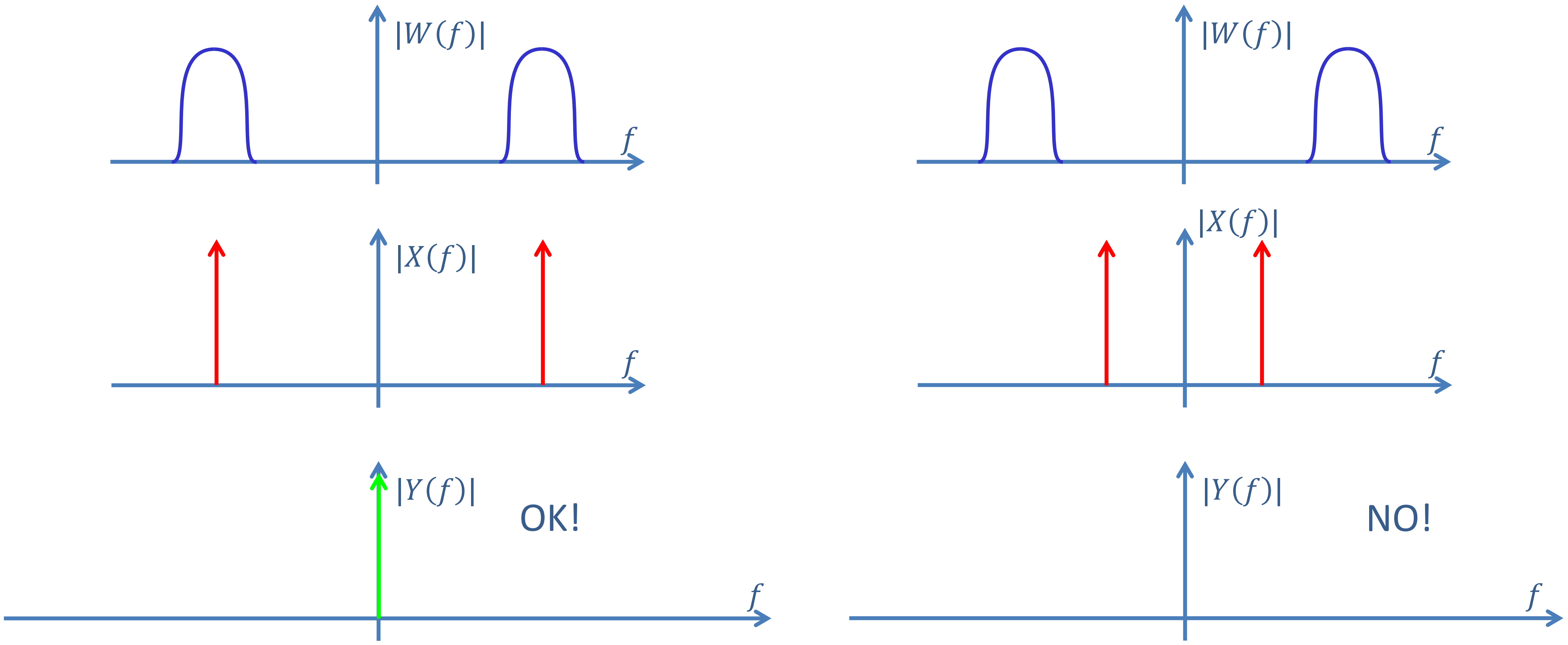
$$Y(f) = H_{LP}(f)\big(X(f) * W_R(f)\big)$$

- If $w_R(t)$ is a periodic signal, $|W(f)|$ represents the frequency components that give contributions in the baseband

Examples



Weighting function interpretation



PSD as optimum filter

- Let's take a simple case of constant signal $\Rightarrow$ the input of the PSD is then $x(t) = A\cos(\omega_r t)$
- If LPF is an LTI integrator

$$w_{LP}(t,\tau) = K\mathrm{u}(t-\tau) \Rightarrow w(t,\tau) \propto x(\tau)$$

$\Rightarrow$ PSD is the optimum filter!

- In the general case, the signal is not constant $\Rightarrow$ the PSD is quasi-optimum, but more flexible
  - Reference frequency set externally
  - BW of LPF can accomodate different signals

Another look at the weighting function

- For the case of the LTI integrator we have

$$y(t) = K\int_{-\infty}^{t} x(\tau)w_R(\tau)d\tau \approx K_{xw_R}(0)$$

- $y(t)$ is an estimate of the cross-correlation between input and reference signals $\Rightarrow$ maximum output is achieved when $x(t)$ has the same frequency and phase as $w_R(t)$
- For, say, a simple RC filter we have

$$y(t) = \frac{1}{T_F}\int_{-\infty}^{t} x(\tau)w_R(\tau)e^{-\frac{t-\tau}{T_F}}d\tau$$

which is again $K_{xw_R}(0)$ estimated over a time T_F

Frequency domain

- In the frequency domain we have

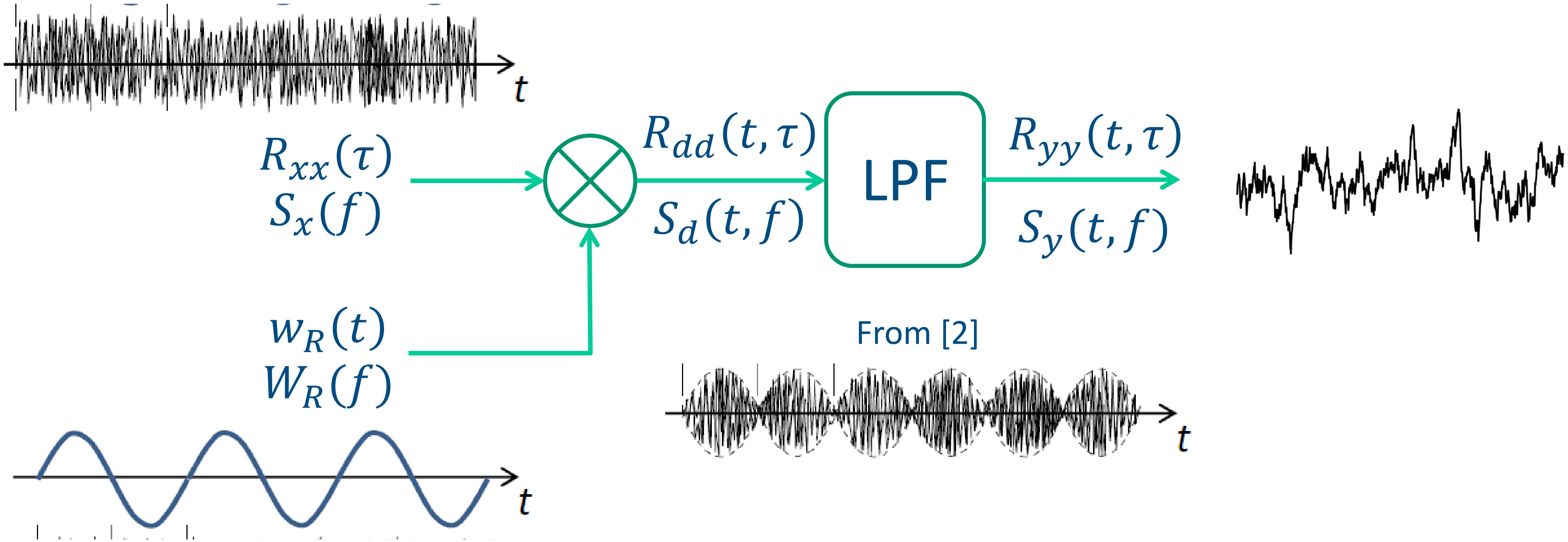
$$X(f) * W_R(f) = \int X(\nu) W_R(f - \nu) d\nu$$

- The output LPF selects the components around $f = 0$, i.e.

$$Y(f) \approx \int X(\nu) W_R(-\nu) d\nu = \int X(\nu) W_R^*(\nu) d\nu$$

We find again the correlation behavior!

Output noise



The output noise of the PSD is non-stationary (actually cyclostationary) even for stationary input noise

Output noise autocorrelation

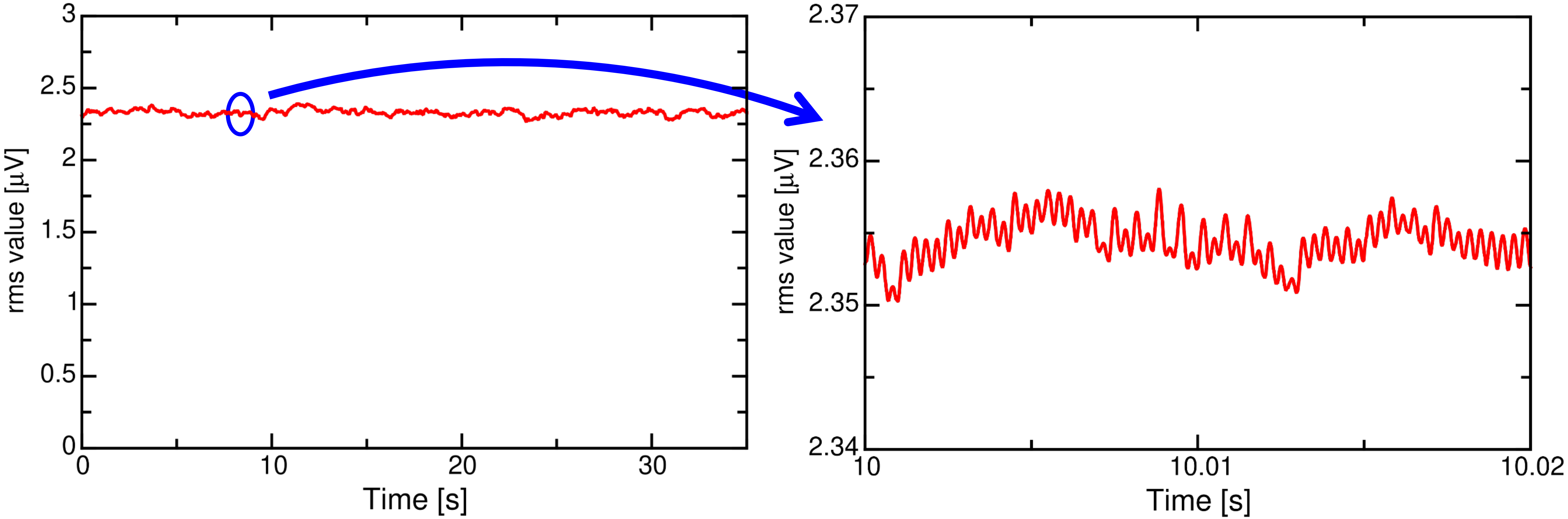
$$R_{dd}(t, t+\tau) = \overline{n_d(t)n_d(t+\tau)} = \overline{n_x(t)n_x(t+\tau)}w_R(t)w_R(t+\tau)$$
$$= R_{xx}(\tau)w_R(t)w_R(t+\tau)$$

- In particular, $\overline{n_d^2(t)} = \overline{n_x^2}w_R^2(t)$
- Since the output filter averages over many periods of w_R, we consider the time average

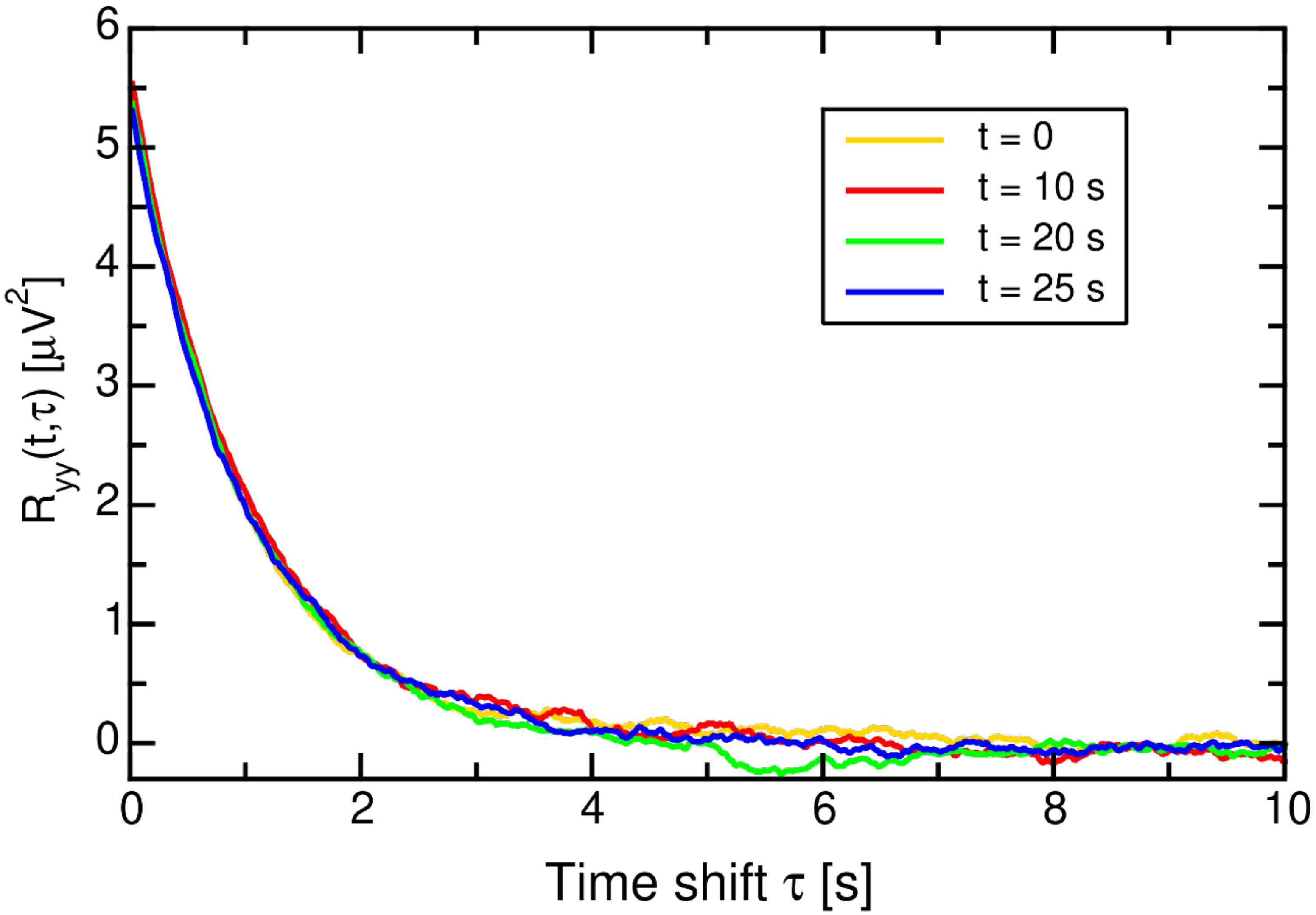
$$\langle R_{dd}(t, t+\tau)\rangle = R_{xx}(\tau)\langle w_R(t)w_R(t+\tau)\rangle$$
$$= R_{xx}(\tau)\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} w_R(t)w_R(t+\tau)dt = R_{xx}(\tau)K_{w_R w_R}(\tau)$$

Output noise rms value

- Input flicker noise, $K = 3.3 \times 10^{-8}$ V^2, $f_r = 1.5$ kHz, $T_F = 1$ s
- Output rms noise (ensemble average over 6000 realizations):



Output noise autocorrelation



Wrap up

- The reference signal is

$$w_R(t) = B\cos(\omega_r t) \Leftrightarrow W_R(f) = \frac{B}{2}\big(\delta(f - f_r) + \delta(f + f_r)\big)$$

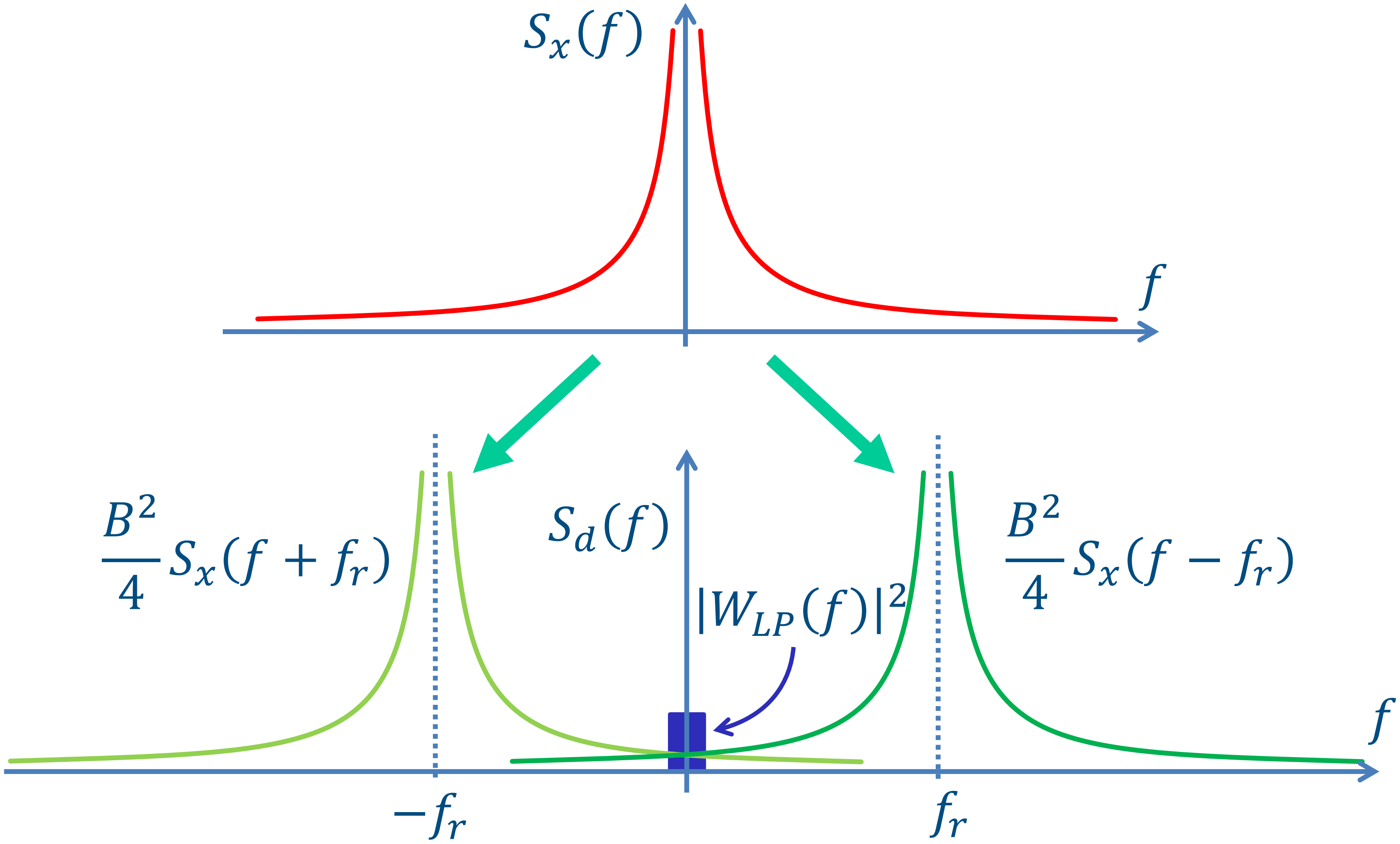
- Its time correlation is

$$K_{w_R w_R}(\tau) = \frac{B^2}{2}\cos(\omega_r \tau) \Leftrightarrow S_{w_R}(f) = \frac{B^2}{4}\big(\delta(f - f_r) + \delta(f + f_r)\big)$$

- The demodulated noise autocorrelation becomes

$$R_{dd}(\tau) = R_{xx}(\tau)\frac{B^2}{2}\cos(\omega_r \tau) \Leftrightarrow S_d(f) = \frac{B^2}{4}\big(S_x(f - f_r) + S_x(f + f_r)\big)$$

Ex: flicker noise spectra (LTI LPF)



Output noise

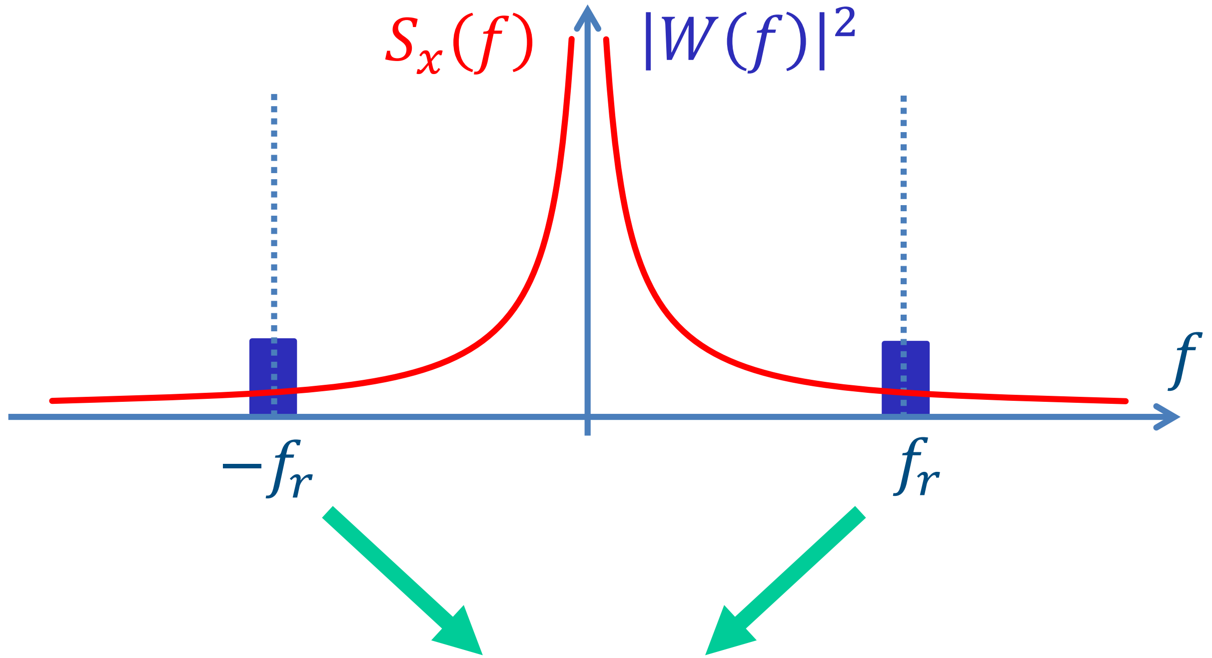
$$\overline{n_y^2} = \int S_d(f)|W_{LP}(f)|^2 df$$

$$= \frac{B^2}{4}\left(\int S_x(f - f_r)|W_{LP}(f)|^2 df + \int S_x(f + f_r)|W_{LP}(f)|^2 df\right)$$

$$= \frac{B^2}{4}\left(\int S_x(f)|W_{LP}(f + f_r)|^2 df + \int S_x(f)|W_{LP}(f - f_r)|^2 df\right)$$

$$= \int S_x(f)|W(f)|^2 df$$

Output noise



These are the input noise components that will contribute to the output noise $\Rightarrow$ remember the meaning of the weighting function!

Output rms noise

- We consider the equivalent noise bandwidth of $W_{LP}(f)$, BW_n
- We approximate $S_d(f)$ with $S_d(0)$ over BW_n (narrow-band output filter)
- The output mean square noise is then

$$\overline{n_y^2} = \int S_d(f)|W_{LP}(f)|^2 df \approx S_d(0)2BW_n$$

$$= 2BW_n \frac{B^2}{4}\left(S_x(-f_r) + S_x(f_r)\right) = B^2 BW_n\, S_x(f_r)$$

Previous example:

- $B = 1$ V, $f_r = 1.5$ kHz, $T_F = 1$ s $\Rightarrow BW_n = 0.25$ Hz
- WN case:

$$\overline{n_y^2} = \lambda\, BW_n = 2.5 \times 10^{-12}\ \text{V}^2 \Rightarrow \sqrt{\overline{n_y^2}} = 1.58\ \mu\text{V}$$

- FN case:

$$\overline{n_y^2} = \frac{K}{f_r}\, BW_n = 5.5 \times 10^{-12}\ \text{V}^2 \Rightarrow \sqrt{\overline{n_y^2}} = 2.35\ \mu\text{V}$$

S/N ratio

- We consider a constant (modulated) signal

$$x(t) = A\cos(\omega_r t)$$

- Output signal and noise are

$$y(t) = \frac{AB}{2} \quad \text{(no phase errors)}$$

$$\overline{n_y^2} = B^2 BW_n S_x(f_r)$$

$$\frac{S}{N} = \frac{A}{\sqrt{4S_x(f_r)BW_n}}$$

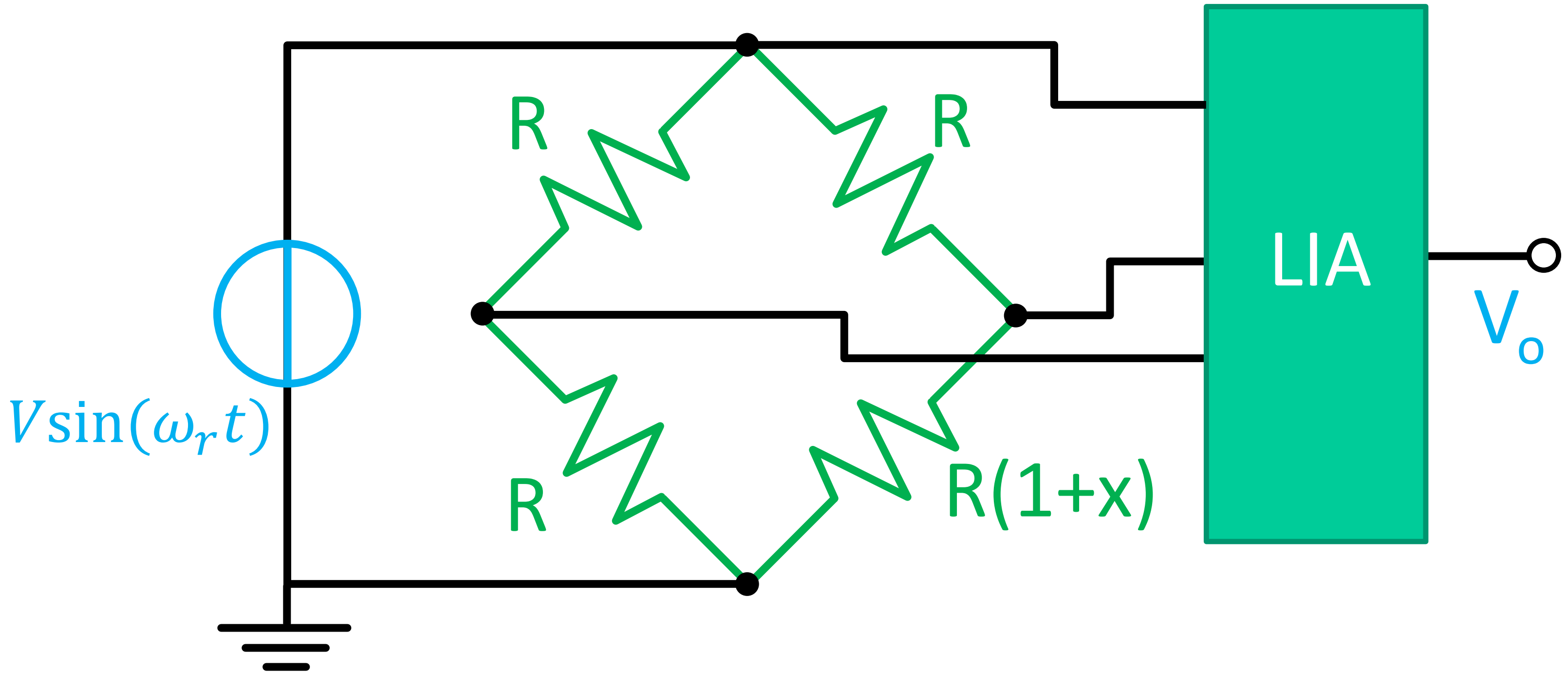
Remarks

- If only LPF were used, S/N would be

$$\frac{S}{N} = \frac{A}{\sqrt{2S_x(0)BW_n}}$$

- Synchronous detection is useful only if $S_x(0) \gg S_x(f_r)$
- The modulation stage must be inserted before the relevant LF noise sources (usually the amplifiers)

Wheatstone bridge with LIA



Low-light measurement with LIA