Electronics – 96032

POLITECNICO DI MILANO

Wheatstone Bridge

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Disclaimer

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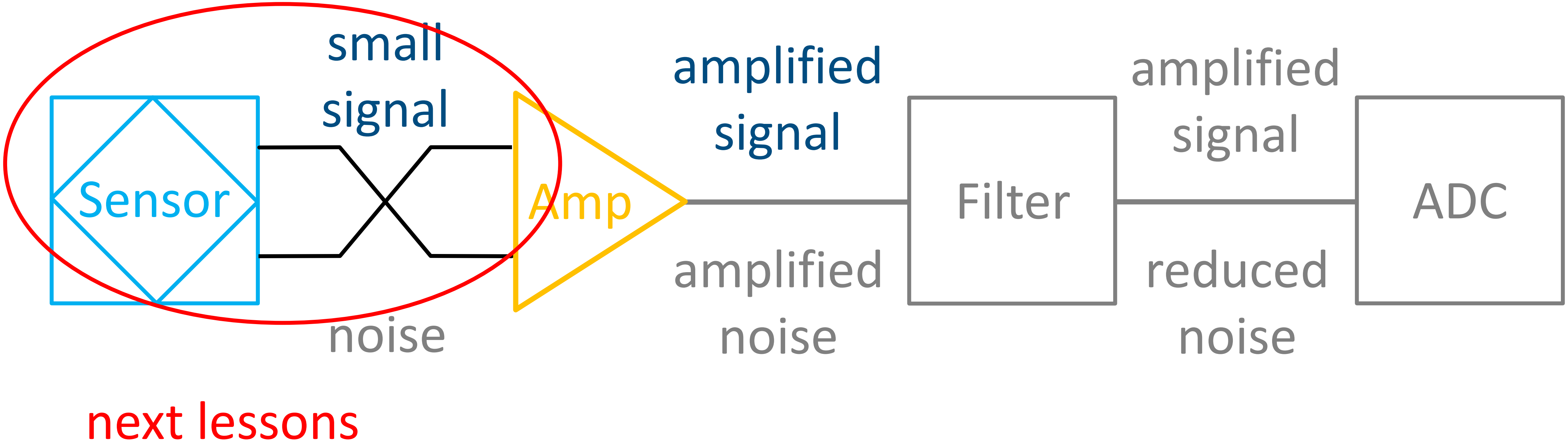
Acquisition chain



Purpose of the lesson

- At this point, we know how to analyze and design simple amplifiers
- Effective amplifier design depend upon the input signal characteristics (impedance, bandwidth,…)
- In this part of the class we discuss the Wheatstone bridge arrangement

Outline

- Wheatstone bridge and sensitivity
- Effect of wire resistance
- Temperature compensation

Resistive sensors

- Resistors whose value changes with variation in a physical quantity *S* (light, heat, stress,…)
- Among the most common in instrumentation
- For small changes in *S*, a linear approximation holds:

$$R = R_0(1 + \alpha\Delta S) = R_0(1 + x)$$

where $\alpha = \frac{1}{R_0}\left.\frac{dR}{dS}\right|_{R_0}$

Common examples

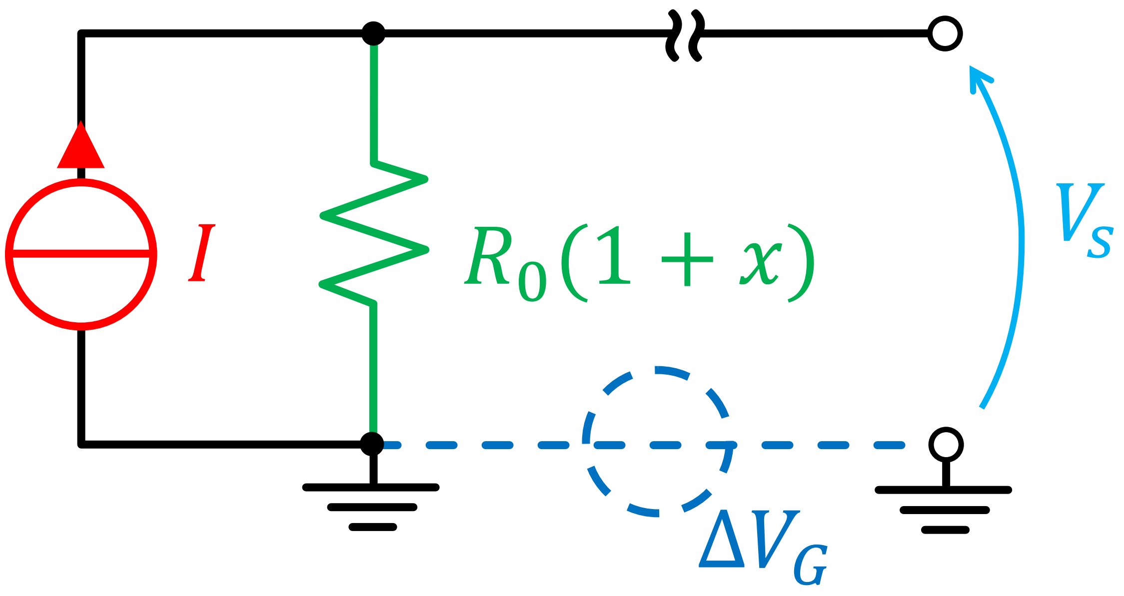
- Temperature sensors

$$\alpha = TCR = \frac{1}{R_0}\frac{\Delta R}{\Delta T}$$

- Deformation sensors

$$\alpha = GF = \frac{1}{R_0}\frac{\Delta R}{\varepsilon} \qquad \left(\varepsilon = \frac{\Delta L}{L} = \text{strain}\right)$$

Single-ended measurements

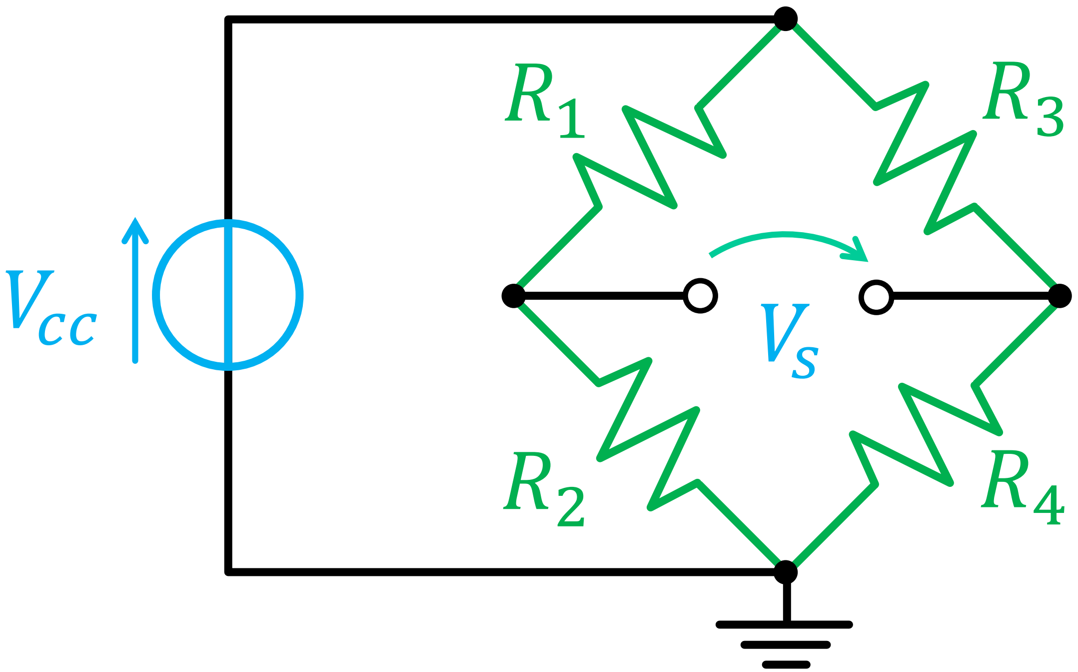


- Noise and fluctuations in ground potential and R_0 degrade performance
- Can be used for high-level signals, low noise, short distance environments

$$V_s = IR_0 + IR_0x + \Delta V_G + I\Delta R_0 + I\Delta R_0 x$$

Offset + Signal + Disturbs

Wheatstone bridge



$$V_s = V_{cc}\left(\frac{R_4}{R_3 + R_4} - \frac{R_2}{R_1 + R_2}\right)$$

Insensitive to ground potential and resistance fluctuations

Bridge balancing

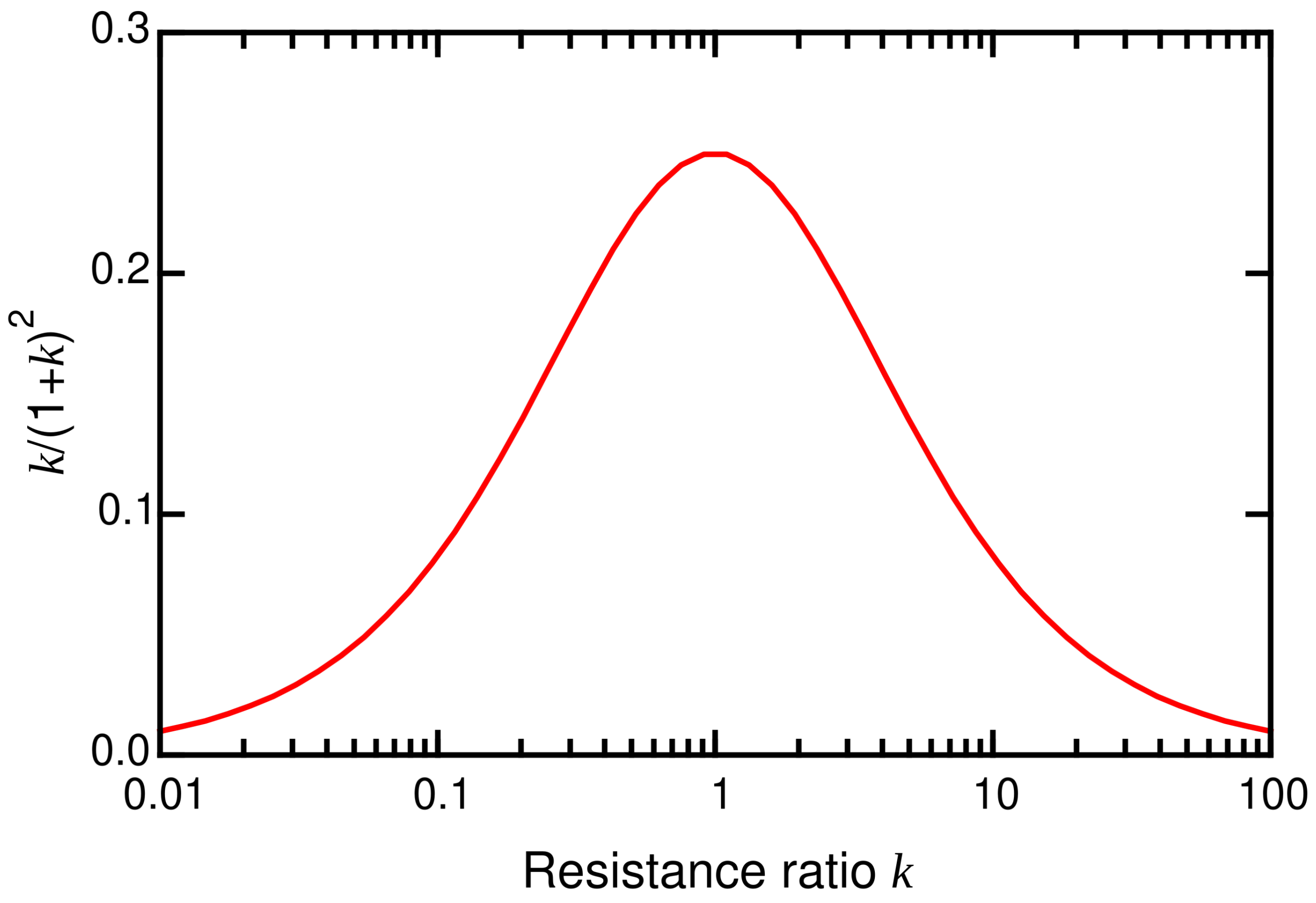
- We set $V_s = 0$ for $x = 0$:

$$\left(\frac{1}{1 + R_3/R_4} - \frac{1}{1 + R_1/R_2}\right) = 0 \Rightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4} = k$$

- We pick k by requiring maximum V_s sensitivity to resistance variation:

$$\frac{dV_s}{dR_1} = \frac{V_{cc}}{\left(1 + \frac{R_1}{R_2}\right)^2} \frac{1}{R_2} = \frac{V_{cc}}{R_1} \frac{k}{(1+k)^2}$$

Bridge sensitivity to k



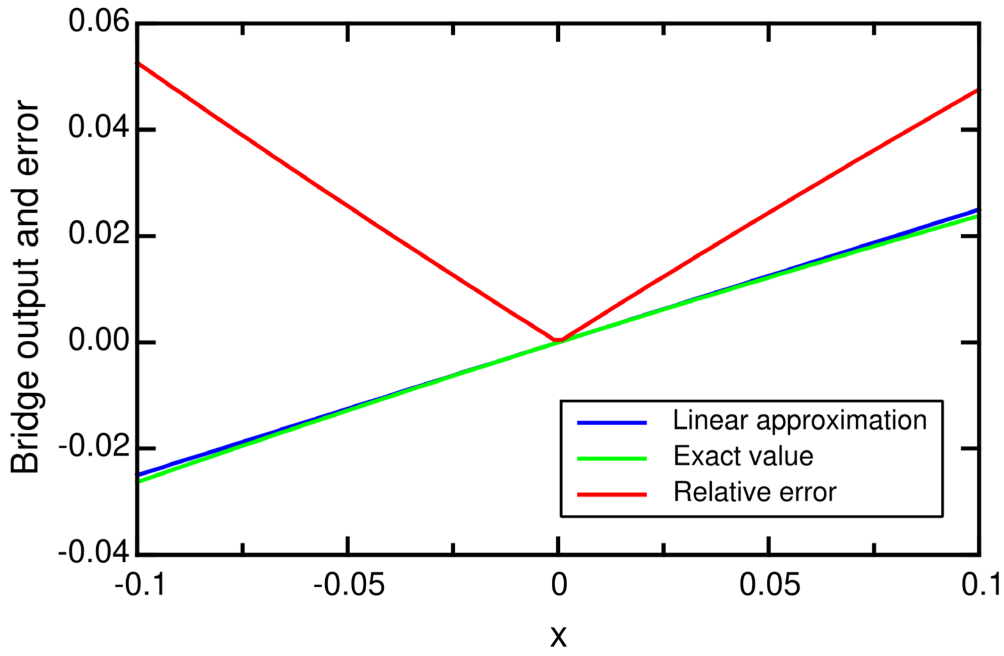
Unbalanced bridge

Let's assume $R_1 = R_2 = R_3 = R$; $R_4 = R(1+x)$. We have:

$$V_s = V_{cc}\left(\frac{R(1+x)}{R(2+x)} - \frac{1}{2}\right) = V_{cc}\frac{x}{2(2+x)} \approx V_{cc}\frac{x}{4}$$

- The non-linearity relative error is

$$\varepsilon_{NL} = \left|\frac{x}{4}\frac{2(2+x)}{x} - 1\right| = \frac{|x|}{2}$$



Example

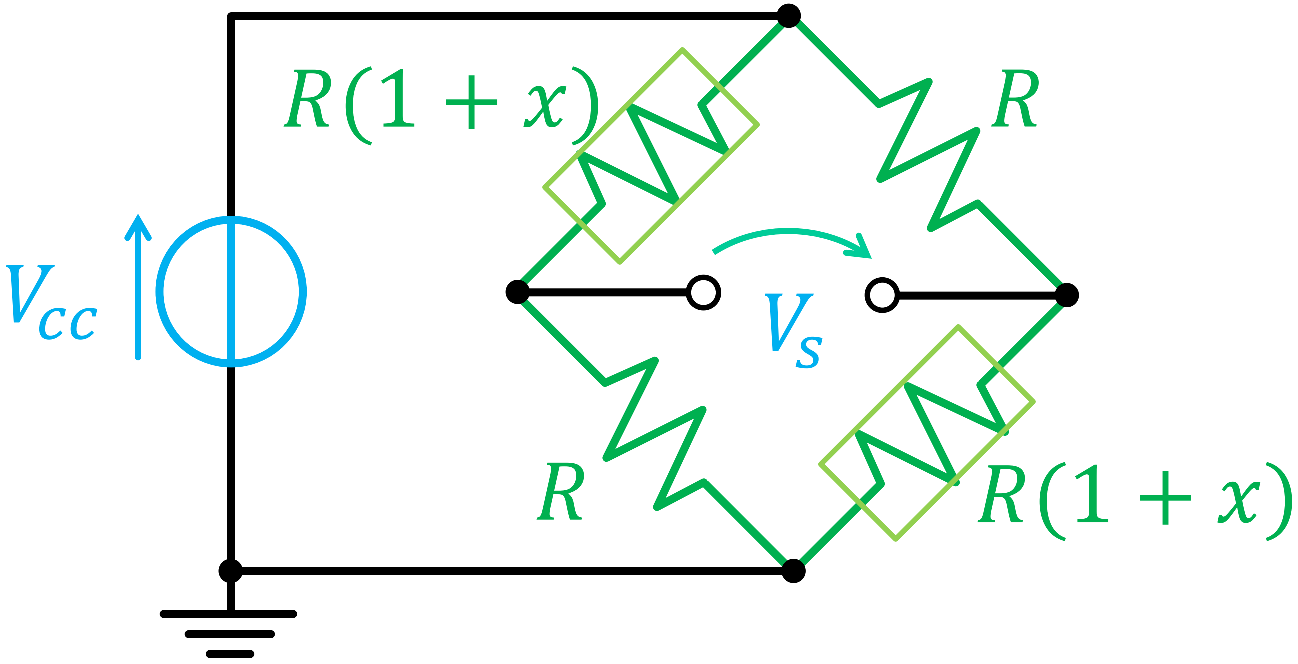
- With $V_{cc} = 5$ V and $x^{max} = 1\%$ (large!), we have

$$V_s^{max} = V_{cc}\frac{x^{max}}{4} = 12.5\ \text{mV} \quad (\approx 12.44\ \text{mV in reality})$$

- The non–linearity error is $\varepsilon_{NL} = \frac{x}{2} = 0.5\%$

- V_{cc} usually cannot be increased (compatibility with other blocks and power dissipation issues) $\Rightarrow$ The output is a small signal!

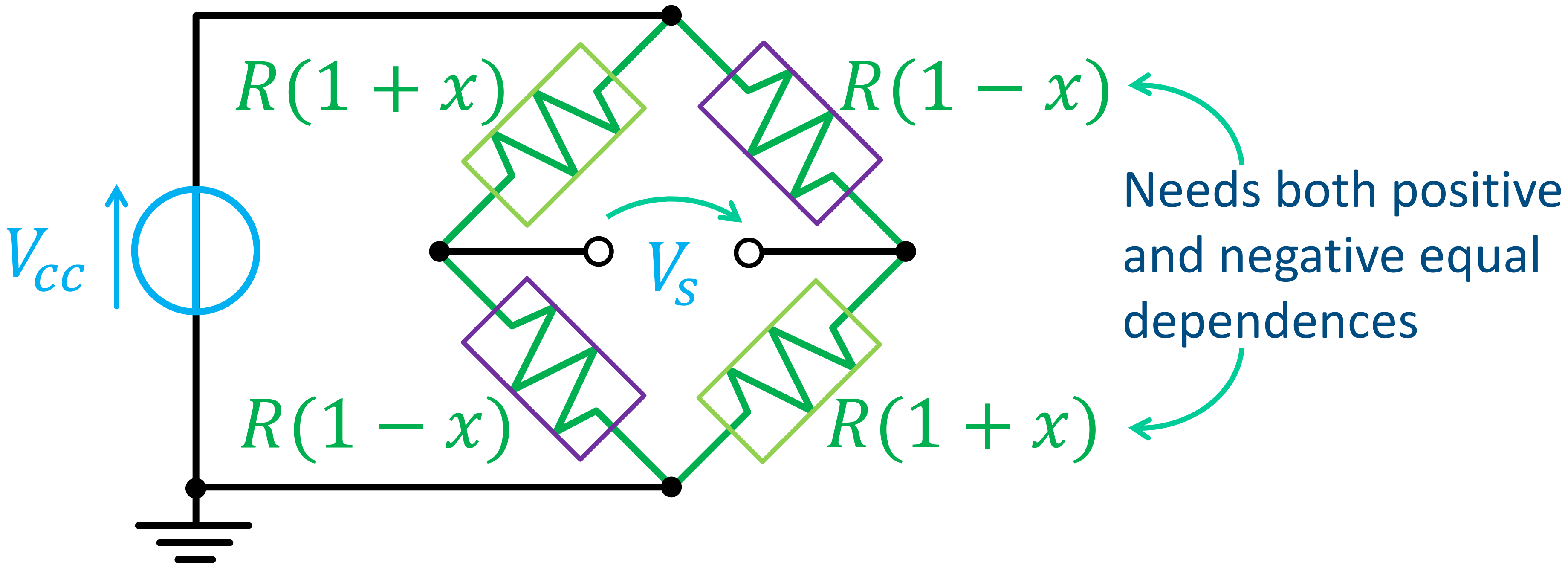
Double sensitivity



$$\varepsilon_{NL} = \frac{x}{2}$$

$$V_s = V_{cc}\left(\frac{R(1+x)}{R(2+x)} - \frac{R}{R(2+x)}\right) = V_{cc}\frac{x}{2+x} \approx V_{cc}\frac{x}{2}$$

Maximum sensitivity

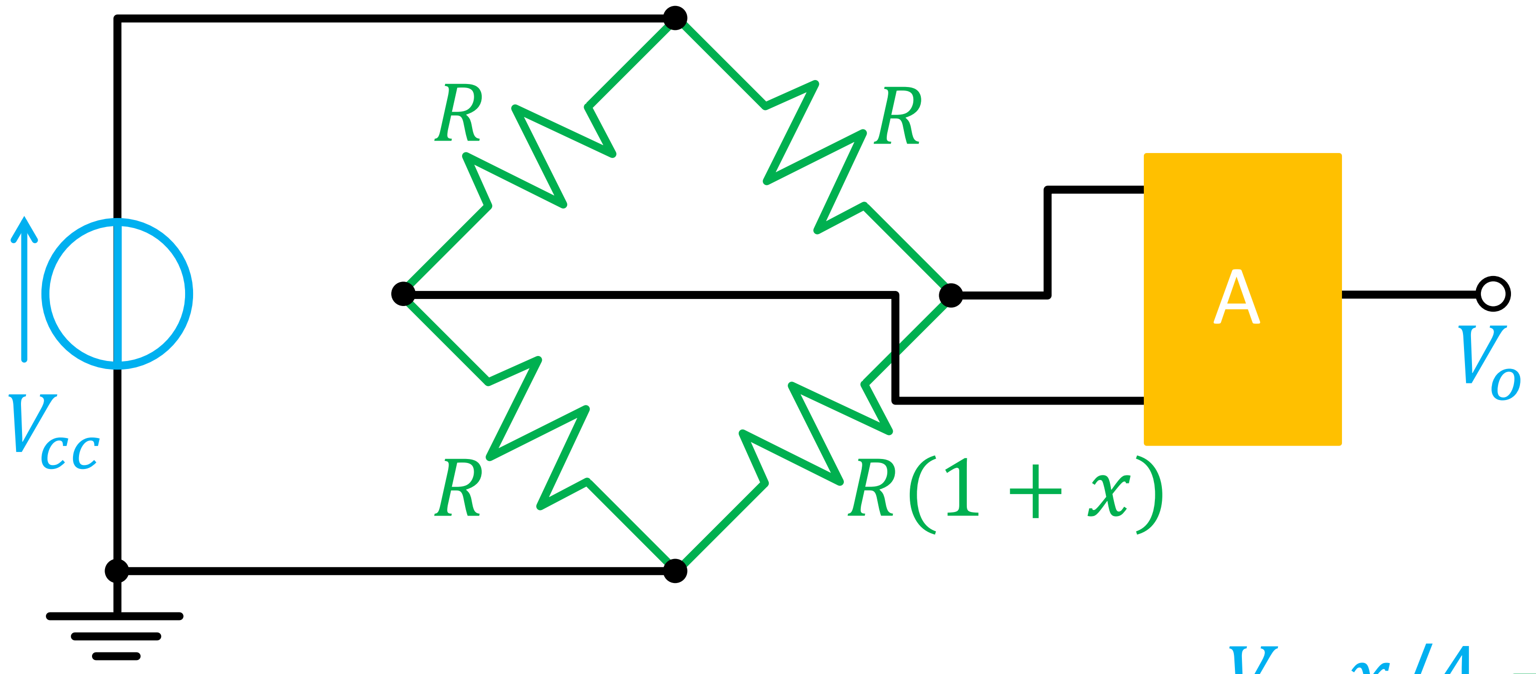


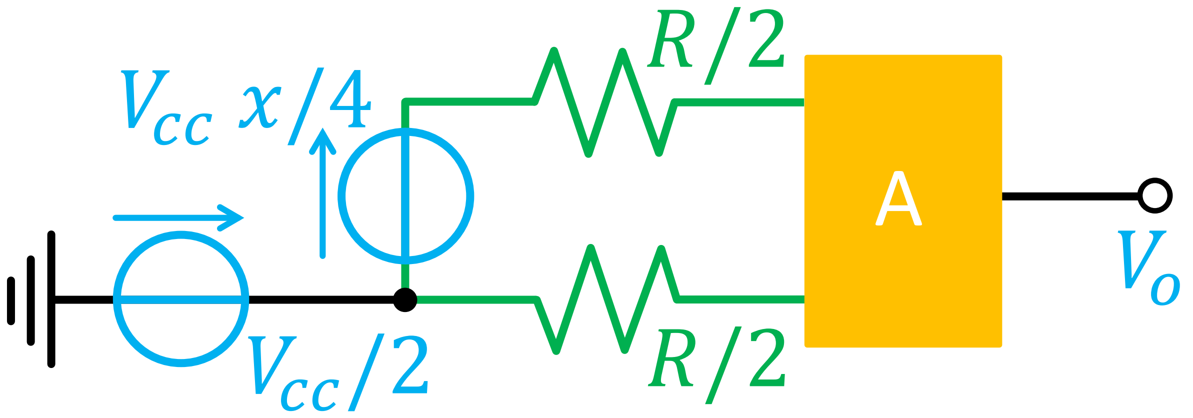
$$V_s = V_{cc}\left(\frac{R(1+x)}{2R} - \frac{R(1-x)}{2R}\right) = V_{cc}x$$

Bridge parameters

- Sensitivity: voltage output when $V_{cc} = 1$ V and $x = x^{max}$. Usually expressed in mV/V (in previous example, $S = 2.5$ mV/V)
- Accuracy: difference with respect to the linear characteristics. Expressed in %
- Resistance: resistance of the bridge measured between the output terminals

Bridge amplifiers





Amplifier requirements (example)

- High gain
  - $V_{cc} = 5$ V, $x^{max} = 1\% \Rightarrow V_s^{max} = 12.5$ mV. If $V_o^{max} = V_{cc} = 5$ V $\Rightarrow$ $G = 400$
- High input resistance
  - $R = 100\ \Omega$ and an error smaller than 1‰ is required $\Rightarrow$ $R_i \geq 1000R = 100\ \text{k}\Omega$
- High CMRR
  - with 8-bit resolution, $V_s^{LSB} = 12.5\ \text{mV}/2^8 \approx 50\ \mu\text{V}$ and $V_{CM} = 2.5$ V $\Rightarrow$ $\text{CMRR} \geq V_{CM}/V_{s,LSB} = 94$ dB

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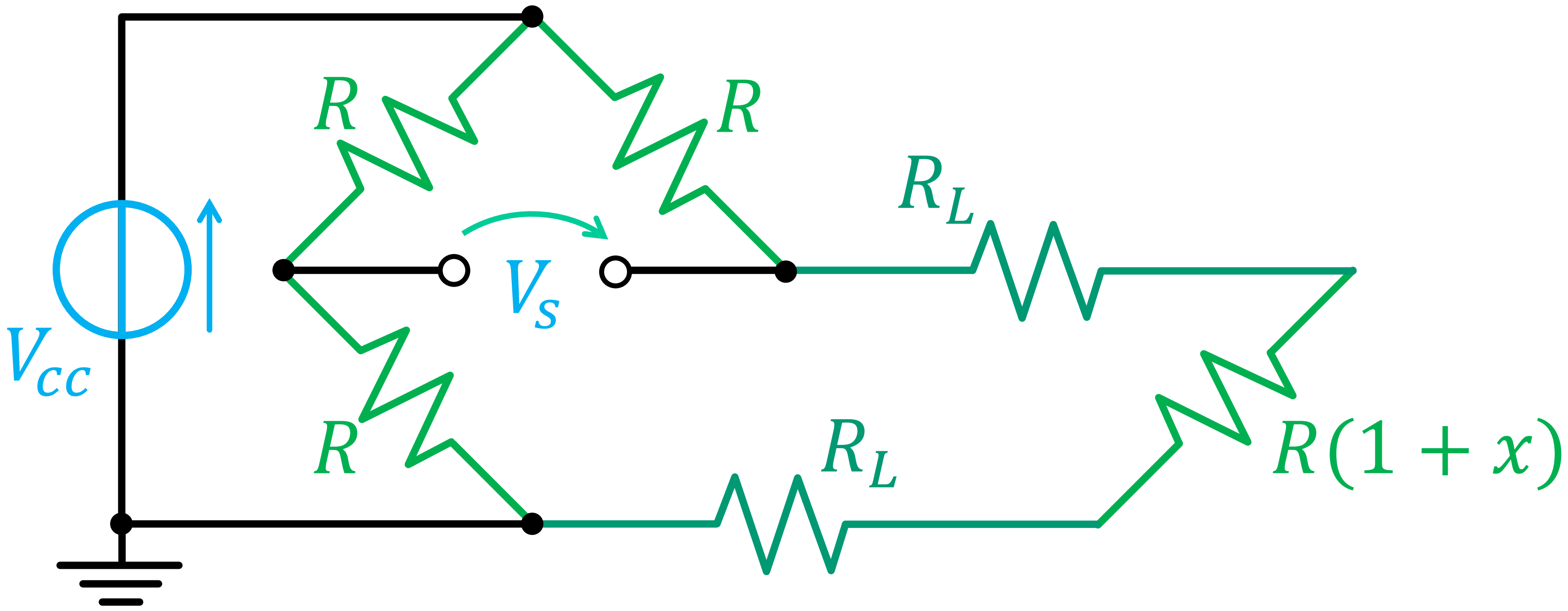
Wiring resistance in remote sensor

- In remotely located bridges, cable resistances and noise pickup are the biggest problems
- Cable resistances give an offest error (which can be compensated), but…
- Changes in cable resistances during operation (*e.g.*, with temperature) produce an error signal (gain error) at the bridge output

2-wire connection



$$V_s = V_{cc}\left(-\frac{1}{2} + \frac{R(1+x)+2R_L}{R(2+x)+2R_L}\right) \approx V_{cc}\left(\frac{x}{4} + \frac{R_L}{2R}\right)$$

Example

- Consider $V_{cc} = 10$ V, $R = 350\ \Omega$, $x^{max} = 1\%$

$$V_o(0) = 0 \text{ and } V_o(x^{max}) = 24.9 \text{ mV}$$

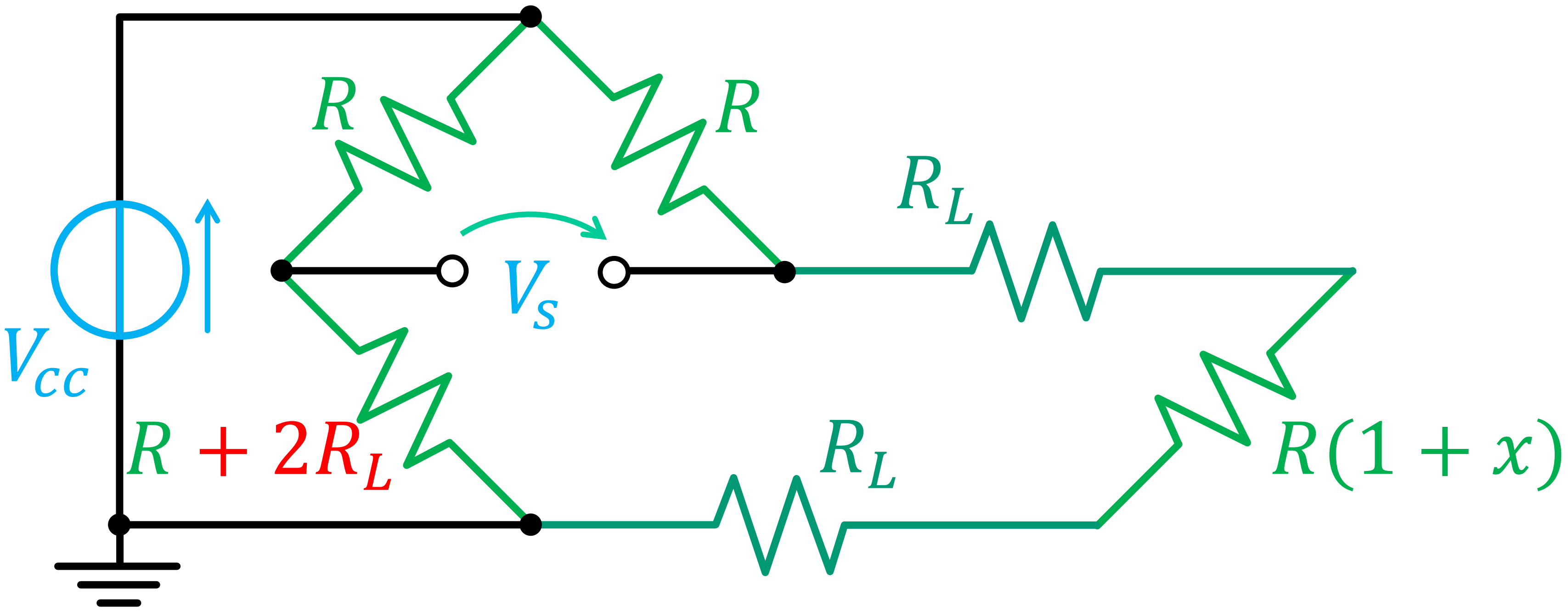
- With $R_L = 10\ \Omega$ we have:

$$V_o(0) = 10\left(\frac{370}{720} - \frac{1}{2}\right) = 138.9 \text{ mV}$$

$$\Delta V = 23.5 \text{ mV}$$

$$V_o(x^{max}) = 10\left(\frac{370 + 3.5}{720 + 3.5} - \frac{1}{2}\right) = 162.4 \text{ mV}$$

What if we compensate?



Adding $2R_L$ in series to the lower-left bridge resistor leads to

$$V_o(0) = 0; \qquad V_o(x^{max}) = 10\left(\frac{370 + 3.5}{720 + 3.5} - \frac{370}{720}\right) = 23.52\ \mathrm{mV}$$

Effect of temperature

- The problem now is the temperature dependence of R_L, expressed by its temperature coefficient

$$TCR = \frac{1}{R}\frac{dR}{dT}$$

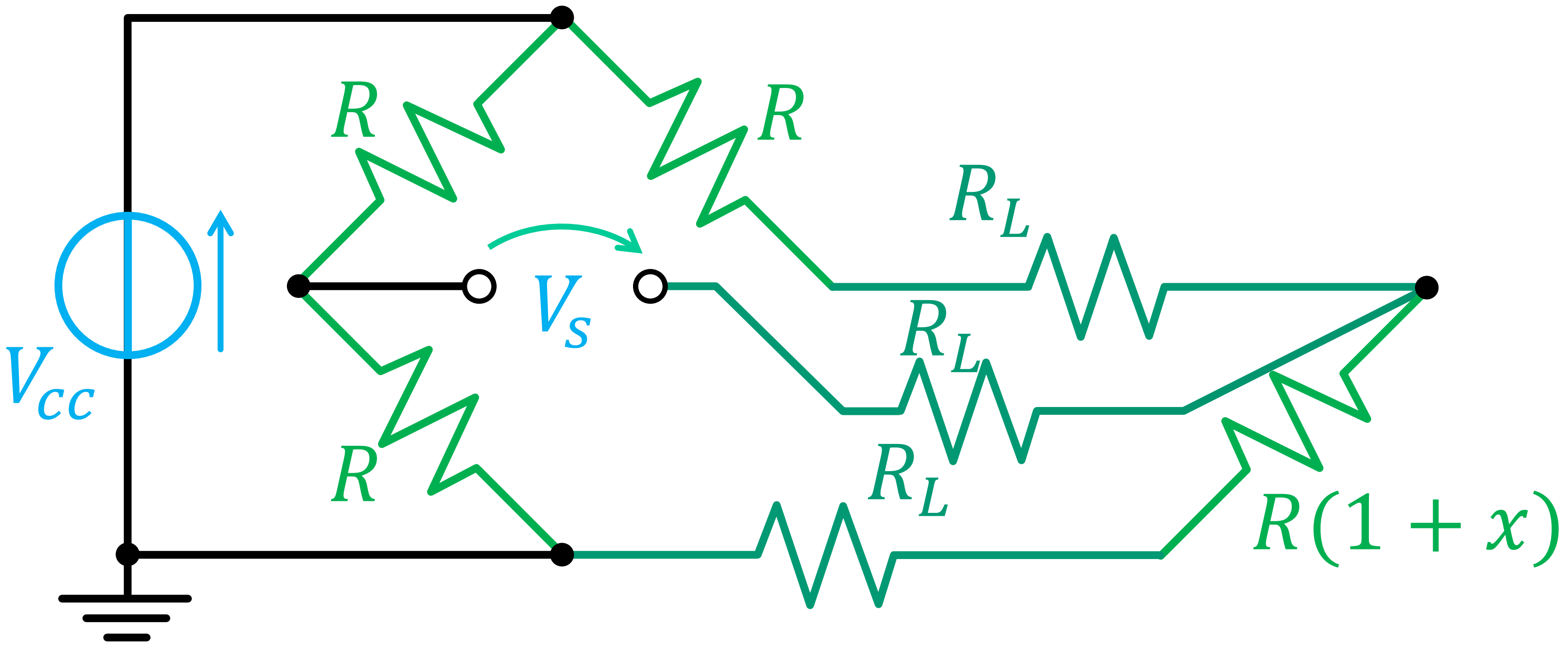
- With $TCR = 0.385\%$ /°C and $\Delta T_{max} = 10$°C $\Rightarrow 2\Delta R_L = 0.77\ \Omega$

$$V_o(0) = 10\left(\frac{370 + 0.77}{720 + 0.77} - \frac{370}{720}\right) = 5.19\ \text{mV}$$

$$\Delta V = 23.47\ \text{mV}$$

$$V_o(x^{max}) = 10\left(\frac{370 + 3.5 + 0.77}{720 + 3.5 + 0.77} - \frac{370}{720}\right) = 28.66\ \text{mV}$$

3-wire connection



$$V_s = V_{cc}\left(-\frac{1}{2} + \frac{R(1+x)+R_L}{R(2+x)+2R_L}\right) \approx V_{cc}\frac{x}{4}\left(1-\frac{x}{2}-\frac{R_L}{R}\right)$$

3-wire connection

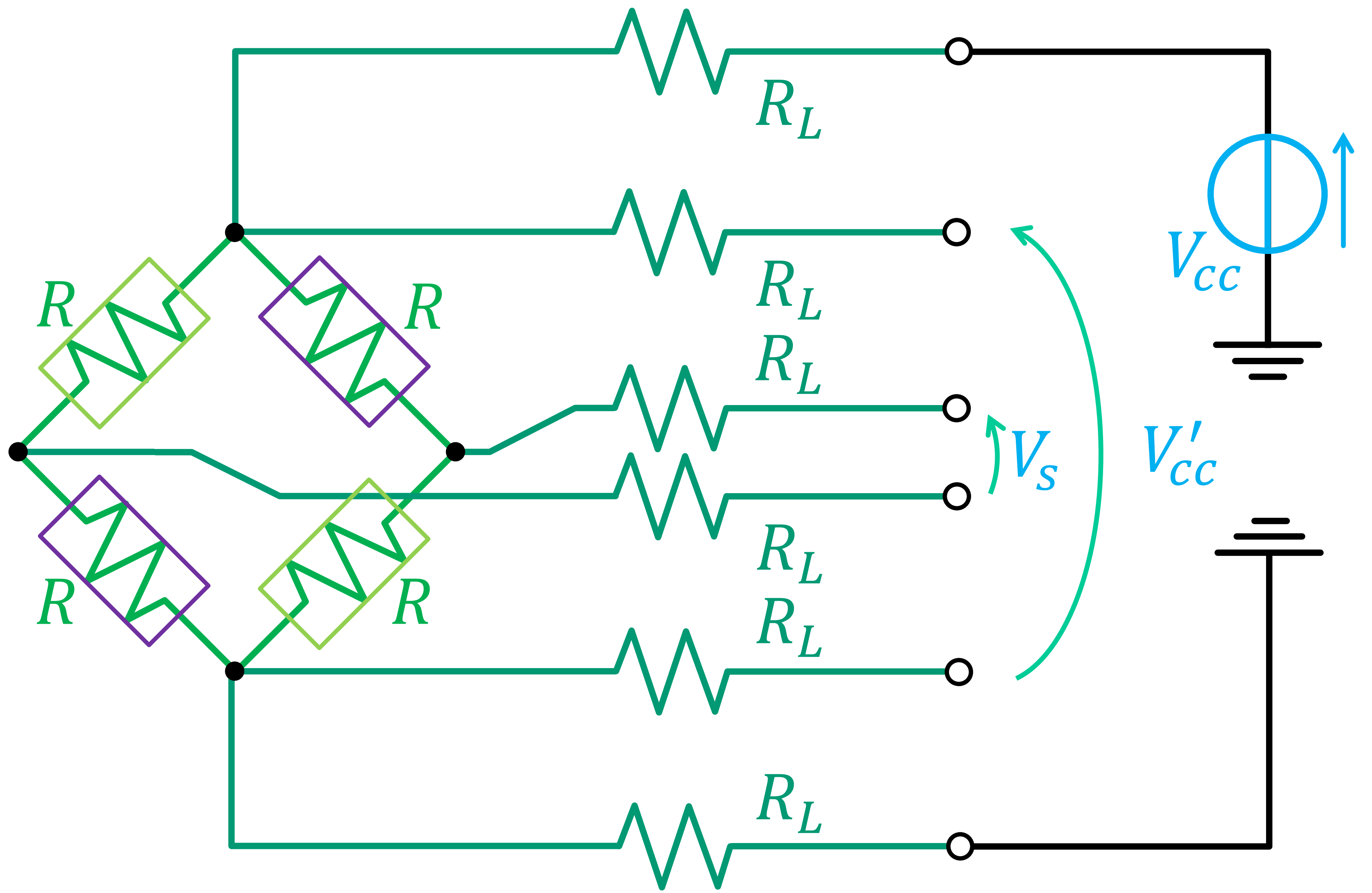
- When $\Delta T = 0$ we have

$$V_o(0) = 0; \quad V_o(x^{max}) = 10\left(\frac{360 + 3.5}{720 + 3.5} - \frac{1}{2}\right) = 24.19\ \text{mV}$$

- When temperature is accounted for, we have $V_o(0) = 0$ and

$$V_o(x^{max}) = 10\left(\frac{360 + 3.5 + 0.385}{720 + 3.5 + 0.77} - \frac{1}{2}\right) = 24.16\ \text{mV}$$

Kelvin (4-wire) connection



Comparison

- The 3-wire method works well for remote elements several tens of meters away
- Connecting wires must have the same characteristics
- Stability of V_{cc} remains a concern
- The 4-wire connection is required for remote bridges, e.g. with 4 active elements
- The Kelvin connection is actually a six-lead assembly. Constant-current excitation can reduce it to 4

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Temperature dependence

- The bridge output is

$$V_s = V_{cc}x = V_{cc}\alpha\Delta S$$

$$\alpha = \frac{1}{R_0}\left.\frac{dR}{dS}\right|_{R_0}$$

- In reality, $\alpha = \alpha(T)$, which introduces inaccuracies in the output (unless we are measuring the temperature)

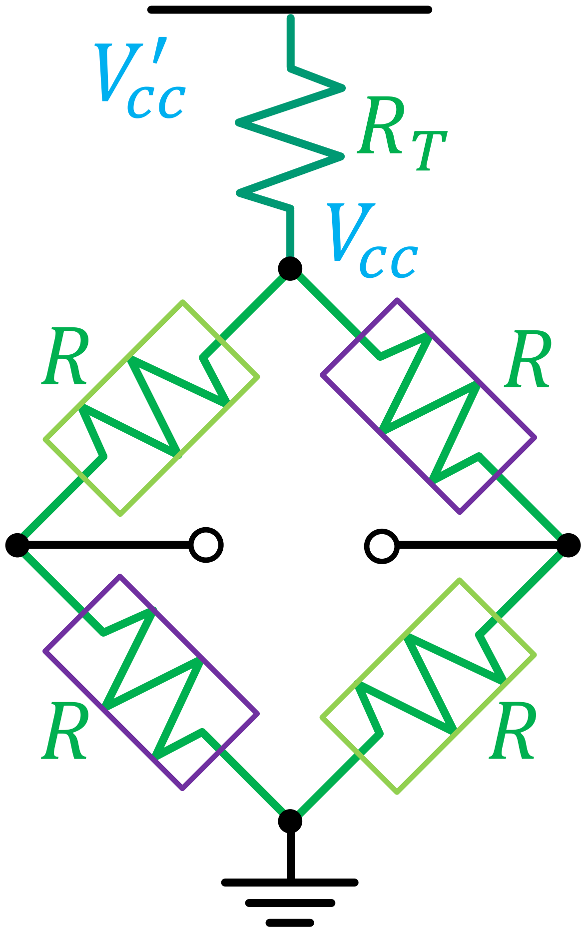
Temperature compensation

$$\frac{dV_s}{dT} = \Delta S\left(\alpha\frac{dV_{cc}}{dT} + V_{cc}\frac{d\alpha}{dT}\right) = 0$$

$$\Rightarrow \frac{1}{V_{cc}}\frac{dV_{cc}}{dT} = -\frac{1}{\alpha}\frac{d\alpha}{dT} = -\beta$$

The bridge excitation voltage must be temperature-dependent and have an opposite rate of variation with respect to α

T-independent resistor in series



$$V_{cc} = V'_{cc}\frac{R_B}{R_B + R_T}$$

$$\frac{dV_{cc}}{dT} = V'_{cc}\frac{R_T}{(R_B + R_T)^2}\frac{dR_B}{dT}$$

$$\frac{1}{V_{cc}}\frac{dV_{cc}}{dT} = \frac{R_T}{R_B + R_T}\frac{1}{R_B}\frac{dR_B}{dT}$$

$= -\beta$

$= \frac{1}{R}\frac{dR}{dT} = TCR$

$$-\beta = TCR\frac{R_T}{R_B + R_T}$$

(for a bridge with equal resistances)

Result

$$R_T = -\frac{\beta}{\beta + TCR} R$$

- Very simple and popular solution, but with a few disadvantages:
  - Only possible if $\beta < 0$ and $TCR > |\beta|$
  - β and TCR must be precisely known
  - Output signal is reduced
- Usually adopted in the range 25 ± 15°C