POLITECNICO DI MILANO

Electronics – 96032

Thermal, Shot and Flicker Noise

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Disclaimer

Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes

Purpose of the lesson

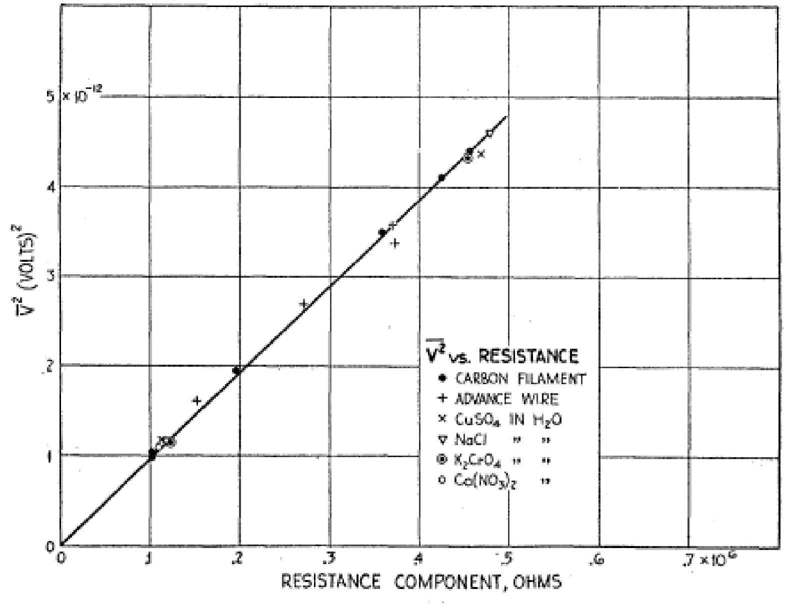
- Up to now we have seen Amplifiers and Sensors and we know how to deal with the problems from the viewpoint of the signal
- In reality, noise is always present and can affect the precision or even overshadow the signal
- In the second half of the class, we will discuss techniques for noise reduction. Before doing this, however, we need to learn:
  - how to mathematically describe noise (previous lesson);
  - what are the origins of noise (this lesson);
  - how circuits respond to noise (next lesson);

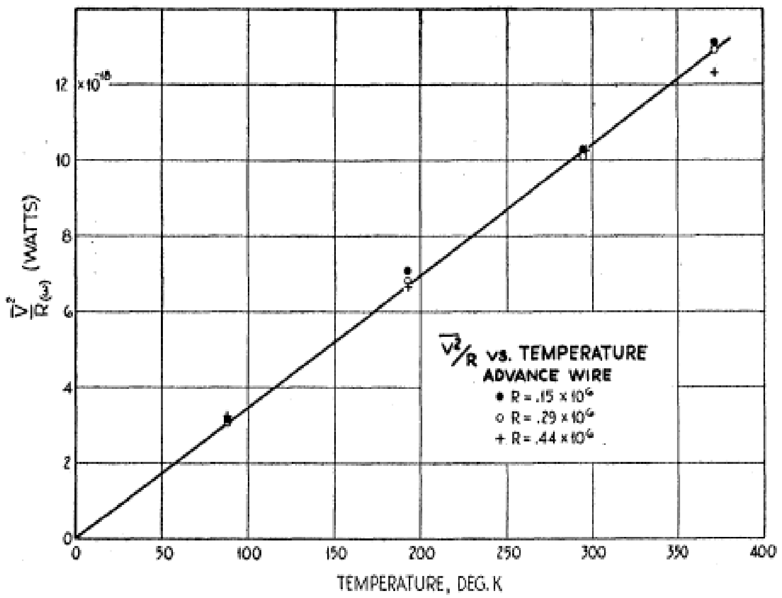
Outline

- Thermal noise
- Shot noise
- Flicker noise

Experimental evidence (Johnson, 1928)

From [1]

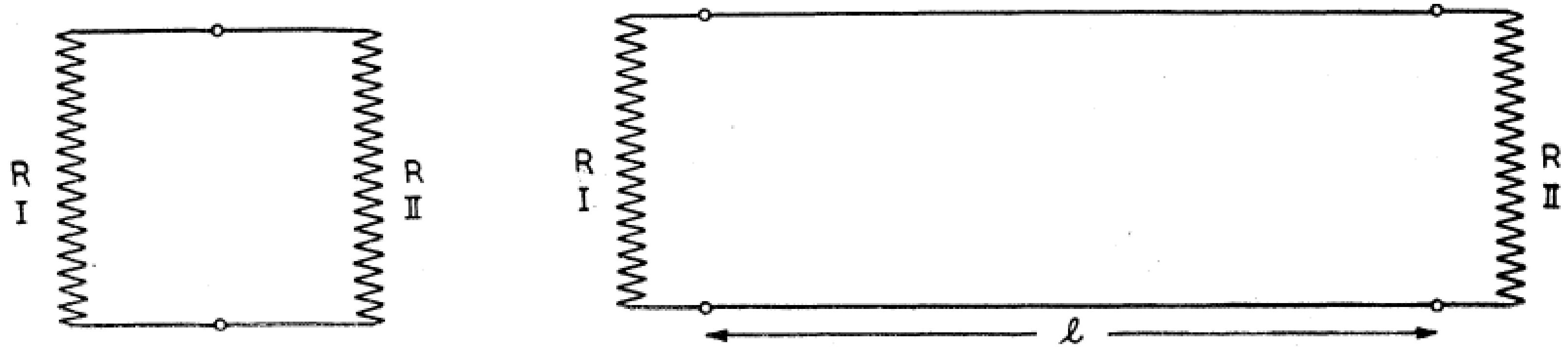




The rms potential fluctuation is proportional to the resistance and the temperature of the conductor

Nyquist derivation (1928)

From [2]



- At thermal equilibrium, the same power is transferred from R_I to R_{II} and vice versa
- A lossless transmission line with characteristic impedance R is inserted
- The line is then short-circuited and the energy is “trapped” into the line

Modes and power density

$$n\lambda = l, \text{ with } \lambda f = v$$

$$n = \frac{l}{\lambda} = \frac{l}{v} f \Rightarrow dn = \frac{l}{v} df$$

An energy $k_B T$ is associated to each mode (two degrees of freedom):

$$E(f)df = k_B T dn = k_B T \frac{l}{v} df,$$

transferred in a time l/v. The power density is

$$P(f)df = \frac{E(f)df}{l/v} = k_B T df$$

Spectral density

The emf V generates a current $V/2R$, dissipating on each resistor a power

$$P = R\overline{I^2} = R\left(\frac{\overline{V^2}}{4R^2}\right) = \frac{\overline{V^2}}{4R} = \int_0^{+\infty} \frac{S_V}{4R} df$$

unilateral

$$\frac{S_V}{4R} = k_B T \Rightarrow S_V = 4k_B TR \text{ (white)}$$

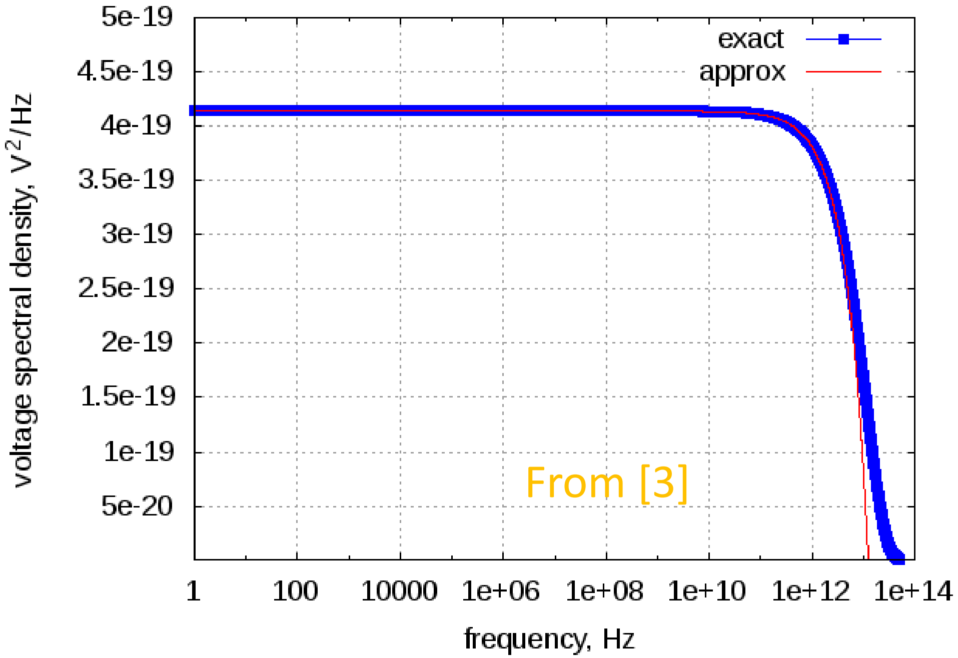
e.g., for $R = 1\ \mathrm{k\Omega}$ $S_V \approx 1.66 \times 10^{-17}$ V²/Hz $= \left(4.07\ \mathrm{nV}/\sqrt{\mathrm{Hz}}\right)^2$

High-frequency behavior

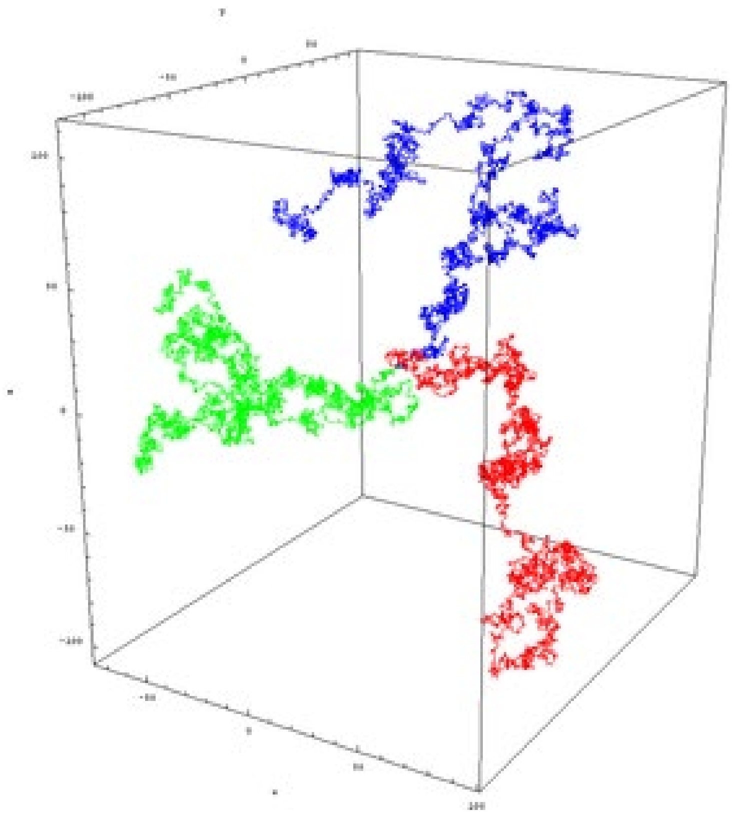
- The Planck equation should be used for the mode energy

$$E = \frac{hf}{e^{hf/k_B T} - 1} \approx k_B T \quad \forall \frac{hf}{k_B T} \ll 1$$

- The spectral density is white up to ≈ 100 GHz
- Zero-point energy can not be used and is not included



Derivation based on random motion



- Thermal noise is seen as a consequence of the random motion of the electrons, *i.e.*, of their velocity fluctuations
- The thermal noise spectrum can be derived from the Brownian motion equation

$$\frac{\partial n}{\partial t} = D\,\frac{\partial^2 n}{\partial x^2}$$

leading to $\overline{x^2(t)} = 2Dt$ (Einstein, 1905)

and to $S_{\Delta v_x} = 4D$ (see Appendix I)

Calculation

- A single electron contribution to the current is

$$i = \frac{q}{L} v_x \Rightarrow \Delta i = \frac{q}{L} \Delta v_x$$

$$R_{\Delta \mathrm{i}} = \frac{q^2}{L^2} R_{\Delta v_x} \Rightarrow S_{\Delta \mathrm{i}} = \frac{q^2}{L^2} S_{\Delta v_x} = \frac{q^2}{L^2} 4D$$

- For $N = nAL$ independent electrons, we have

$$S_I = \frac{q^2}{L^2} 4DnAL = \frac{4nAq^2}{L} \frac{k_B T}{q} \mu = 4k_B T \frac{qn\mu A}{L} = 4k_B T \frac{\sigma A}{L} = \frac{4k_B T}{R}$$

$$\Rightarrow S_V = S_I R^2 = 4k_B T R$$

Outline

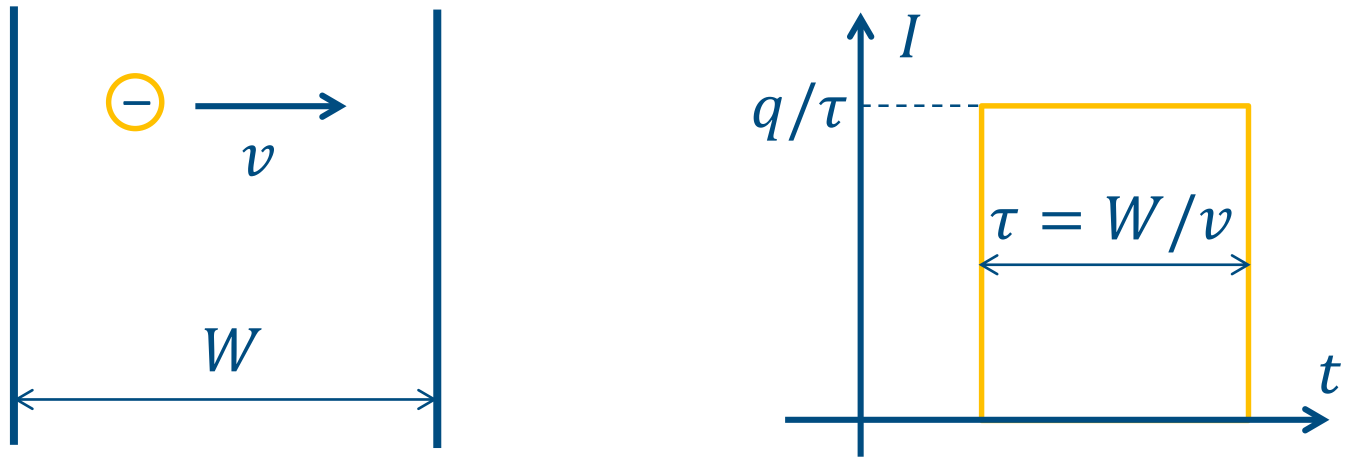
- Thermal noise
- Shot noise
- Flicker noise

Generalities

- Observed by Walter Schottky in 1918 while studying current fluctuations in vacuum tubes
- It is related to fluctuation in the number of charge carriers, rather than of their velocity

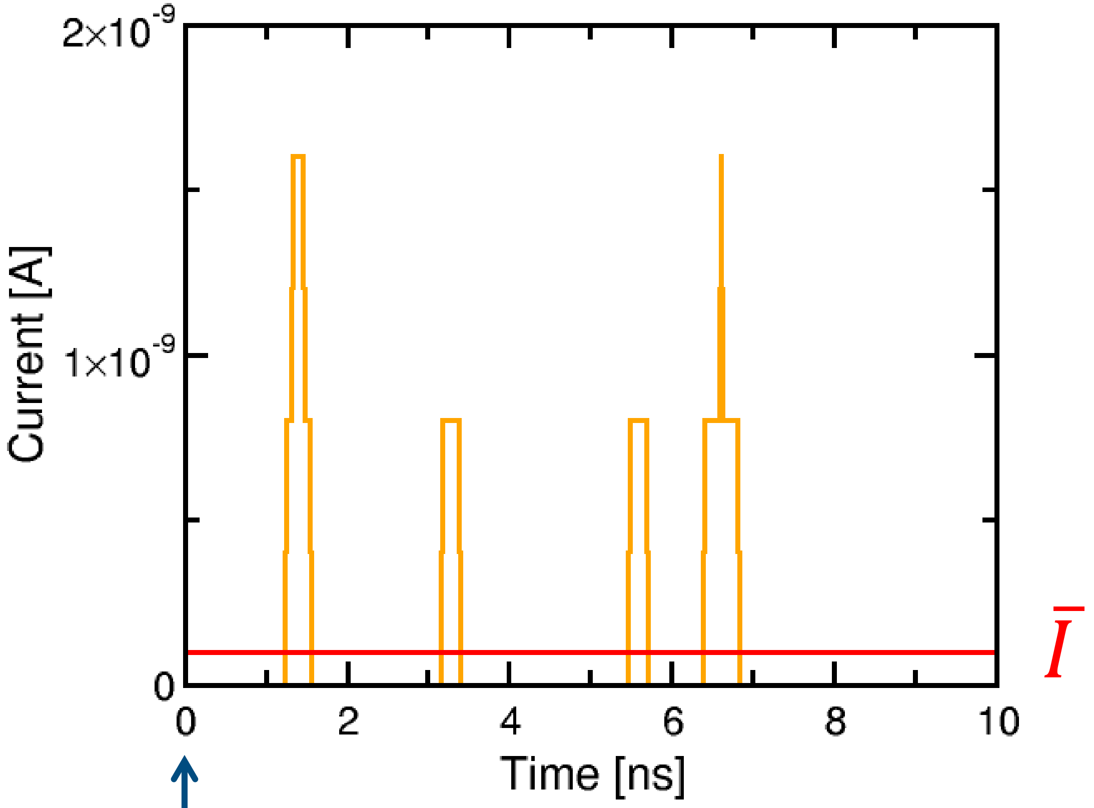
Example – one electron

- Consider an electron flowing through the depletion region W of a semiconductor:



- What happens when we have more than one electron flowing?

Very few electrons

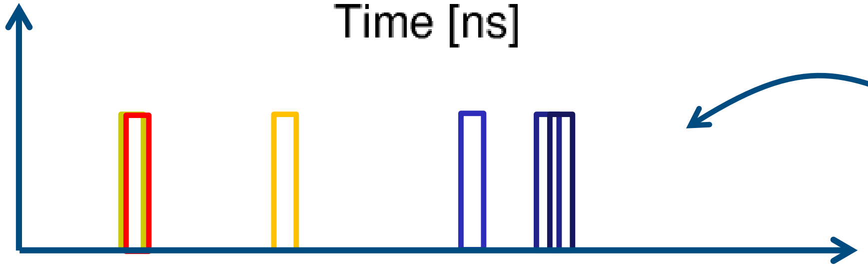


$\tau = 100$ ps

$\bar{I} = 10^{-10}$ A

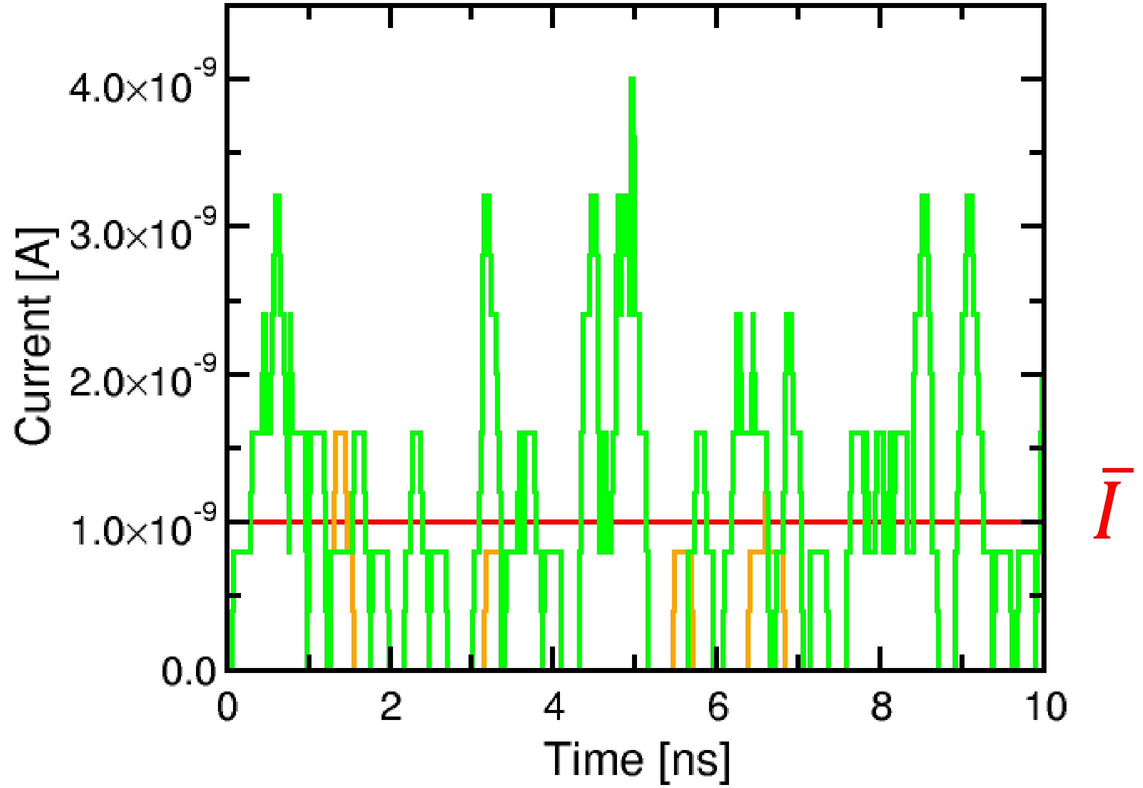
$\bar{n} \approx 6.2 \times 10^{8}$ s^{-1}

(= 6.2 in the observation time)



6 electrons actually observed

Some more…



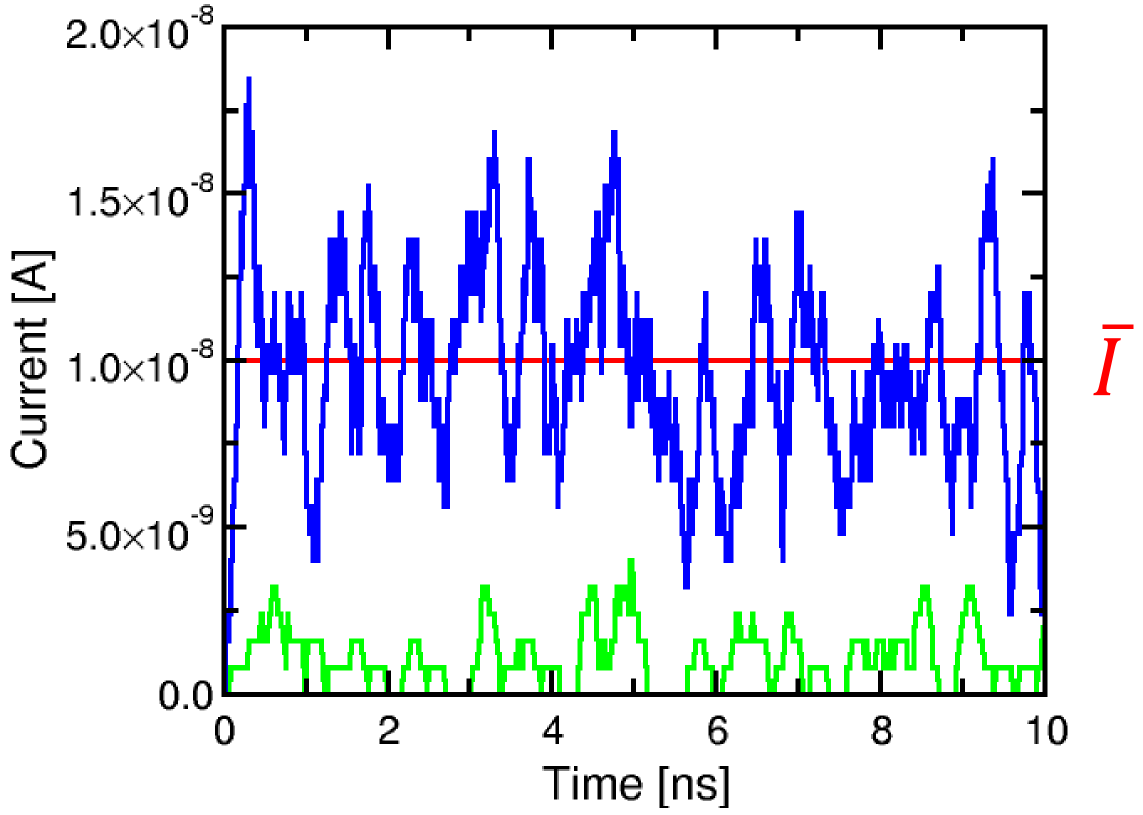
$\tau = 100$ ps

$\bar{I} = 10^{-9}$ A

$\bar{n} \approx 6.2 \times 10^{7}$ s^{-1}

(= 62 in the observation time)

Even more…



$\tau = 100$ ps

$\bar{I} = 10^{-8}$ A

$\bar{n} \approx 6.2 \times 10^{6}$ s^{-1}

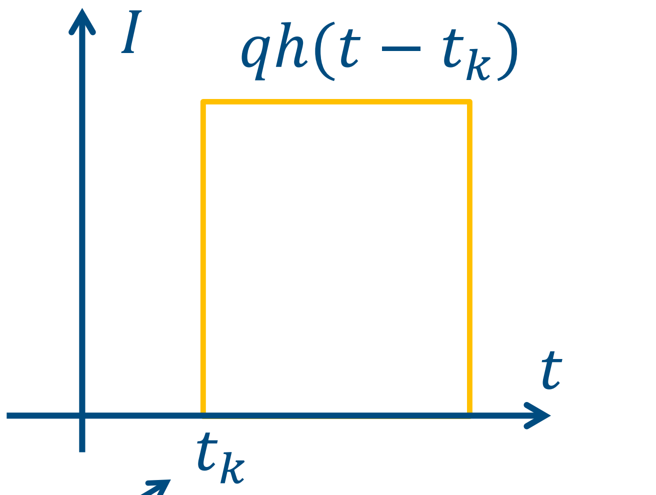
($= 620$ in the observation time)

Shot noise process

$$x(t) = \sum_k qh(t - t_k)$$

h is the current pulse due to a single electron, with unit area

Starting time of the k[th] electron (random variable)



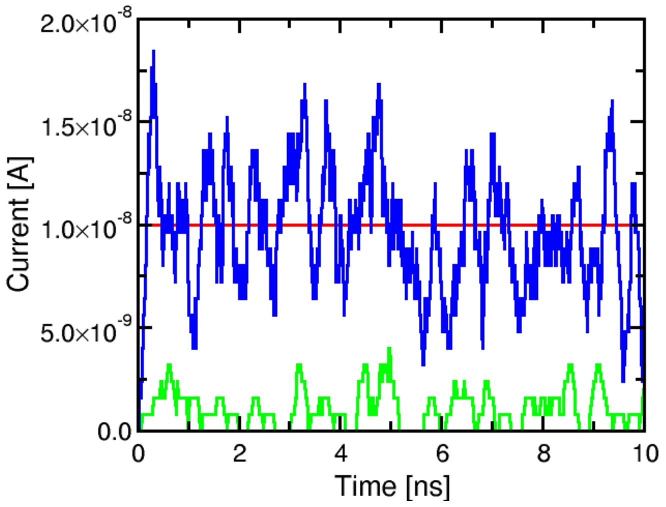
$$S(f) = \lim_{T\to\infty} \frac{1}{2T} \overline{|X_T(f)|^2} \quad ; \qquad X_T(f) = qH(f) \sum_k e^{-j2\pi f t_k}$$

$$S(f) = q^2 |H(f)|^2 \lim_{T\to\infty} \frac{1}{2T} \overline{\left(\sum_k e^{-j2\pi f t_k}\right)\left(\sum_m e^{j2\pi f t_m}\right)}$$

Shot noise PSD

The final result is (see Appendix II)

$$S(f) = \bar{I}^2\delta(f) + q\bar{I}|H(f)|^2$$



Due to the non-zero average value $\bar{I}$ (DC value $\Rightarrow \delta(f)$)

This is the PSD of the random component. The unilateral PSD (white up to the reciprocal of the transit time) is then

$$S(f) = 2q\bar{I}$$

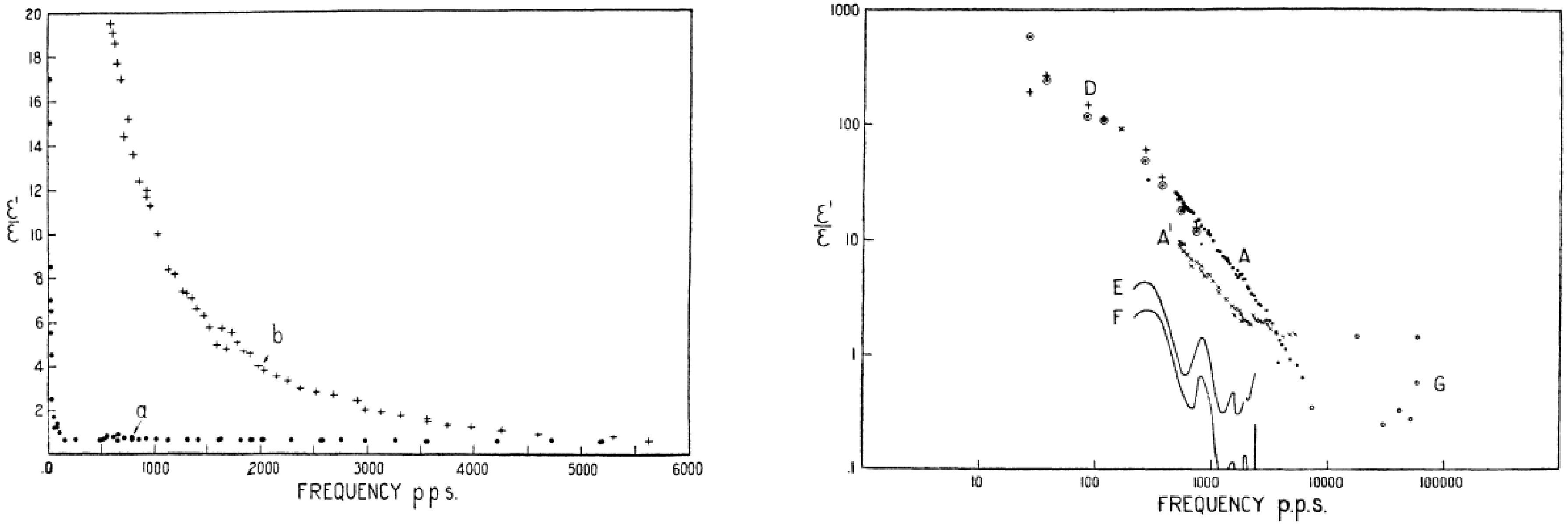
(remember that $H(0) = 1$)

Outline

- Thermal noise
- Shot noise
- Flicker noise

Experimental evidence

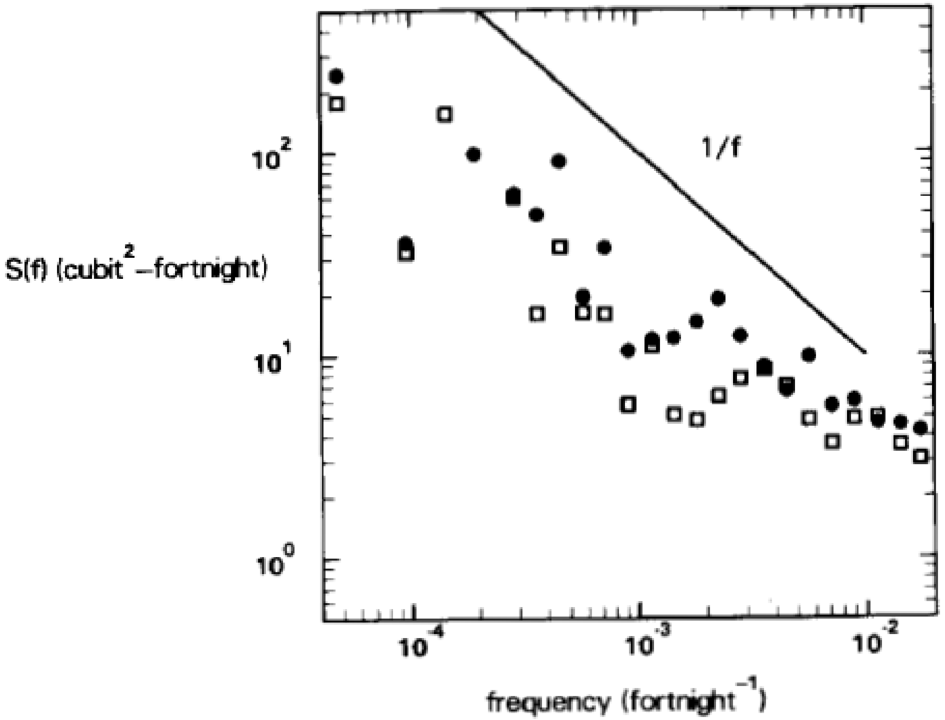
From [4]



- Discovered by J. Johnson in 1925, while investigating shot noise in vacuum tubes
- Found in all other electron devices

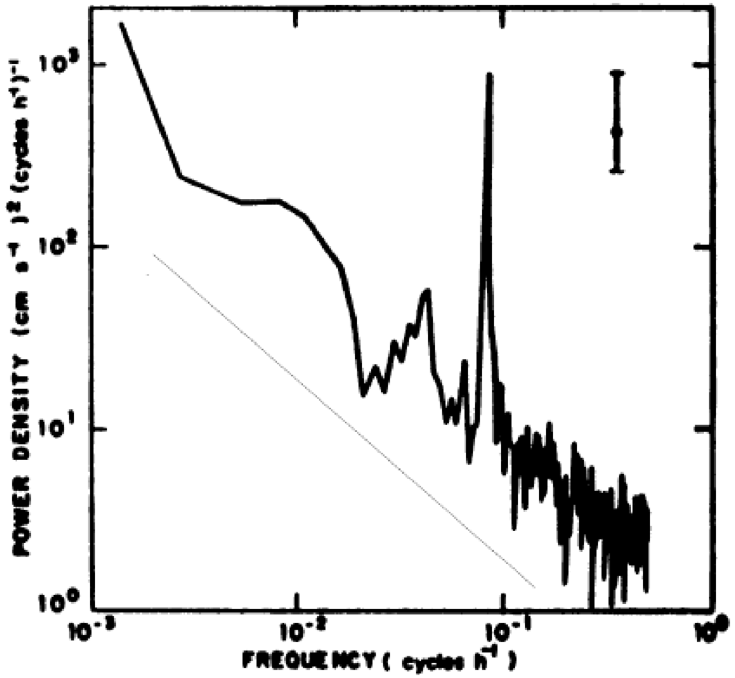
$1/f$ pervasivity: geophysics

From [5]



Nile river flood levels

From [6]



Ocean current velocity

$1/f$ pervasivity: music

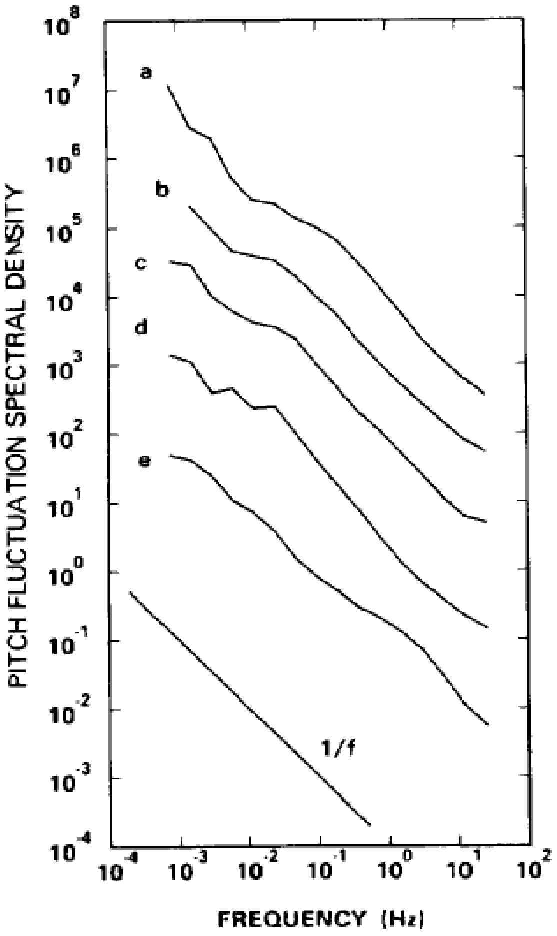
a) Medieval

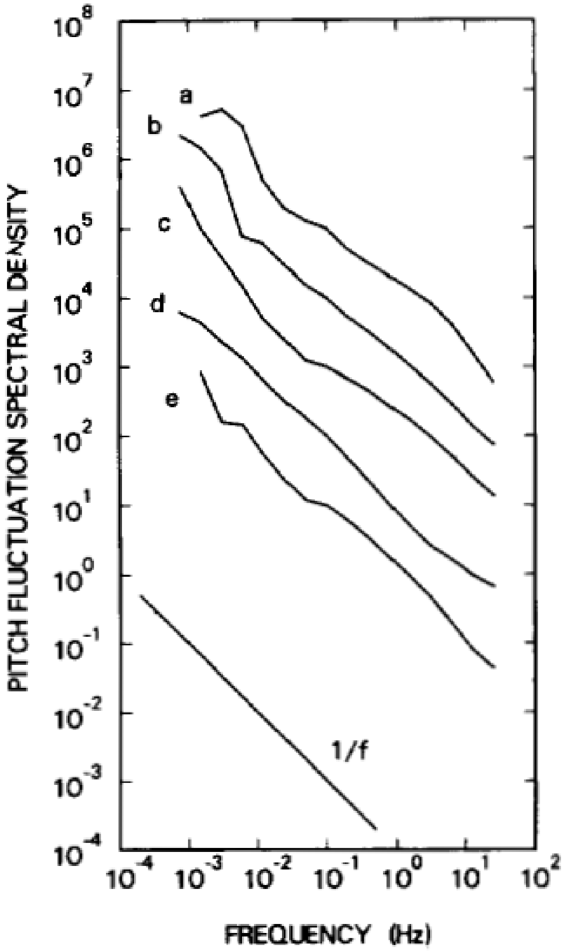
b) L. Beethoven

c) C. Debussy

d) R. Strauss

e) The Beatles





a) Pigmies

b) Japan

c) India

d) Russia

e) Blues

From [5]

From www.zazzle.com

Scale invariance

$$P = \int_{f_L}^{f_H} \frac{K}{f} df = K \ln\left(\frac{f_H}{f_L}\right)$$

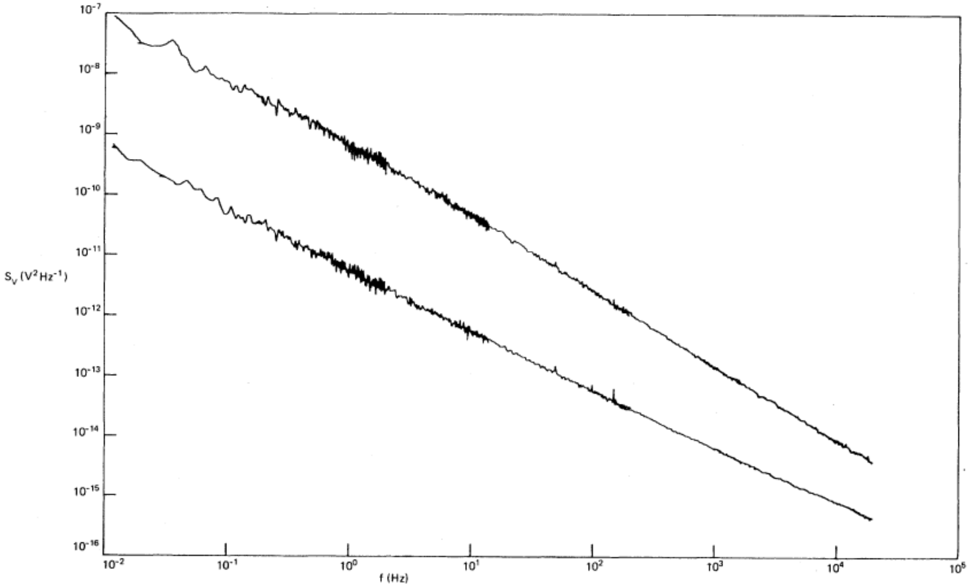
- The total noise power between – say – 0.1 Hz and 1 Hz and between 100 kHz and 1 MHz is the same (for white noise, power in the second interval is 1 million times larger)
- Power diverges in both the high- and low-frequency limit

Stationarity

- A band-limited 1/f noise is stationary
- In practice, frequency cutoff always occurs:
  - High frequency: response time ($1\ \mathrm{ps} \Rightarrow 160\ \mathrm{GHz}$; $\lambda/c \Rightarrow 10^{21}$ Hz)
  - Low frequency: observation time ($1\ \mathrm{day} \Rightarrow 10^{-5}$ Hz; $1\ \mathrm{year} \Rightarrow 3 \times 10^{-8}$ Hz; age of the universe $\Rightarrow 10^{-17}$ Hz)
- The maximum $1/f$ power is only 38 times the power in the $1-10$ Hz bandwidth!

Minimum frequency

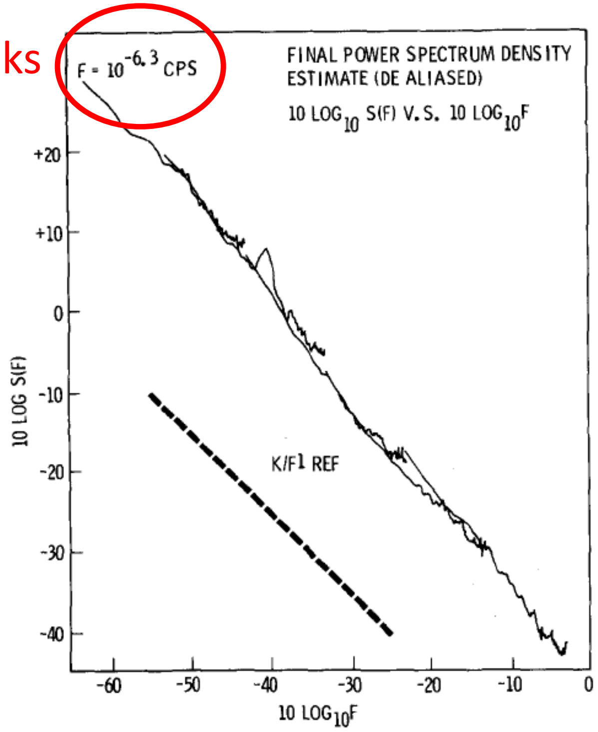
From [7]



10^{-2} Hz 10^{4} Hz

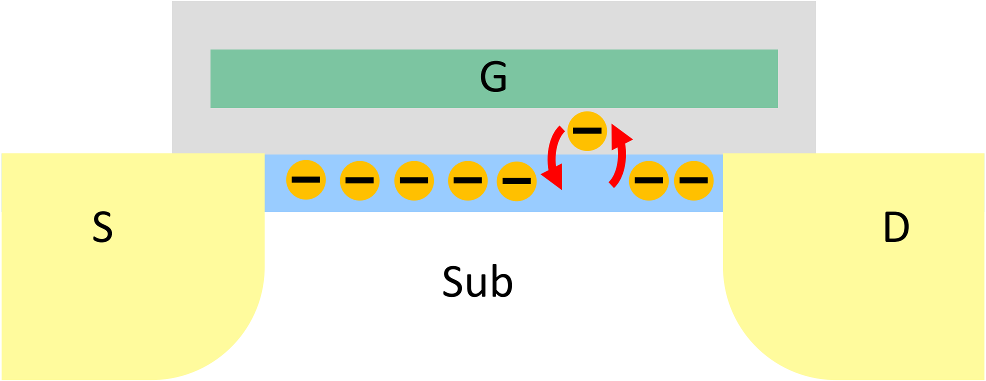
From [8]

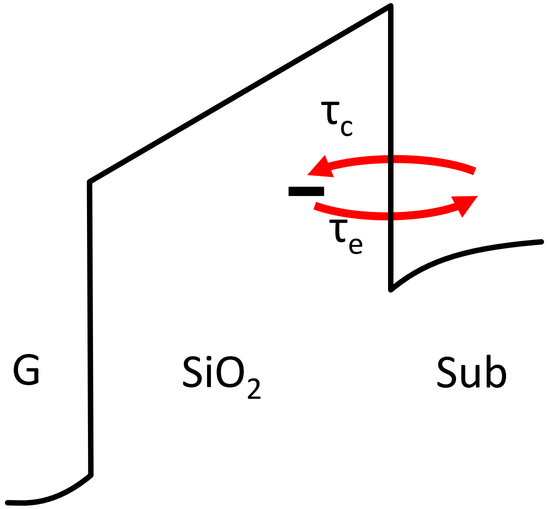
≈3 weeks



Every model predicts the existence of a minimum frequency, but this is always extremely small and has never been observed

Random telegraph noise (RTN)

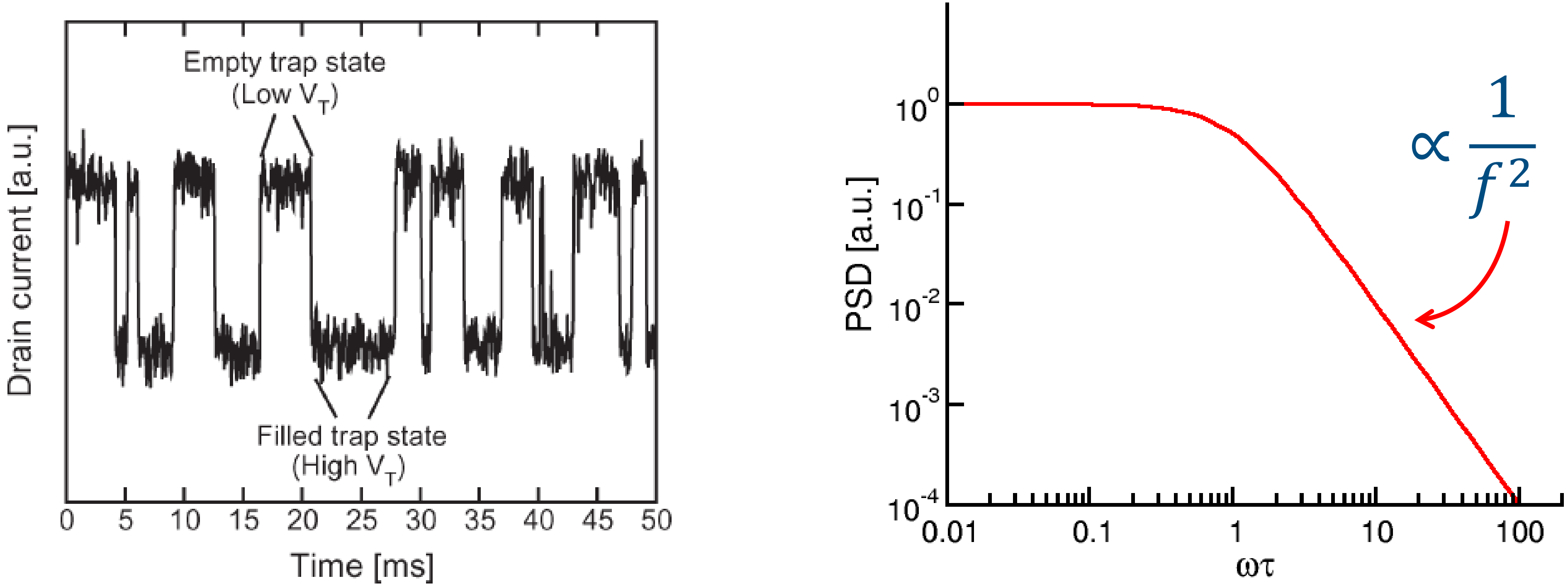




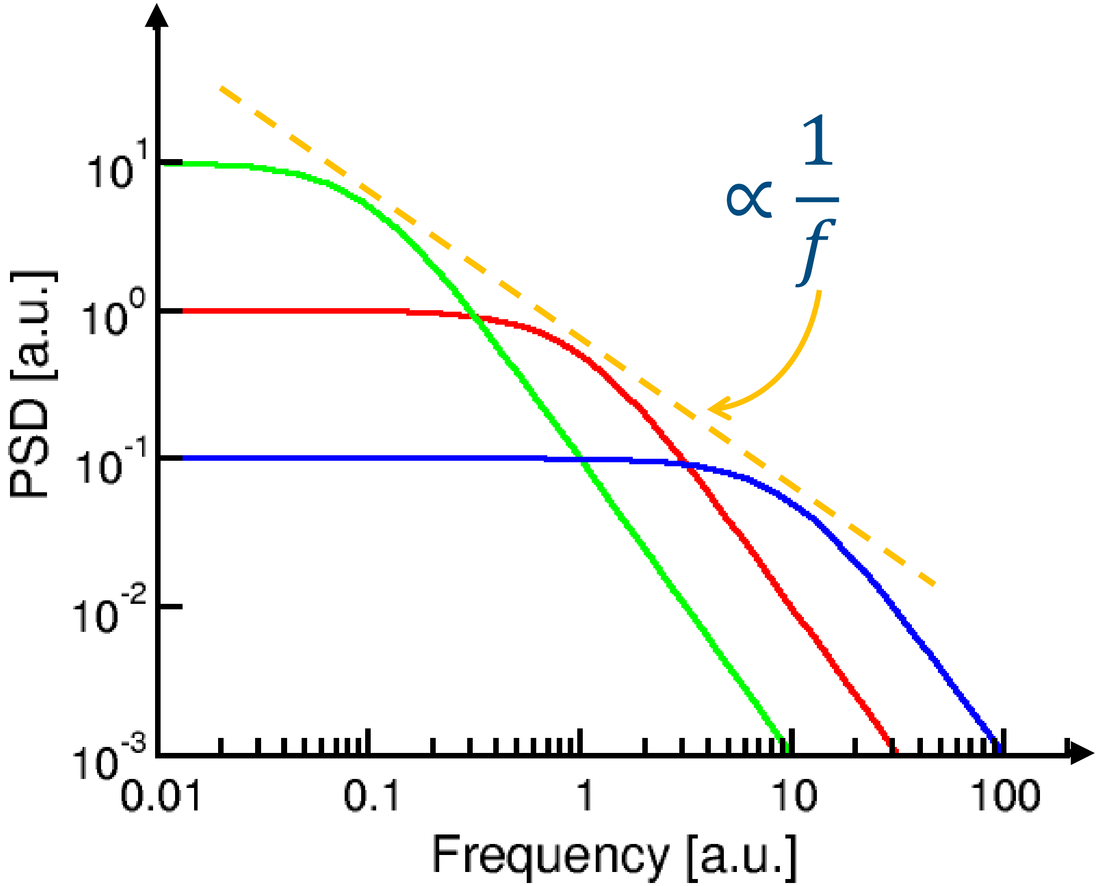
- Capture/emission of single electrons by oxide traps, with time constants τ_c and τ_e
- Fluctuations in the drain current (or threshold voltage)

RTN PSD

From [9]



Multitrap composition

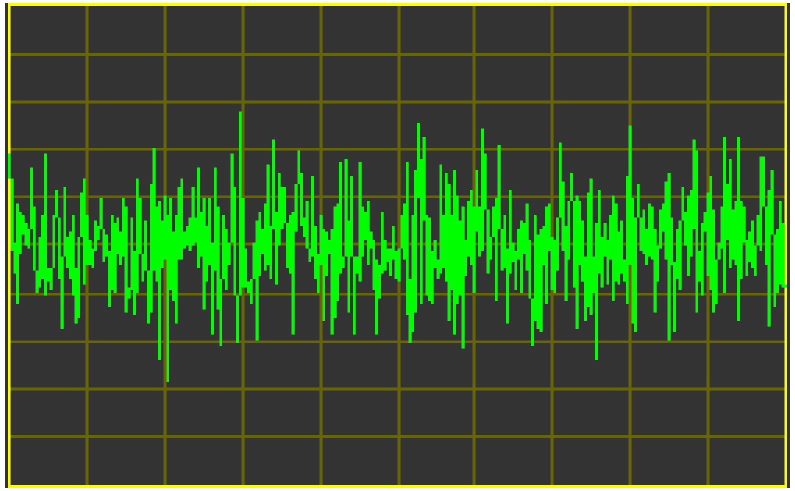


$1/f$ noise in semiconductors

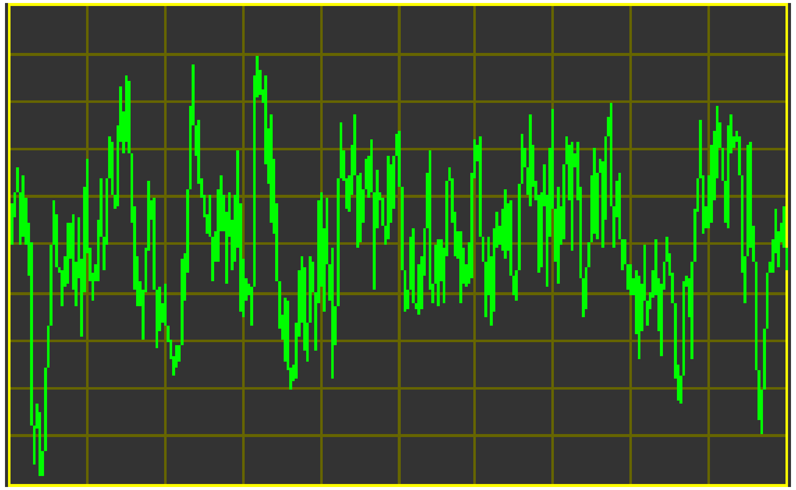
- Flicker noise in semiconductors is mainly due to distributed capture-emission processes
- MOS devices have a high flicker noise due to the high trap density at the silicon-oxide interface
- JFETs and BJTs have better performance (c-e in the space-charge regions)

Time behavior

From [10]



White noise



$1/f$ noise

References

1. J. Johnson, Phys. Rev. 32, 97 (1928)
2. H. Nyquist, Phys. Rev. 32, 110 (1928)
3. http://www.dsplog.com/2012/03/25/thermal-noise-awgn/
4. J. Johnson, Phys. Rev. 26, 71 (1925)
5. K. Voss, Proc. ASFC, 40 (1979)
6. http://arxiv.org/ftp/physics/papers/0204/0204033.pdf
7. B. Pellegrini et al., Phys. Rev. B 27, 1233 (1983)
8. M. A. Caloyannides, J. Appl. Phys. 45, 307 (1974)
9. C. Monzio Compagnoni et al., IEEE TED 55, 388 (2008)
10. http://wwwusers.ts.infn.it/~milotti/1overf/1overf_noise.html

Appendix I: velocity PSD at low frequencies

$$S_{\Delta v_x} = \lim_{T\to\infty} \frac{1}{2T} \overline{|X_T(f)|^2} = \lim_{T\to\infty} \frac{1}{2T} \overline{\left| \int_{-T}^{T} \Delta v_x e^{-j2\pi f t} dt \right|^2}$$

$$= \lim_{T\to\infty} \frac{1}{2T} \overline{\left| \int_{-T}^{T} (v_x - \overline{v_x}) dt \right|^2} = \lim_{T\to\infty} \frac{1}{2T} \overline{(x_{2T} - \overline{x_{2T}})^2}$$

$$= \lim_{T\to\infty} \frac{1}{2T} 2D(2T) = 2D$$

The unilateral PSD becomes then $S_{\Delta v_x} = 4D$

Appendix II: shot noise PSD calculation

$$S(f) = q^2|H(f)|^2 \lim_{T\to\infty} \frac{1}{2T} \overline{\left(\sum_k e^{-j2\pi f t_k}\right)\left(\sum_m e^{j2\pi f t_m}\right)} =$$

$$= q^2|H(f)|^2 \lim_{T\to\infty} \frac{1}{2T} \sum \overline{e^{-j2\pi f (t_k - t_m)}}$$

$$\sum_{k=m} + \sum_{k\neq m}$$

$$\sum_{k=m} \overline{e^{-j2\pi f (t_k - t_m)}} = N_{2T}$$

$$S(f)_{k=m} = q^2|H(f)|^2 \lim_{T\to\infty} \frac{N_{2T}}{2T} = q^2\bar{n}|H(f)|^2 = q\bar{I}|H(f)|^2$$

Appendix II: the term with $k \neq m$

$$S(f)_{k\neq m} = q^2|H(f)|^2 \lim_{T\to\infty} \frac{1}{2T} \sum\nolimits_{k\neq m} \overline{e^{-j2\pi f t_k}}\,\overline{e^{j2\pi f t_m}}$$

Since t_k are uniformly distributed in –T-T:

$$\overline{e^{-j2\pi f t_k}} = \frac{1}{2T}\int_{-T}^{T} e^{-j2\pi f t_k} = \frac{\sin(2\pi f T)}{2\pi f T}$$

$$S(f)_{k\neq m} = q^2|H(f)|^2 \lim_{T\to\infty} \frac{1}{2T} \sum\nolimits_{k\neq m} \frac{\sin^2(2\pi f T)}{(2\pi f T)^2}$$

$$= q^2|H(f)|^2 \lim_{T\to\infty} \frac{1}{2T} N_{2T}(N_{2T}-1) \frac{\sin^2(2\pi f T)}{(2\pi f T)^2} = q^2|H(f)|^2 \lim_{T\to\infty} \left(\frac{N_{2T}}{2T}\right)^2 \frac{1}{2\pi^2 T} \left(\frac{\sin^2(2\pi f T)}{f^2}\right)$$

$$= q^2\bar{n}^2\delta(f) = \bar{I}^2\delta(f)$$

Recalling that $\lim\limits_{a\to\infty} a\,\mathrm{sinc}^2(\pi a f) = \delta(f)$