Electronics – 96032

POLITECNICO DI MILANO

Lock-in Amplifiers

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Disclaimer

Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes

Purpose of the lesson

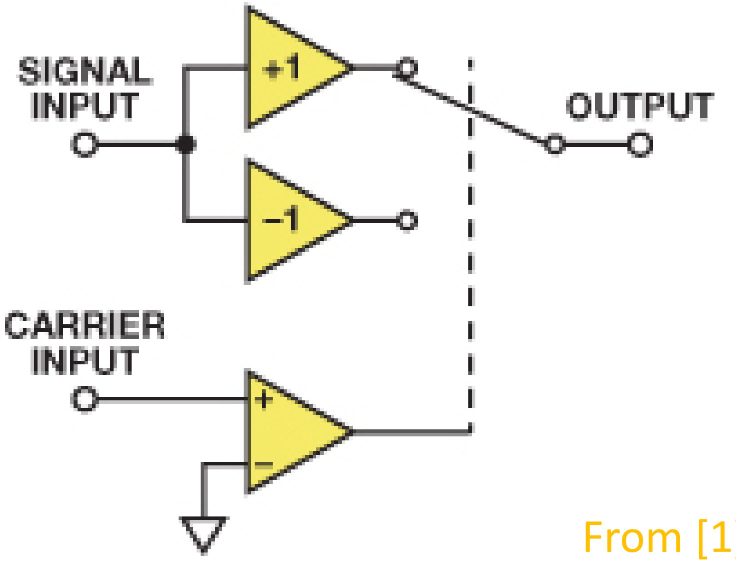
- To discuss how real signal recovery systems based on amplitude modulation are designed
- To highlight a few parameters and performance

What LIAs are

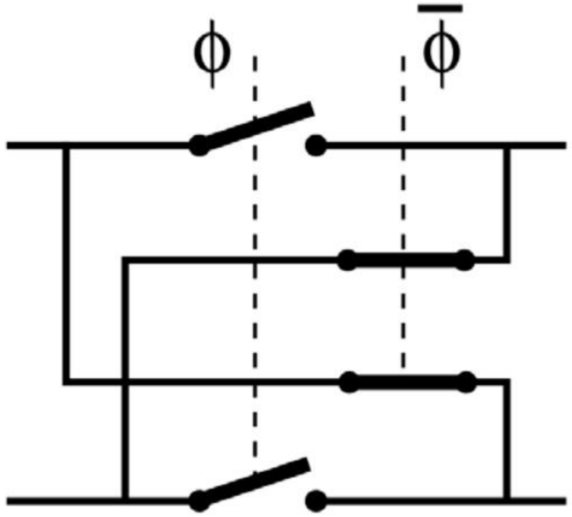
- LIAs are powerful tools that make use of a PSD to recover weak modulated signals buried in noise
- Several modifications are made to the basic scheme in order to reach high performance

Switching demodulators

- Analog multipliers are expensive and prone to nonlinearity errors
- Modulators/mixers are rather used, where the signal is multiplied by the sign of the carrier (i.e., by a square wave at f_r)



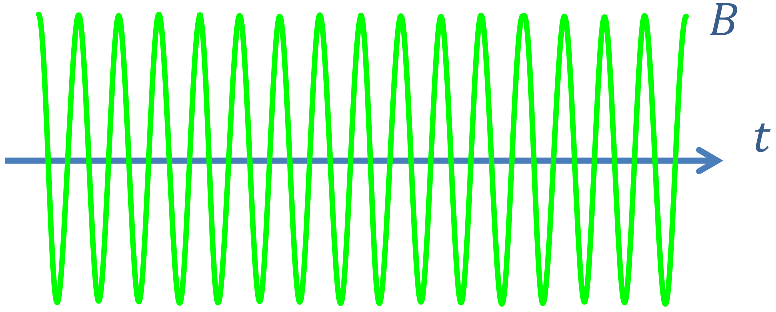
From [1]



Advantages

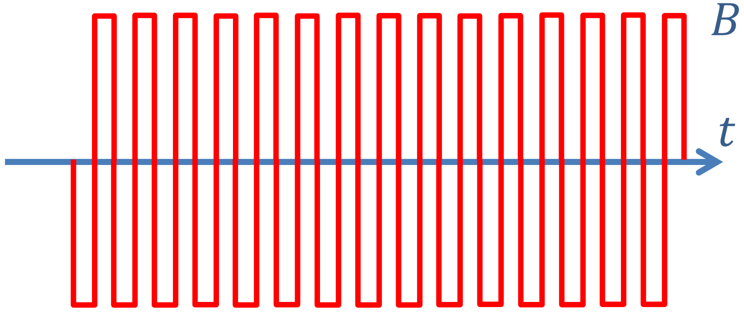
- Any noise or amplitude variation on the reference can be ignored
- The noise is mainly due to the switches and is usually lower than in multipliers
- Easier to design and manufacture

Reference functions – time domain



Sinusoidal case:

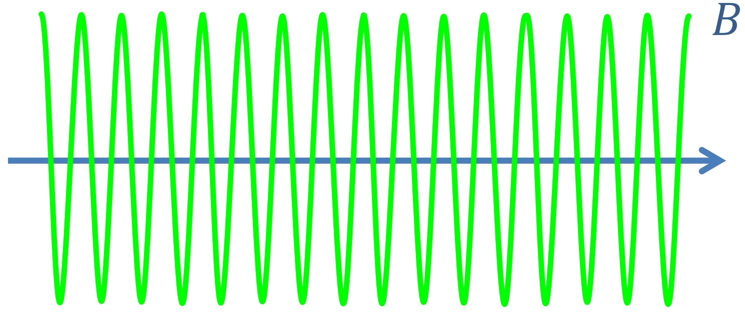
$$w_R(t) = B\cos(\omega_r t)$$

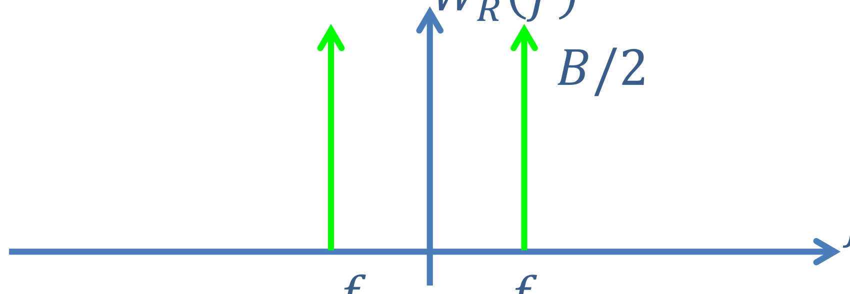


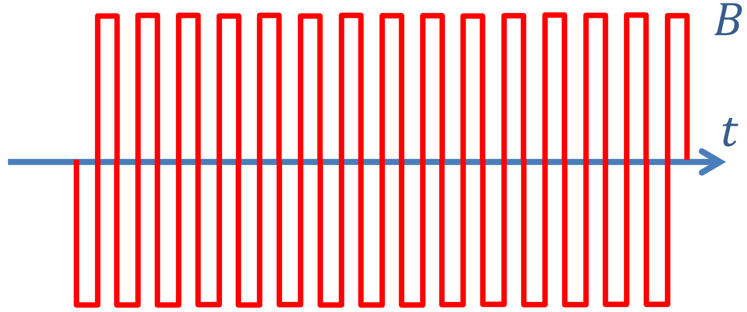
Square-wave case:

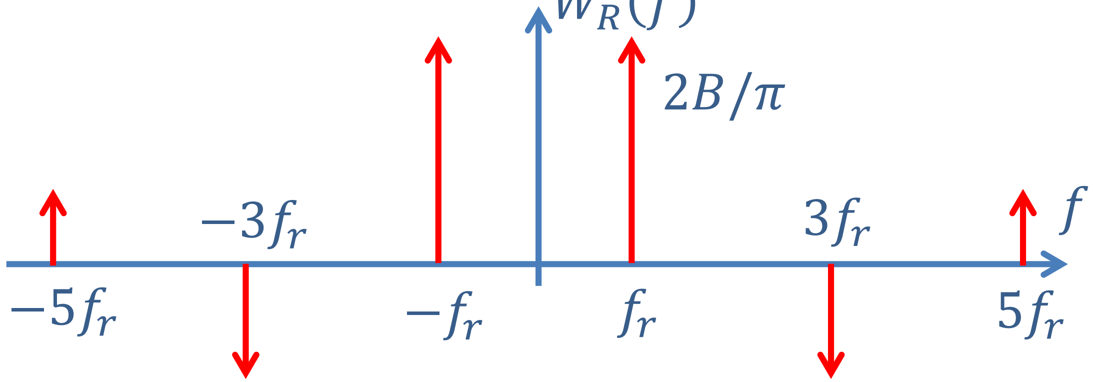
$$w_R(t) = \frac{4B}{\pi}\sum_k \frac{(-1)^k}{2k+1}\cos\big((2k+1)\omega_r t\big)$$

Reference functions – frequency domain



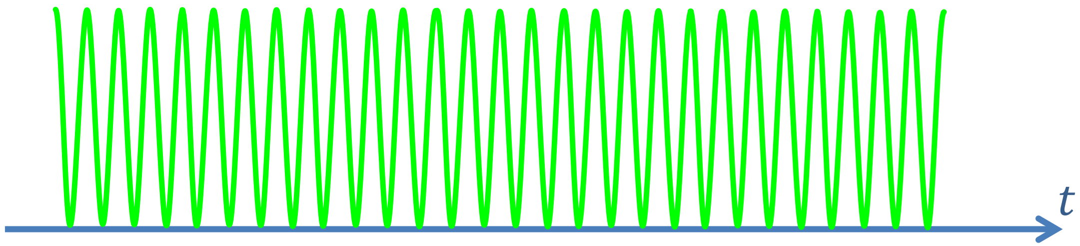






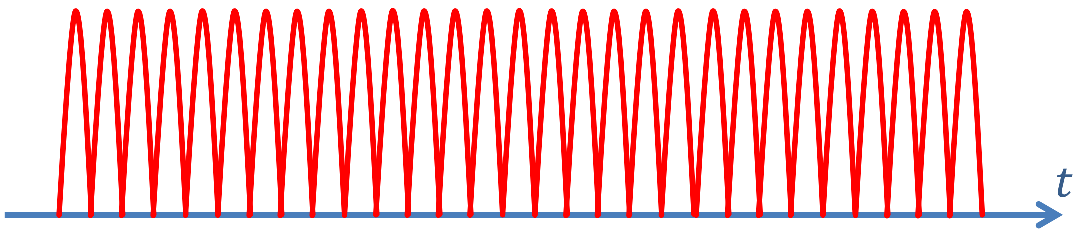
$$W_R(f) = \frac{2B}{\pi} \sum_k \frac{(-1)^k}{2k+1} \left(\delta(f - (2k+1)f_r) + \delta(f + (2k+1)f_r) \right)$$

Mixer output signals



Sinusoidal demodulation

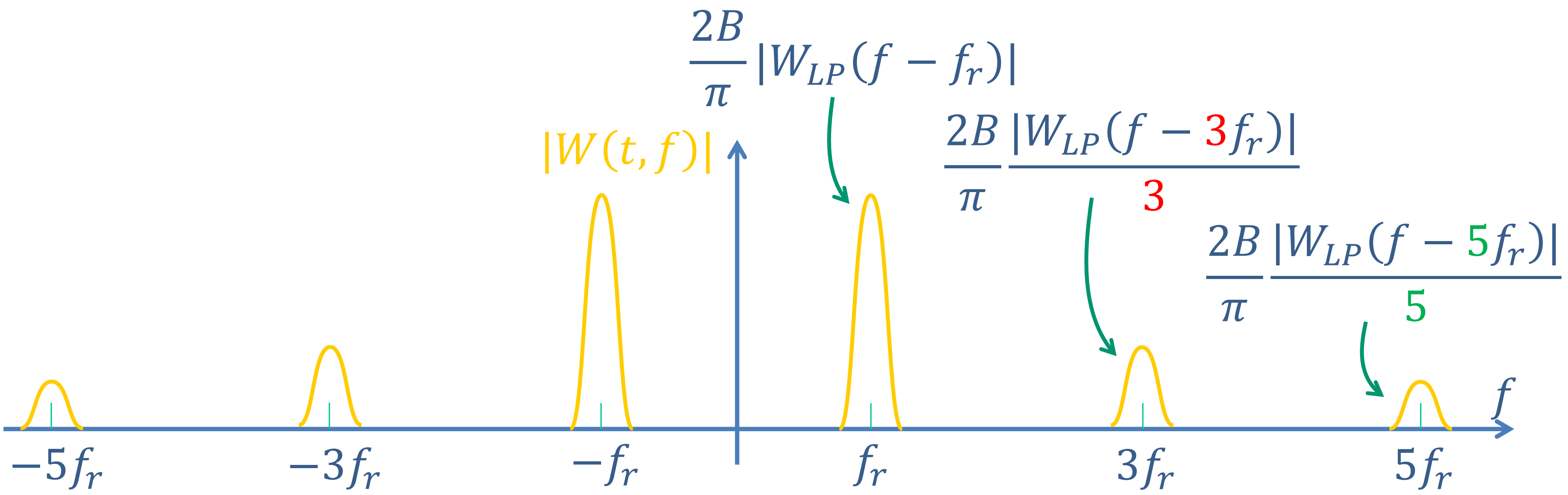
$$d(t) = AB\cos^2(\omega_r t)$$



Square-wave demodulation

$$d(t) = AB|\cos(\omega_r t)|$$

Switching PSD spectral response



$$W(t,f) = W_R(f) * W_{LP}(f)$$

Time correlation of reference signal

$$w_R(t) = \sum_k 2B_k \cos(k\omega_r t) \Leftrightarrow$$

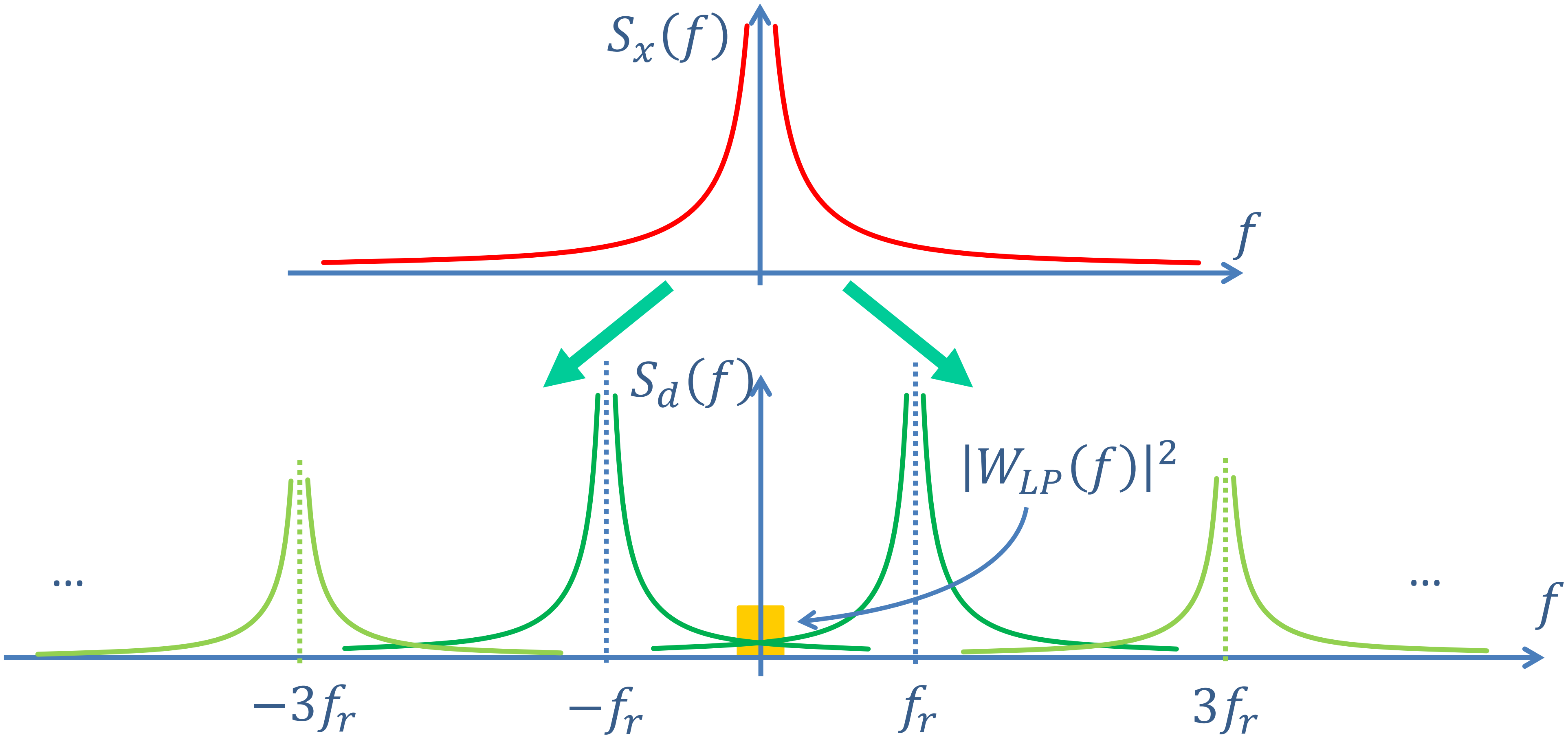
$$W_R(f) = \sum_k B_k\left(\delta(f - kf_r) + \delta(f + kf_r)\right)$$

$$K_{w_R w_R}(\tau) = \sum_k 2B_k^2 \cos(k\omega_r \tau) \Leftrightarrow$$

$$S_{w_R}(f) = \sum_k B_k^2\left(\delta(f - kf_r) + \delta(f + kf_r)\right)$$

B_k = amplitude of delta functions (Fourier coefficients)

Ex: input LF noise



$$S_d(f) = S_x(f) * S_{w_R}(f) = \sum_k B_k^2 \left(S_x(f - kf_r) + S_x(f + kf_r) \right)$$

Output rms noise

The output filter collects noise from all replicas:

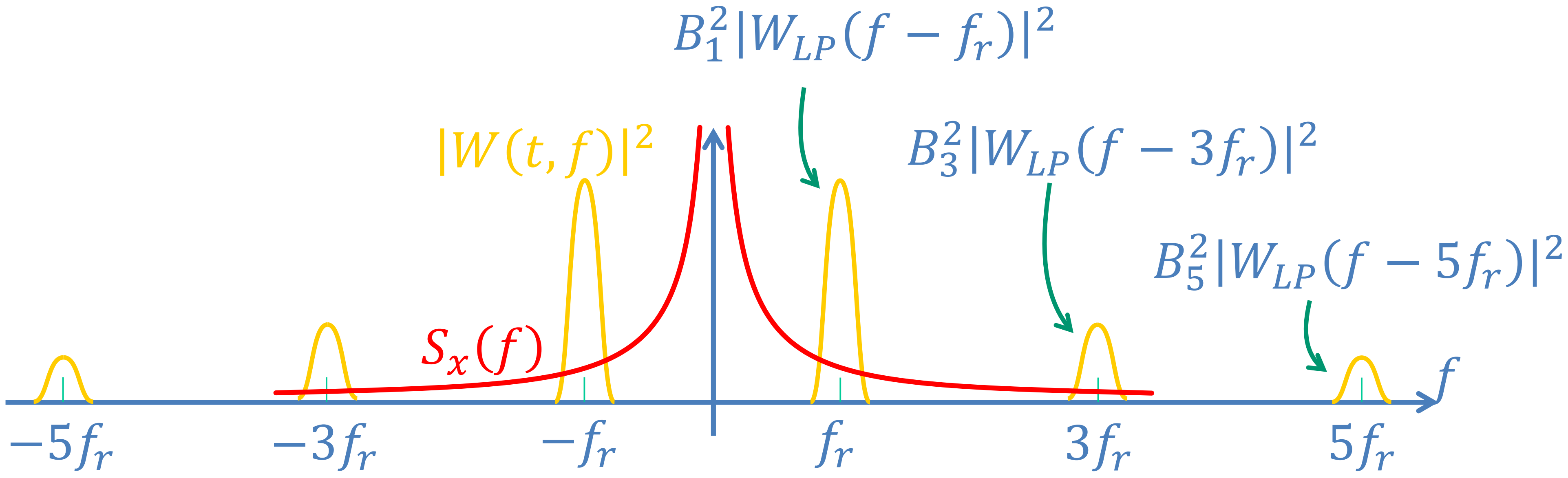
$$\overline{n_y^2} = \int S_d(f)|W_{LP}(t,f)|^2 df \approx S_d(0) 2BW_n$$

$$= 2BW_n \sum_k B_k^2 \left(S_x(-kf_r) + S_x(kf_r)\right) = 4BW_n \sum_k B_k^2 S_x(kf_r)$$

$$= 4BW_n B_1^2 S_x(f_r) \sum_k \frac{B_k^2}{B_1^2} \frac{S_x(kf_r)}{S_x(f_r)}$$

Extra noise collected by the harmonics

Another look at noise collection



The output noise is controlled by the spectral response of the PSD

S/N ratio (square wave)

- We consider a constant (sin-modulated) signal $x(t) = A\cos(\omega_r t)$
- Output signal and noise are

$$y(t) = A\frac{2B}{\pi} \qquad \overline{n_y^2} = 4BW_n\left(\frac{2B}{\pi}\right)^2 \sum_k \frac{S_x((2k+1)f_r)}{(2k+1)^2}$$

$$\frac{S}{N} = \frac{A}{\sqrt{4BW_n \sum_k \frac{S_x((2k+1)f_r)}{(2k+1)^2}}} = \frac{A}{\sqrt{4BW_n S_x(f_r) \sum_k \frac{S_x((2k+1)f_r)}{S_x(f_r)(2k+1)^2}}}$$

A few cases – 1

- If we are modulating beyond the noise corner frequency, $S_x((2k+1)f_r)$ is white (constant) and

$$\sum_k \frac{1}{(2k+1)^2} = \frac{\pi^2}{8} \approx 1.2337$$

$$\left(\frac{S}{N}\right)_{\text{sqd}} = \frac{1}{\sqrt{1.2337}}\left(\frac{S}{N}\right)_{\text{sin}} = \frac{1}{1.11}\left(\frac{S}{N}\right)_{\text{sin}}$$

- S/N is (slightly) degraded because of the extra noise collected at the higher harmonics, where there is no signal

A few cases – 2

- If instead there is a pure $1/f$ input noise spectrum, we have $S_x((2k+1)f_r) = \frac{Const}{(2k+1)f_r}$ and the extra noise becomes

$$\sum_k \frac{1}{(2k+1)^3} \approx 1.0518$$

$$\left(\frac{S}{N}\right)_{\text{sqd}} = \frac{1}{\sqrt{1.0518}}\left(\frac{S}{N}\right)_{\text{sin}} = \frac{1}{1.0256}\left(\frac{S}{N}\right)_{\text{sin}}$$

A few cases – 3

- If the signal is also modulated by a square wave (full-square demodulation), we get (white noise case)

$$y(t) = AB = \frac{\pi}{2}\big(y(t)\big)_{\text{sin}} \qquad \overline{n_y^2} = \frac{\pi^2}{8}\left(\overline{n_y^2}\right)_{\text{sin}}$$

$$\left(\frac{S}{N}\right)_{\text{fsq}} = \frac{\pi}{2}\frac{\sqrt{8}}{\pi}\left(\frac{S}{N}\right)_{\text{sin}} \approx 1.41\left(\frac{S}{N}\right)_{\text{sin}}$$

- Note that in the sinusoidal case, the signal is the first harmonic of the square wave

A few cases – 4

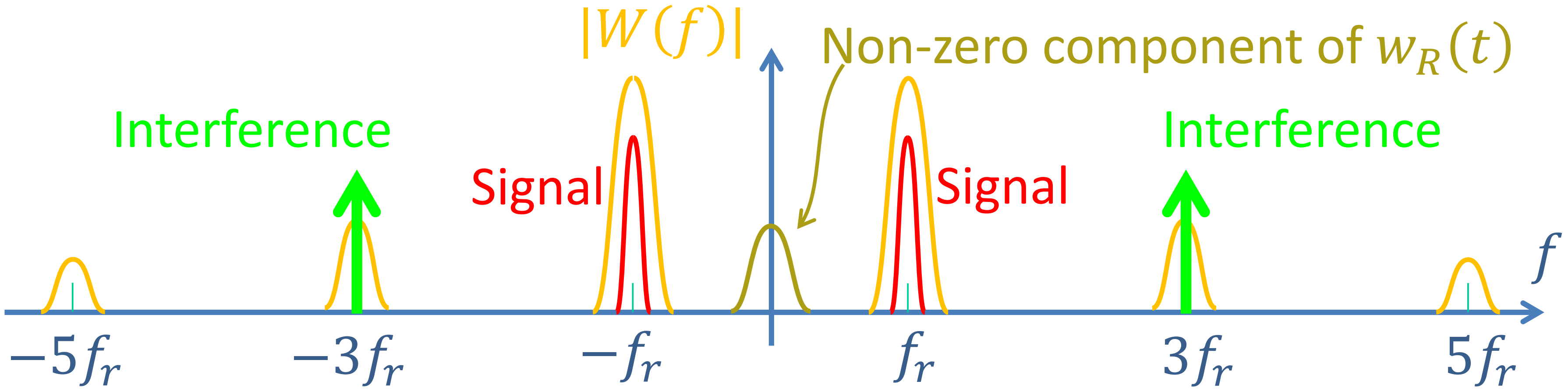
- For square-wave signal and sinusoidal demodulation we would get

$$y(t) = \frac{4}{\pi}\frac{AB}{2} = \frac{4}{\pi}\big(y(t)\big)_{\text{sin}} \qquad \overline{n_y^2} = \left(\overline{n_y^2}\right)_{\text{sin}}$$

$$\left(\frac{S}{N}\right)_{\text{sqs}} = \frac{4}{\pi}\left(\frac{S}{N}\right)_{\text{sin}} \approx 1.27\left(\frac{S}{N}\right)_{\text{sin}}$$

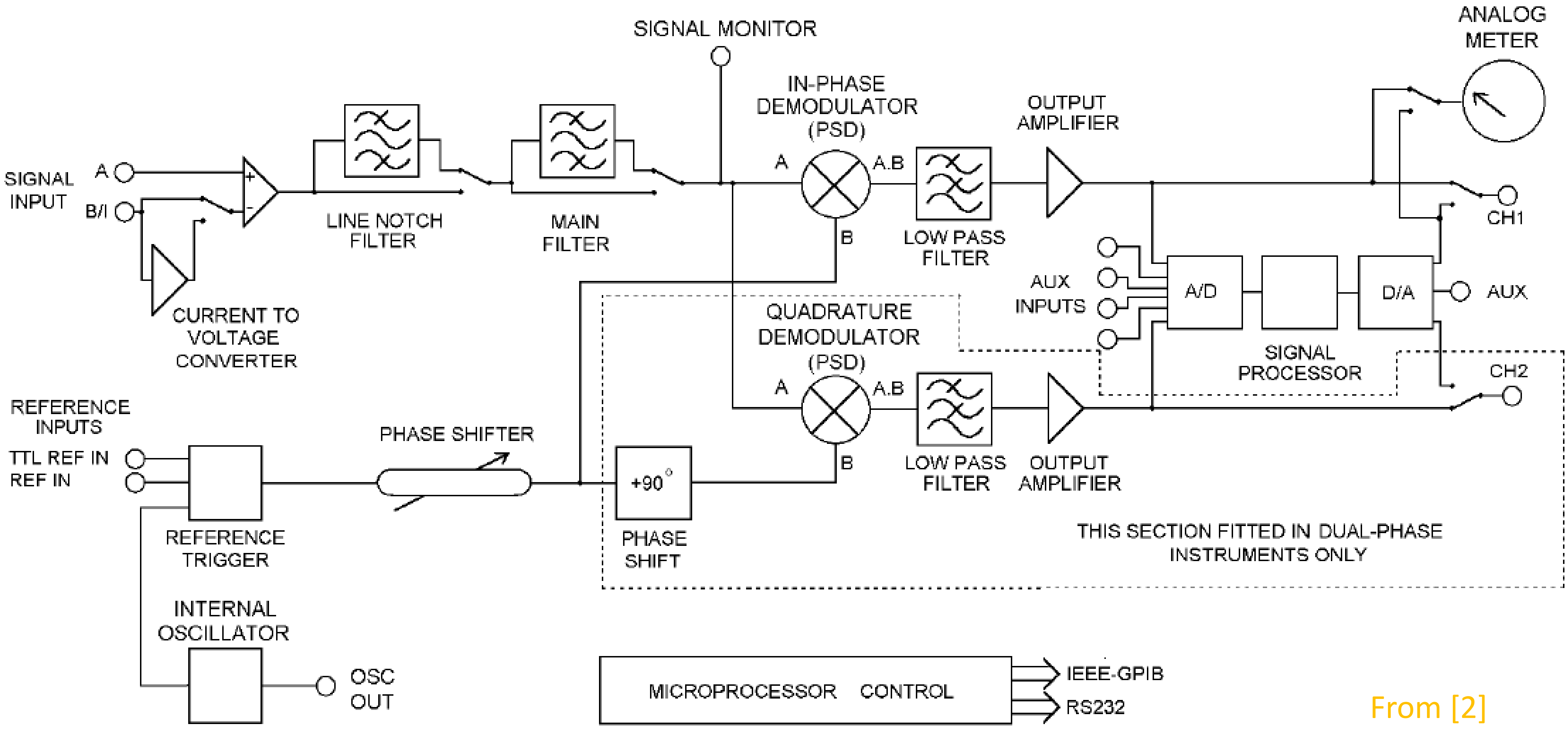
- Note that the sinusoidal demodulation only detects the first harmonic of the square wave, with amplitude $4A/\pi$

Switching PSD issues



- If $\langle w_R(t) \rangle \neq 0$ the lock-in will collect low-frequency noise, wasting all efforts (also true for sinusoidal modulation)
- The output is sensitive to localized interferences at the harmonics of $f_r \Rightarrow$ keep f_r away from multiples of 50 (or 60) Hz

Actual structure



From [2]

Signal channel components

- Notch filter: removes 50/60 Hz and/or 2nd harmonics from signal (check Q value)
- Main filter: either LP, BP or notch
  - BP is usually employed (cleans up the signal improving dynamic reserve – see later)
  - LP can be used for low f_r
  - Flat response (no filter) can be selected for non-demanding applications
  - Always check the roll-off of the filter

Ref. channel components and PSD

- Reference trigger: generates a trigger signal (e.g. sqw) synchronous with the reference (or at $2f_r$)
- Phase shifter: adjusts the phase of the reference to track the signal
- PSD: usually a square-wave mixer. More refined implementations (e.g. suppressing 3rd harmonic response) may be employed
- Two channels working in quadrature are always employed

Output stage components

- Low-pass filter: either first- or second-order filters, usually specified via the time constant
  - 1st order may be not good enough in rejecting aliases
  - 2nd order may give problems if LIA is used within a feedback loop
- Output amplifier: provides gain increasing the sensitivity. Since it is DC-coupled, its drift and noise must be controlled

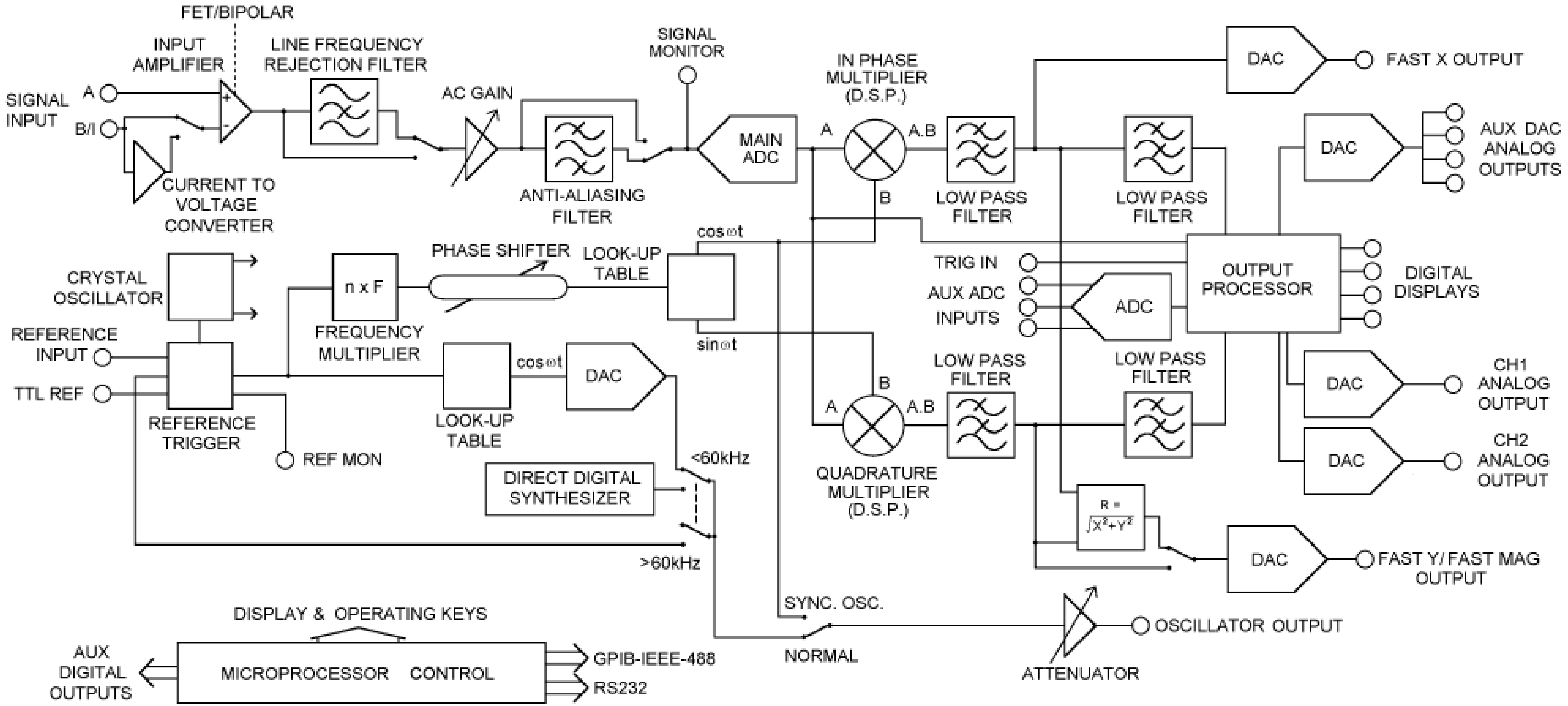
Main parameters

- Frequency range: typically from a fraction of Hz to a few MHz (200 MHz for RF LIAs)
- Output time constants: ms to thousands of s
- Dynamic reserve: is the ratio of the largest “tolerable” noise signal to the full scale signal, expressed in dB. Typical values are $\leq$ 60 dB

More on the dynamic reserve

- “tolerable” noise refers to a specified precision in the measurement of the signal
- DR depends on the frequency of the noise. The quoted value is the maximum value, achievable when noise is away from f_r
- DR is eventually limited by PSD non-linearity and overload
- More information in [3]

Digital/DSP LIAs



From [4]

The PSD is implemented in digital circuitry

Advantages

- Reference signals are pre-computed with high precision ⇒ better PSD performance (offset, harmonics,…). DR of 100 dB can be achieved
- Much lower f_r (mHz) are allowed
- More flexibility in the (digital) LPF design (higher orders, longer time constants,…)
- No output amplifier ⇒ better stability

What is left for analog LIAs?

- Demodulation at high frequencies (MHz), since f_r is limited to a few MHz for digital LIAs
- Applications requiring short time constants at mid-range frequencies (≈ 100 kHz)
- Feedback loops employing LIAs

- More discussion on analog and digital LIAs in [5]

A comparison chart

Make/Model	Features	Price Range	Remarks
SRS Model SR830	• 1 mHz to 102.4 kHz frequency range • 256 kHz front-end sampling rate • > 100 dB dynamic reserve (5 ppm/C stability) • Auto gain, phase, reverse, and offset • 0.01 deg phase resolution • Time constant from 10us to 30 ks • GPIB, and RS-232 interfaces	~ $5,200	DSP Lock-in Amplifier with 41/2-digit LED display. Power 40 W 100/120/220/240 VAC, 50/60Hz
SRS Model SR850	• 1 mHz to 102.4 kHz frequency range • CRT display, and built-in data analysis • 64,000 point data buffer • Chart recording, numeric display, polar plots • Smoothing, curve fitting, and statistics • Direct printing, and plotting • 0.001 deg phase resolution • >100 dB dynamic reserve • GPIB, and RS-232 interfaces	~ $8,700	DSP Lock-in Amplifier. Power 60W 100/120/220/240 VAC, 50/60Hz
SRS Model SR844	• 25 kHz to 200 MHz frequency range • 80 dB dynamic reserve (5 ppm stability) • Auto gain, phase, reverse, and offset • Internal or external reference • Time constant from 100us to 30 ks • No Time Constant mode • GPIB, and RS-232 interfaces	~ $9,200	RF Lock-in Amplifier For high frequency applications.
Perkin Elmer/ EG&G Model 7265	• 0.001 Hz to 250 kHz frequency range • Voltage and current mode inputs • 10 us to 100 ks output time constants • Quartz crystal stabilized internal oscillator • Harmonic measurements to 65,536F • Dual reference, Dual harmonic, and Virtual Reference modes • Spectral display modes	~$5,300	Dual Phase DSP Lick-in Amplifier
Perkin Elmer/ EG&G Model 7280	• 0.5 Hz to 2 MHz frequency range • Voltage and current mode inputs • 7.5 MHz main ADC sampling rate • 1 us to 100 ks output time constants • Quartz crystal stabilized internal oscillator • Harmonic measurements to 32F • Dual reference, Dual harmonic, and Virtual Reference modes • Spectral display mode	~$12,500	Dual Phase wide bandwidth DSP Lock-in Amplifier. Power 200 VA max 100/120/220/240 VAC, 50/60Hz

From [6]

References

1. http://www.analog.com/library/analogdialogue/archives/47-06/multipliers_modulators.html
2. http://cpm.uncc.edu/sites/cpm.uncc.edu/files/media/tn1002.pdf
3. http://www.thinksrs.com/downloads/PDFs/ApplicationNotes/AboutLIAs.pdf
4. http://mrflip.com/papers/LIA/LIAProgram/Support/References/AmetekSR-TechNote1003.pdf
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