Electronics - Tutorage 2

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30th April, 2025

Stability criterion for negative feedback systems

For a negative feedback system characterized by a general transfer function of the kind $\frac{G_{ol}}{1-G_{loop}}$ it is possible to infer the stability of the system by simply looking at G_{loop} (or better, at $-G_{loop}$). Indeed, in most of the cases of interest, we can say that a system is stable if:

- G_{loop} has poles with a real part negative or, at least, equal to zero
- There is only one frequency called f_{0dB} at which $|G_{loop}| = 1$
- The phase of $\angle[-G_{loop}(f_{0dB})] > -180°$

Or equivalently:

- G_{loop} has poles with a real part negative or, at least, equal to zero
- There is only one frequency called f_{180} at which $\angle[-G_{loop}(f_{0dB})] = -180°$
- The loop gain at f_{180} is such that $|G_{loop}(F_{180})| < 1$

These two requirements ensure stability and are typically referred to as Bode stability criterion. In general, two quantities can be defined to provide further information on the stability behavior of the circuit. These are the phase margin ϕ_m and the gain margin G_m, that can be easily computed as:

- $\phi_m = 180 - \angle[-G_{loop}(f_{0dB})]$
- $G_m|_{dB} = -|G_{loop}(f_{180})|_{dB}$

For a stable system, both ϕ_m and G_m are positive. For practical applications, ϕ_m determines the eventual settling time necessary to stabilize the output of the stage. For this reason, a phase margin of 45° or 60° is usually a common request.

Minimum phase systems

In many cases, the stages that we will analyze are so called minimum phase systems, *i.e.* systems characterized by having all the poles and zeros with a strictly negative real component. For these systems, a simplified version of the Bode stability criterion can be derived, which states that:

- If $|G_{loop}|$ intersects the $0dB$ axis with a slope of $-20dB/dec.$, then the system is stable
- If $|G_{loop}|$ intersects the $0dB$ axis with a slope of $-40dB/dec.$, then the system is unstable
- If at the cross-point between the $0dB$ axis and $|G_{loop}|$ the slope of $|G_{loop}|$ changes from $-20dB/dec.$ to $-40dB/dec.$ (or from $-40dB/dec.$ to $-20dB/dec.$) then the system is stable with a phase margin of 45°

Extracting values from the magnitude Bode plot

In many situation it is useful to determine with simple calculations the coordinate of a point in a magnitude Bode plot. To do so, there exist some tricks that directly comes from the fact that on the general plot $|G|_{dB}$ vs. $log_{10}(f)$ our transfer functions will be piece-wise curves with a slope that is always a multiple of $20dB/dec.$. Specifically, there is a general formula that allows to calculate any coordinate on a region of the transfer function where the slope is fixed by simply knowing the slope and the coordinates of a point in the same region. Indeed, said (f_1, G_1) the known point and being n the slope of the curve (in terms of multiple of $20dB/dec.$), it holds for any point (G, f) on the same curve that:

$$\frac{G}{(f)^n} = \frac{G_1}{(f_1)^n}$$

Notice that for a region where the slope is $-20dB/dec.$, $n = -1$ and the expression turns into the well known $G \cdot f = G_1 \cdot f_1$. Figure 1 reports some of the most common cases that can be encountered in real situations, with the generalized expression adapted for the specific one.

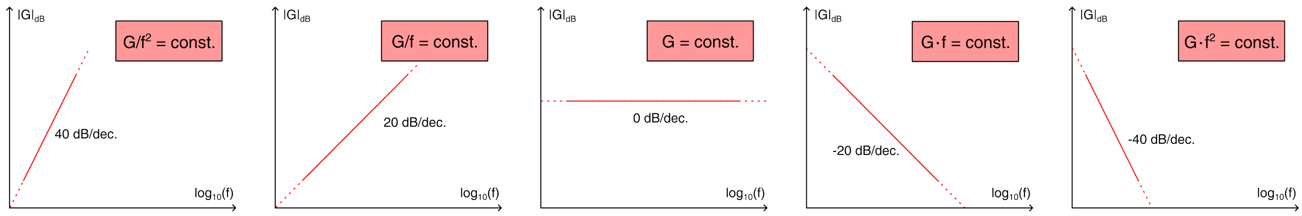

Figure 1: Relevant cases of transfer functions with the respective expression necessary to calculate any coordinate point given one.

List of relevant exams

In almost all the exams the student is asked to calculate G_{loop}, thus stability can be easily determined by looking at the transfer function. In the following list, instead, are reported exercises in which the student is asked to compensate for unstable stages or to determine the conditions for which the circuit is stable:

- Exam of 16$^{\text{th}}$ February 2015, exercise 1 (Q.2)
- Exam of 10$^{\text{th}}$ July 2015, exercise 1 (Q.2)
- Exam of 21$^{\text{st}}$ July 2016, exercise 1 (Q.2)
- Exam of 23$^{\text{rd}}$ September 2016, exercise 1 (Q.2)
- Exam of 15$^{\text{th}}$ February 2017, exercise 1 (Q.2)
- Exam of 6$^{\text{th}}$ July 2017, exercise 1 (Q.4)
- Exam of 20$^{\text{th}}$ July 2017, exercise 1 (Q.2)
- Exam of 21$^{\text{st}}$ June 2018, exercise 1 (Q.2 and Q.4)
- Exam of 16$^{\text{th}}$ January 2019, exercise 1 (Q.2)
- Exam of 19$^{\text{th}}$ June 2019, exercise 1 (Q.4)
- Exam of 19$^{\text{th}}$ July 2019, exercise 1 (Q.2)
- Exam of 13$^{\text{th}}$ February 2020, exercise 1 (Q.2)
- Exam of 18$^{\text{th}}$ June 2020, exercise 1 (Q.2 and Q.4)
- Exam of 11$^{\text{th}}$ September 2020, exercise 1 (Q.2 and Q.4)
- Exam of 22$^{\text{nd}}$ January 2021, exercise 1 (Q.2)
- Exam of 18$^{\text{th}}$ February 2021, exercise 1 (Q.2)
- Exam of 23$^{\text{rd}}$ June 2021, exercise 1 (Q.2)
- Exam of 22$^{\text{nd}}$ July 2021, exercise 1 (Q.2)
- Exam of 21$^{\text{st}}$ January 2022, exercise 1 (Q.2)
- Exam of 23$^{\text{rd}}$ June 2022, exercise 1 (Q.2)
- Exam of 17$^{\text{th}}$ February 2023, exercise 1 (Q.4)
- Exam of 15$^{\text{th}}$ January 2024, exercise 1 (Q.2)
- Exam of 7$^{\text{th}}$ February 2024, exercise 1 (Q.2)
- Exam of 11$^{\text{th}}$ September 2024, exercise 1 (Q.4)