Electronics - Tutorage 6

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High-pass filters (HPF)

High-pass filter are LTI filters which can be described in terms of a general transfer function $H(s)$ given by:

$$H(s) = \frac{sT_f}{1 + sT_F} \tag{1}$$

In their simplest implementation they are the reciprical of a low-pass filter, *i.e.* made up by the series of a capacitor C and a resistor R, where the signal is applied to one of the capacitor ends and the output is taken across the resistor. In that situation, the filter time constant T_F is simply given by the product CR.
From a time-domain perspective, the filter can be described in terms of it's pulse response $h(t)$, given by:

$$h(t) = \delta(t) - \frac{1}{T_F} e^{-\frac{t}{T_F}} u(t) \tag{2}$$

It is worth noticing that the integral of the pulse response is identically equal to zero. This property can be inferred also recalling the initial value theorem and observing that the transfer function has a zero at $0Hz$. From the theory, we know that the weighting function of an LTI system is the pulse response shifted and reversed, *i.e.* $w(t, \tau) = h(\tau - t)$, from which we can calculate the time-correlation of the weighting function (which will be equal to the auto-correlation of the pulse response):

$$K_{hh}(\tau) = \delta(t) - \frac{1}{2T_F} e^{-\frac{|t|}{T_F}} \tag{3}$$

Let's now exploit this last result to understand how noise can be affected by the HPF. Let's assume in general that we are given a noise input auto-correlation $R_{xx}(\tau) = \overline{n_x^2} \cdot \rho(\tau)$, where $\rho(\tau)$ is by definition a normalized auto-correlation such that $\rho(0) = 1$ and $\rho(|\tau| > 0) \leq 1$. Then:

$$\overline{n_y^2} = \overline{n_x^2} \left(1 - \int \frac{\rho(\tau)}{2T_F} e^{-\frac{|t|}{T_F}} d\tau \right) \tag{4}$$

As the integral of the exponential contribution of the HPF k_{hh} is equal to one, we can understand that a large noise cancellation can be achieved if we are dealing with a noise that has a large time correlation compared to T_F. Indeed, if the input noise auto-correlation decays within a certain time T_N such that $T_N \gg T_F$, we can assume $\rho(\tau)$ to be almost constant and equal to 1 in our integral, thus obtaining a complete noise removal. This is what would have happened if the noise was a static baseline: due to the filter zero at $0Hz$, the baseline would have been completely removed!
This example highlights the capability of HPFs to remove low-frequency noise. Clearly, it has to be used cautiously, depending on the signal we want to read. Indeed, if the signal is mainly a low-frequency signal submerged in low-frequency noise, the adoption of a HPF is not ideal and better choices could be made. For this reason, let's now study how a square pulse of amplitude A and duration T, which is our prototypical high-frequency signal, is passed through a HPF. Let's start with out input signal, which is denoted as $x(t) = A \cdot Rect_T(t)$. During the ramp-up front, the filter output will be given by its step response, *i.e.* $y(t) = Ae^{-\frac{t}{T_F}} u(t)$. Once the falling edge is reached, then, the filter output results in:

$$y(t) = Ae^{-\frac{t}{T_F}} u(t) - Ae^{-\frac{t-T}{T_F}} u(t - T) \tag{5}$$

It is easy to verify that the time-integral of $y(t)$ is equal to zero, due to the transfer function zero in the origin. Moreover, we can immediately observe that the output is characterized by a response with a long negative tail. If we have a single pulse at the input, or multiple pulses distanced by a sufficient time, this will not be a problem. Otherwise, if the pulse repetition rate is high, we may end up facing pile-up problems, *i.e.* the new pulse is processed while the previous pulse output has not vanished yet.

Baseline restorers (BR)

The baseline restorer (BR) is a LTV filter that, similar to the HPF, can be successfully employed when a high-frequency signal has superimposed a low-frequency noise. The simplest implementation is the one of a HPF with a switch between the resistor and GND, as shown in Fig. 1(left). The weighting function can be easily determined observing that, if the switch is open, any delta function applied to the input will be passed to the output immediately (the capacitor can be assumed as a short-circuit). When the switch is closed, instead, the weighting function has the shape of a negative exponential (you can easily realise this by writing the output as the difference between the input and the voltage drop on the capacitor). Fig. 1(right) shows the weighting function of the BR.

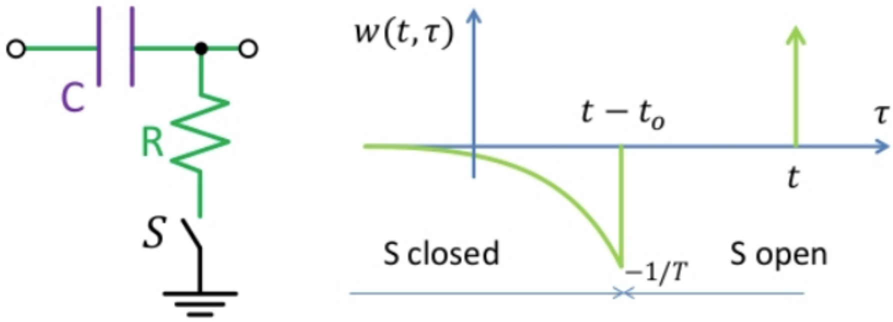

Figure 1: (Left) A simple implementation of a baseline restorer with a CR network and a switch. (Right) The baseline restorer weighting function.

To understand how noise is affected by the BR, it is necessary to calculate the time-correlation of the weighting function. The calculations are easy, but it is more interesting to represent it graphically in two conditions, *i.e.* when $T_F \gg t_0$ and when $t_0 \gg T_F$, being $T_F = RC$ and t_0 the time difference between the switch opening and the signal evaluation. Fig. 2 reports the weighting function time-correlation in these two conditions.

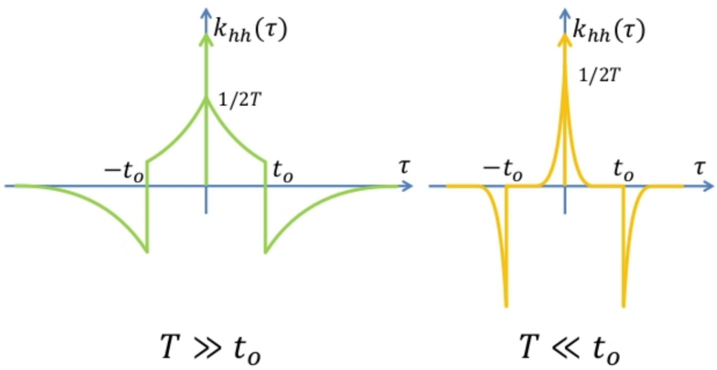

Figure 2: Time-correlation of the BR weighting function in the cases in which (Left) $T_F \gg t_0$ and (Right) $T_F \ll t_0$.

The important point here is that the filter is able to correctly reduce the input noise if and only if the noise correlation time T_N is somehow larger that t_0. This is intuitive: what's physically happening in the filter is that the noise alone is sampled when the switch is closed and the value is kept as a voltage drop on the capacitor C. When the switch is opened and the measurement is taken, the net directly subtract the previously integrated noise value from the sample. If the noise is correlated, *i.e.* if the time elapsed between the switch opening and the sample is shorter than T_N, we are effectively able to remove a certain noise component. Otherwise, we are just summing uncorrelated noise, thus increasing the variance with respect to a single signal+noise sample.

List of relevant exams

- Exam of 3rd March 2016, exercise 2
- Exam of 21th July 2016, exercise 2
- Exam of 20th July 2017, exercise 2
- Exam of 19th July 2019, exercise 2
- Exam of 23rd June 2021, exercise 2
- Exam of 6th September 2023, exercise 2